Information-Based Pricing in Specialized Lending*

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Abstract

We study specialized lending in a credit market competition model with private information. Two banks, equipped with similar data processing systems, possess "general" signals regarding the borrower's quality. However, the specialized bank gains an additional advantage through further interactions with the borrower, allowing it to access "specialized" signals. In equilibrium, both lenders use general signals to screen loan applications, and the specialized lender prices the loan based on its specialized signal conditional on making a loan. This private-information-based pricing helps deliver the empirical regularity that loans made by specialized lenders have lower rates (i.e., lower winning bids) and better ex-post performance (i.e., lower non-performing loans). We show the robustness of our equilibrium characterization under a generalized information structure, endogenize the specialized lending through information acquisition, and discuss its various economic implications.

JEL Classification: G21, L13, L52, O33, O36

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1 Introduction

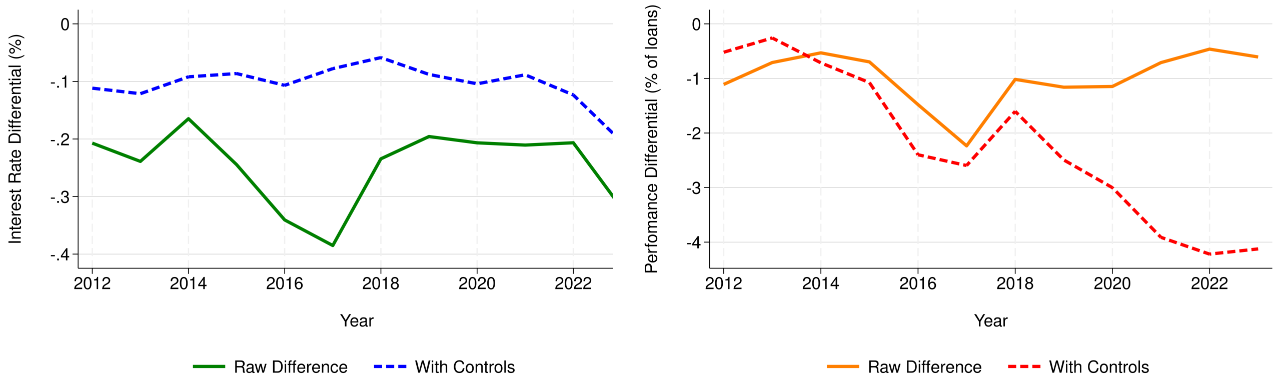
Banks are crucial intermediaries in modern economies, serving as the main conduit between savers and borrowers. One of their primary functions is to choose in which borrowers to invest, and as it has long been recognized by the literature (e.g., Broecker, 1990; Riordan, 1993; Hauswald and Marquez, 2003), competition among informed financial intermediaries in the credit market is central to the stability and efficiency of financial systems.

Of significant importance, banks hold a diverse array of lending-related information, including financial data on customers, collateral evaluations, and market and economic trends, not to mention state-of-the-art data analytics. Moreover, banks can choose to acquire specific types of information, enabling them to attain economies of scale and expertise in certain domains. The accumulation of these economies of scale and specialized knowledge is often achieved by concentrating on lending to particular industries, through the acquisition and analysis of diverse information on the business practices of individual firms and industries, beyond mere macroeconomic factors.

Despite the remarkable technological advancement that could significantly impact the industrial landscape of the banking sector, the prevailing literature (Marquez, 2002; Hauswald and Marquez, 2003; He, Huang, and Zhou, 2023) on information-based credit market competition predominantly focuses on binary signal realizations, overlooking the nuances of the aforementioned intricate economics. To this goal, this paper studies credit market competition with specialized lending, where one (specialized) lender with general and specialized signals competes against another (non-specialized) lender with a general signal only. Importantly, the specialized lender's extra continuous signal is crucial in setting its equilibrium fine-tuned loan pricing. This novel multi-dimensional information setting, incorporated into an otherwise classic credit market competition model (a la Broecker, 1990), allows us to study private-information-based pricing in specialized lending.

As motivation, we perform a simple empirical exercise based on regulatory loan data. Using supervisory information on the commercial loans of stress-tested banks, we can compare interest rates charged by those banks that are the most specialized in an industry with the rates charged by all other banks.[1] For a discussion of specialization in banking, please refer to Blickle, Parlatore, and Saunders (2023). We calculate the rate differential — i.e. the difference in interest rates charged by the specialized banks vs. non-specialized banks — over the past decade in 1. As shown in Panel (a), the loan rates by specialized lenders are lower than those of non-specialized lenders; this is true even when accounting for loan-specific characteristics (dashed line). In our sample, specialized lenders consistently charged around 10 basis points or less for ostensibly similar loans, and we emphasize that this difference is on "winning bids"— our loan-level data is on rates but not offers. Equally importantly, when comparing the likelihood at which loans become non-performing, panel

[1] Data is taken from the Y14-Q Schedule H database maintained by the Federal Reserve System for stress testing purposes. It covers a total of 40 banks the largest in the U.S. over the period of 2012-2023.



(a) **Interest rates: specialized vs. non-specialized**

(b) **Loan performance: specialized vs. non-specialized**

Figure 1: **Loan rates and performance in specialized vs non-specialized lenders.** We compare loans from specialized vs non-specialized lenders made to similar firms within a two-digit NAICS industry. We measure lender specialization as deviations from a diversified portfolio, where deviations from a diversified portfolio are measured by: $\frac{LoanAmount_{b,i,t}}{\sum_s LoanAmount_{b,i,t}} - \frac{LoanAmount_{i,t}}{\sum_i LoanAmount_{i,t}}$ for bank b in industry i at time t. We then define specialized lenders as those with more than 10% over-investment in an industry. Panel (a) plots the difference between the loan rates by specialized lenders and those by non-specialized lenders in the same industry. The solid line plots the raw difference with equal weights, while the dashed line groups loans by industry and lender specialization and also by loan purpose, observable risk, maturity (in deciles), collateral, and loan size (in deciles) also with equal weights. Panel (b) plots the difference in loan performance measured by the share of loans defaulting, falling in arrears, or requiring renegotiation, with raw differences in the solid line and differences accounting for loan characteristics in the dashed line; a negative number implies a higher quality of specialized lending. The empirical patterns are robust to various specifications say classification of specialized lenders and volume-based weights. See Blickle, Parlatore, and Saunders (2023) for a more in-depth discussion of measures of bank specialization.

(b) of Figure 1 shows that specialized lenders are less likely to encounter issues of non-performing loans, regardless of whether we account for loan characteristics or not.[2] If we account for loan characteristics, loans by specialized lenders are up to 5 percentage points less likely to become non-performing towards the end of our sample. The empirical regularity in specialized lending shown in Figure 1 suggests that specialized lenders are better informed about the borrower's quality private-information-based loan pricing to identify better borrowers and "undercut" the non-specialized opponent lenders in their specialized industries.

The existing information-based models—exemplified by (Broecker, 1990; Marquez, 2002) fail to deliver the above empirical regularity. There, each lender with binary signals actively competes only upon receiving the positive signal realization, offering interest rates that are outcomes of a completely randomized mixed strategy—that is to say, the interest rate per se carries no infor-

[2] Non-performing loans are those that fall into arrears, are not paid down by the end of their maturity, default or require renegotiation due to covenant violation issues. The average non-performance rate of loans throughout our sample is around 5%.

mation. In fact, as we will show in Section 4.1, a stark information rent effect dominates in that canonical setting, under which the loans on the book of a stronger lender (modeled as a greater precision) tend to have higher interest rates. This prediction is counterfactual in light of Figure 1.

In our model, outlined in Section 2, a specialized bank competes with a non-specialized bank. Each lender has a "general" information signal on the loan quality—i.e., the success probability of the borrower—from data processing. Moreover, the specialized lender has access to an additional signal coming from "specialized" information about the borrower, based on which the lender decides on the offered interest rate. We further assume that, while the general signal is binary and is decisive in that each lender makes an offer only if it receives a positive general signal, the specialized signal—which differentiates our paper from existing models—is continuous and guides the fine-tuned interest rate offering—we call this private-information-based loan pricing.[3]

As we highlight in Section 2.2, our analysis focuses on a multiplicative structure. More precisely, the success of the project requires the success of multiple fundamental states. For instance, the project success may require two distinct fundamental states—say "general" and "specialized"—to be favorable, and the above-mentioned two types of signals, i.e., general and specialized signals, inform the lenders regarding these two states, respectively. However, our equilibrium characterization also applies to the case where the general and specialized fundamental states overlap and hence correlated. To the extreme, the general and specialized fundamental states coincide entirely, and our model becomes the standard setting where one single fundamental state dictates the overall quality of the project.

In Section 3, we fully characterize in closed form the competitive credit market equilibrium with specialized lending, with the specialized bank's interest rate schedule *decreasing* in its specialized signal. Since the successful project's payoff is capped, our specialized bank—even conditional on a positive general signal—withdraws itself from the competition after receiving a sufficiently unfavorable specialized signal. In contrast, the non-specialized bank behaves just like in Broecker (1990) with interest rate offering fully randomized. Therefore by incorporating both general and specialized signals, our model delivers the key result of private-information-based pricing.[4]

Our model features a unique credit market equilibrium, which can fall into two distinct categories depending on whether the non-specialized bank makes zero profits or not as a result of

[3] Besides analytical convenience, this loan-making rule of the specialized bank matches well with the lending practices observed in the real world. Essentially, in our model, the specialized bank acquires two signals, one being "principal" while the other being "supplementary;" the former determines whether to lend while the latter affects the detailed pricing terms. Alternatively, the principal signal represents the result of a credit screening test, while the supplementary signal serves the role of internal ratings (of borrowers who are qualified for credit).

[4] Conceptually, this is similar to Milgrom and Weber (1982), in which the informed buyer who privately observes a continuum of signal realizations in a common value auction bids monotonically based on its own private information; see literature review for more details. In addition, one could extend the range of quoted interest rates by borrowers to include infinity and interpret $r = \infty$ as "rejection/withdrawal;" this way the lenders in the classic credit market competition model in Broecker (1990) and Hauswald and Marquez (2003) also have private-information-based pricing. However, Figure 1 is constructed based on interest rates of granted loans, and therefore loan rejection with $r = \infty$ cannot help explain the empirical regularity of lower interest rates of loans granted by specialized lenders.

competition. In the first category of equilibria, the winner's curse dominates and pushes the non-specialized "weak" bank to earn zero profits—therefore we call it a zero-weak equilibrium. In this case, the non-specialized bank randomly withdraws when receiving a positive general signal, which increases the specialized lender's monopoly power and information rent. In the second category of equilibria, the winner's curse is less severe and the non-specialized bank makes a positive profit in equilibrium (therefore always participates upon a positive general signal)—we call it a positive-weak equilibrium. The private-information-based pricing effect tends to dominate in this case, as the specialized bank with less monopoly power makes more aggressive offers to get good borrowers.[5]

We discuss the model's implications in Section 4. We focus on the empirical regularity that loans of specialized lenders have lower rates, which we simply call the "negative interest rate wedge." First, we highlight the difference between *bids* (i.e., offered interest rates) and *winning bids* (offered rates that are accepted by the borrower); this distinction is crucial when loan rejections are an important part of equilibrium strategies, as is typical in credit competition models. Although the standard winner's curse effect pushes a weaker lender to quote higher prices, as shown in He, Huang, and Zhou (2023) in credit market competition models the weak lender also responds by rejecting loan applications, favoring the strong lender to have a higher expected winning bids. This intuition is consistent with the fact that the aforementioned information rent effect dominates in canonical credit competition models a la Broecker (1990), giving rise to the counterfactual implications that the specialized bank's loans have higher rates.

In contrast, by explicitly incorporating specialized lenders' "undercutting" to win creditworthy borrowers against their competitors, the private-information-based pricing highlighted in our model helps deliver a lower interest on loans granted by specialized banks. We show that a negative interest rate wedge is more likely to occur in the positive-weak equilibrium where the private-information-based pricing effect takes precedence. This economic mechanism is different from Mahoney and Weyl (2017) and Crawford, Pavanini, and Schivardi (2018) who do not differentiate bids and winning bids. As we explain in the literature review, in that literature market power (of lenders) and adverse selection (of borrowers) are treated as two distinct market frictions, while our model features a winner's curse faced by asymmetrically informed lenders as the only underlying force.

We then study several model extensions in Section 4. First, we show that our equilibrium characterization is robust to a generalized information structure where the general and specialized states have some overlaps which induce correlated general and specialized signals. The key to our analytical tractability is the multiplicative structure and its resulting "conditional independence," i.e., general and specialized signals are independent conditional on project success. Second, we

[5]Consistent with information-based pricing, Butler (2008) finds local investment banks charge lower fees and issue municipal bonds at lower yields than non-local underwriters. On the other hand, Degryse and Ongena (2005) finds that local banks charge higher interest rates to small firms, consistent with local banks' strong monopolistic power over hard-to-evaluate captive borrowers.

endogenize the information structure in the baseline model by considering two ex-ante symmetric banks that compete on two firms/industries. Lenders can invest in the general information technology (which has a lump-sum fixed cost and produces a binary signal of borrower quality in either firm); in addition, they can acquire firm-specific specialized information (which is a continuous signal and costly for each firm) and hence becoming specialized. We provide conditions that support the "symmetric" specialization equilibrium where, as in our baseline model, each industry supports one specialized lender and another non-specialized lender.

The remainder of the paper is organized as follows. After a brief literature review, Section 2 presents the baseline model. Section 3 characterizes the credit market equilibrium and Section 4 explores the economic implications of our model, with several extensions. Section 5 concludes.

Literature Review

Lending market competition and common-value auctions. Our paper is built on Broecker (1990) who studies lending market competition with screening tests with symmetric lenders (i.e., with the same screening abilities). Hauswald and Marquez (2003) studies the competition between an inside bank that can conduct credit screenings and an outside bank without such access. He, Huang, and Zhou (2023) considers competition between asymmetric lenders with different screening abilities under open banking when borrowers control access to data. Asymmetric credit market competition can also naturally arise from the bank-customer relationship, as a bank knows its existing customers better than a new competitor does.[6] In these models, for analytical tractability it is often assumed that private screening yields a binary signal and lenders participate in bidding only following the positive signal realization. In contrast to these papers, we consider competition between asymmetrically informed lenders with multiple information sources.

Theory-wise, credit market competition models are an application of common-value auctions, and notably, the auction literature typically allows for general signal distributions (other than the binary signal in the aforementioned papers).[7] For instance, Riordan (1993) extends the N-symmetric-lender model in Broecker (1990) to a setting with continuous private signals. In terms of modeling, our framework can be viewed as a combination of Broecker (1990) (general information) and Milgrom and Weber (1982) (specialized information); to analyze competition among specialized lenders, having asymmetric information technologies is crucial. It is worth highlighting that lenders are each privately informed with general information; this hence breaks the Blackwell ordering of the information of two lenders in Milgrom and Weber (1982), resulting in a problem that is

[6]This idea was explored by a two-period model in Sharpe (1990) where asymmetric competition arises in the second period (with the corrected analysis of a mixed-strategy equilibrium offered by Von Thadden (2004)). A similar analysis is present in Rajan (1992).

[7]The early papers on this topic include Milgrom and Weber (1982) and Engelbrecht-Wiggans, Milgrom, and Weber (1983), and later papers such as Hausch (1987); Kagel and Levin (1999) explore information structures where each bidder has some private information, which is the information structure adopted in Broecker (1990).

considerably more challenging.[8] What is more, the economics revealed by a setting with multi-dimensional information can be fundamentally different, as highlighted by the distinction between information precision and information span emphasized by He, Huang, and Parlartore (2024).

Specialization in lending. There is a growing literature documenting specialization in bank lending; the early work includes Acharya, Hasan, and Saunders (2006). Paravisini, Rappoport, and Schnabl (2023) shows that Peruvian banks specialize their lending across export markets benefiting borrowers who obtain credit from their specialized banks. Based on data for US stress-tested banks, Blickle, Parlatore, and Saunders (2023) documents that specialization is linked with lower interest rates and better performance in the industry of specialization, pointing to a strong link between specialization in lending and informational advantages.[9] Our paper contributes to this literature by providing a framework that can rationalize these patterns, allowing us to understand the economic mechanisms behind them and their implications more deeply.[10]

The connection to imperfect competition and adverse selection in the IO literature. The empirical pattern and our theoretical analyses on the negative interest rate wedge between asymmetrically informed lenders are connected to the industrial organization (IO) literature on imperfect competition and adverse selection (Mahoney and Weyl, 2017; Crawford, Pavanini, and Schivardi, 2018; Yannelis and Zhang, 2023). As we explain in detail toward the end of Section 4.1, different from the IO literature which takes market power (of lenders) and adverse selection (of borrowers) as two independent market frictions, our theory is based on "information asymmetry" which is a more primitive assumption, with winner's curse faced by asymmetrically informed lenders as the only underlying economic force. Although one could link market power and adverse selection to unobservable borrower types, strictly speaking there is no "market power" enjoyed by the specialized lender as money from any funding source is perfectly fungible; and, there is no "adverse selection" from borrowers either, because both types of borrowers will take loans at any interest rate.[11]

[8] More precisely, one bidder knows strictly more than the other bidder. In this setting, one can show that the under-informed bidder always makes zero profit; see also Engelbrecht-Wiggans, Milgrom, and Weber (1983).

[9] Similarly, Gopal (2021) shows that some banks specialize in terms of the collateral they employ.

[10] Our paper also connects to the growing literature on fintech disruption; see Berg, Fuster, and Puri (2021); Vives (2019), for instance, for a review of fintech companies competing with traditional banks in originating loans. Along the line of our model with different dimensions of information, Huang (2023) developed a theoretical framework wherein the importance of information concerning underlying qualities varies between collateral-backed bank lending and revenue-based fintech lending such as Square.

[11] Our paper is also related to the literature on the nature of information in bank lending. Berger and Udell (2006) provide a comprehensive framework of the two fundamental types of bank lending technology, i.e., relationship lending and transactions lending, in the SME lending market; these two types of lending are related to the role played by information as highlighted by Stein (2002); Paravisini and Schoar (2016). Recently, based on Harte Hanks data, He, Jiang, Xu, and Yin (2023) show a significant rise in IT investment within the U.S. banking sector over the past decade, particularly among large banks, and their causal link between communication IT spending and the enhancement of banks' capacity in generating and transmitting soft information motivates our modeling of the specialized signal as the outcome of interactions with borrowers.

2 Model Setup

We lay out the model setup in this section and define the equilibrium accordingly.

2.1 General Setting

We consider a credit market competition model with two dates, one good, and risk-neutral agents (two lenders and one borrower). There are two ex-ante symmetric lenders (banks) indexed by $j \in \{A, B\}$. In the baseline model, we consider only one borrower (firm) where, say, Bank A (B) is the specialized (non-specialized) lender; in an extension of lenders specializing in different industries, we introduce a second firm where banks switch their respective roles.

Project. At $t = 0$, the firm needs to borrow one dollar to invest in a (fixed-scale) risky project that pays a random cash flow y at $t = 1$. The cash flow realization y depends on the project's quality denoted by $\theta \in \{0, 1\}$. For simplicity, we assume that

$$y = \begin{cases} 1 + \overline{r}, & \text{when } \theta = 1, \\ 0, & \text{when } \theta = 0, \end{cases} \tag{1}$$

where $\overline{r} > 0$ is exogenously given, i.e., only the good project has a positive NPV. We will later refer to $\overline{r}$ as the interest rate cap or the return of a good project. The project's quality θ is the firm's private information at $t = 0$, and the prior probability of a good project is $q \equiv \mathbb{P}(\theta = 1)$.

Credit market competition. At date $t = 0$, each bank j can choose to make a take-it-or-leave-it interest rate offer r^j of a fixed loan amount of one to the borrower firm or to make no offer (i.e., exit the lending market), which we normalize as $r^j = \infty$. The borrower firm accepts the offer with the lowest rate if it receives multiple offers.

Information technology. Although the project qualities are not observed by the banks, banks have access to information about the borrower's project quality before choosing to make an offer. We assume that both lenders have access to "general" data (say financial and operating data), which they can process to produce a *general-information*-based private signal g^j for the firm. We call these information "general" signals. For simplicity, we assume that these general signals are binary, i.e., $g^j \in \{H, L\}$, with a realization H (L) being a positive (negative) signal; and that, conditional on the (relevant) state, general signals are independent across lenders. Besides following the traditional structure presented in Broecker (1990), this modeling of general signals also captures the coarseness with which some general information is used in practice.[12]

[12]For example, as a leading example of "general information," credit scores are binned in five ranges even though scores are computed at a much granular level and go from 300 to 850.

Additionally, we endow Bank A with a signal s, which captures the bank being "specialized" in the firm. As the major departure from the existing literature, this additional signal is as a *specialized-information*-based private signal, which is collected, for example, after due diligence or face-to-face interactions with the borrower after on-site visits. We assume that the specialized signal s is continuous, and its distribution is characterized by the Cumulative Distribution Function (CDF) $\Phi(s)$ and probability density function (pdf) $\phi(s)$. Besides mathematical convenience, the continuous distribution captures "specialized" information resulting from research tailored to the particular borrower and, therefore, allows for a more granular assessment of the borrower's quality.

The information structure is incomplete unless we specify the correlations between the fundamental states and the two types of signals, to which we turn in the next subsection.

Remark 1. *Endogenous information structure.* In our main analysis, we take the lenders' information technologies—specifically, Bank A being the specialized lender—as given. Section 4.3 endogenizes this "asymmetric" information technology in a "symmetric" setting with two firms, a and b, where Bank A (B) endogenously becomes specialized by acquiring both "general" and "specialized" signals of the firm a (b), while non-specialized Bank B (A) only acquires the "general" signal of the firm a (b). There, the key difference between general and specialized information is that a lender j only needs to invest once—say installing IT equipment and software—to get two general signals, one for each firm, while specialized information needs to be collected individually for each firm.

2.2 The Setting with a Multiplicative Structure

General and specialized fundamental states. Our main analysis focuses on the specific setting with a multiplicative structure for the state θ, so that

$$\theta \equiv \theta_g \theta_s \equiv \begin{cases} 1, & \text{when } \theta_g = \theta_s = 1, \\ 0, & \text{when either } \theta_g = 0 \text{ or } \theta_s = 0. \end{cases} \tag{2}$$

Here, θ_g captures the "*g*eneralized" state and θ_s captures the "*s*pecialized" state, where $\theta_g \in \{0,1\}$ and $\theta_s \in \{0,1\}$ follow Bernoulli distributions and jointly determine the project's success θ, in that the project fails when *either* state fails.

We further assume that general and specialized states are independent, so that the prior probability of the state being "1" is simply $q = q_g q_s$ with $q_g \equiv \mathbb{P}(\theta_g = 1)$ and $q_s \equiv \mathbb{P}(\theta_s = 1)$. This independence, together with the independence of the noise across signals, implies complete independence between the generalized and specialized signals (for Bank A). Although the independence assumption across states simplifies the exposition quite a bit, this is for convenience and Section 4.2 shows that it is not necessary to ensure tractability. In fact, in a companion paper that highlights the idea of "big data hardening the soft information," He, Huang, and Parlartore (2024) study a setting where the two states are potentially correlated; see more details in Remark 3.

The distribution of the signals conditional on the state reflects the information technology. We assume that, conditional on the state, the signal realizations are independent across borrowers. For general information signals, which are assumed to be binary, we adopt the following notation,

$$\mathbb{P}\left(g^j = H\,|\theta_g = 1\right) = \alpha_u,\ \mathbb{P}\left(g^j = L\,|\theta_g = 0\right) = \alpha_d, \quad \text{for} \quad j \in \{A, B\}\,. \tag{3}$$

Here, the information technology is not indexed by lender j as we assume that lenders have the same technology to process general information that comes from "general" sources like financial statements, an assumption that we relax later in Section 4.2; and $1-\alpha_u$ and $1-\alpha_d$ capture the probabilities of Type I and Type II errors, respectively. Implicitly we impose that lenders have the same technology to process general information, an assumption that we relax later in Section 4.2. The bad-news signal structure in He, Huang, and Zhou (2023) corresponds to $\alpha_u = 1$ and a symmetric signal structure has $\alpha_u = \alpha_d = \alpha \in (0.5, 1]$ as in Hauswald and Marquez (2003) and He, Jiang, and Xu (2024). Our main numerical illustration focuses on the latter case, although the equilibrium characterization does not rely on any specific structure.

For the continuous specialized signal, without loss of generality, we directly work with the posterior probability of the specialized state being good $\theta_s = 1$ given its signal realization, i.e.,

$$s = \Pr[\theta_s = 1|s] \in [0, 1]. \tag{4}$$

Recall that the pdf of s is $\phi(s)$, so we have $\int_0^1 s\phi\,(s)\,ds \equiv q_s$ to satisfy prior consistency. Although our theoretical characterization works for any smooth density function $\phi(\cdot)$, most of our numerical illustrations use the specification that Bank A's specialized signal is a noisy version of the underlying state θ_s, with the signal-to-noise ratio being captured by the precision parameter τ (for more details, see Section 3.3).

The specialized Bank A has both general and specialized signals $\{g^A, s\}$ while Bank B only has a general signal g^B. Throughout we assume that the general signal is "decisive" for lending: Bank j bids only if it receives $g^j = H$. This implies that for the specialized Bank A, the general signal serves as "pre-screening," in the sense that the bank rejects the borrower upon receiving an L signal, while upon an H signal, it makes a pricing decision based on its specialized signal s.

Remark 2. *Principal and supplementary signals and relation to the literature.* The equilibrium loan-making rule of the specialized bank is practically relevant. Essentially, the specialized bank has two signals, one being "principal" which determines whether to lend, and the other being "supplementary" which helps its loan pricing.[13] This is in sharp contrast to the existing literature

[13] Alternatively, the principal signal represents the result of a credit screening test, while the supplementary signal

mentioned in the introduction where lenders make loan offers randomly only conditional on the most favorable realization of their (binary) signals. As shown in Section 4.1, our setting—by decoupling the lender's *ex-post* loan assessment from its *ex-ante* technology strength—helps deliver the empirical regularity of lower observed loan rates granted by specialized banks.

Remark 3. *Multi-dimensional information structure and its general applications.* We take the interpretation of "general" and "specialized" states when we specify $\theta = \theta_g \theta_s$, but this setting with multiple states admits many other interpretations. In fact, one can easily generalize it to the following multi-dimensional multiplicative setting,

$$\theta = \overbrace{\prod_{n=1}^{\hat{N}} \theta_n}^{\theta_g} \cdot \overbrace{\prod_{n=\hat{N}+1}^{N} \theta_n}^{\theta_s}, \tag{5}$$

with independent binomial states (or characteristics) $\theta_n \in \{0, 1\}$ where $n \in \{1, 2, ..., N\}$. Our setting with independent general (specialized) fundamental states is equivalent to setting $\theta_g \equiv \prod_{n=1}^{\hat{N}} \theta_n$ and $\theta_s \equiv \prod_{n=\hat{N}+1}^{N} \theta_n$. In fact, one can always "relabel" the signals to suit the context of a specific application. In a companion paper, He, Huang, and Parlartore (2024) interpret $\prod_{n=1}^{\hat{N}} \theta_n$ and $\prod_{n=\hat{N}+1}^{N} \theta_n$ as the "hard" and "soft" fundamental states, respectively, which are denoted by θ_h and θ_s. Similarly to this paper, lenders can acquire "hard" and "soft" signals, that are informative about the respective fundamental states. The expansion of the scope of hard information, presumably driven by the recent advance in big data technology, is modeled by a greater cut-off state $\hat{N}$, so θ_h covers more fundamental states (that are critical for project success) and hence may overlap with the soft state θ_s. This is the key idea of "hardening soft information" in He, Huang, and Parlartore (2024). There, we solve the model in closed form despite the potential correlation between θ_h and θ_s. We revisit this issue towards the end of Section 4.1.

Parametric assumptions. To ensure that the pre-screening general signal is "decisive," throughout the paper we impose the following parameter restrictions.

Assumption 1. ***(Decisive general signals)***

i) Bank A rejects the borrower upon an L general signal, regardless of any specialized signal s:

$$q_g (1 - \alpha_u) \overline{r} < (1 - q_g) \alpha_d. \tag{6}$$

serves the role of internal ratings (of borrowers who are qualified for credit). This ranking portrays the key role played by hard information for large banks when dealing with new borrowers. Indeed, as documented in page 1677 of Crawford, Pavanini, and Schivardi (2018), Italian large banks list the factors they consider in assessing any new loan applicant's creditworthiness, with the following order of importance: i) hard information from the central bank (financial statement data); ii) hard information from Credit Register; iii) statistical-quantitative methods; iv) qualitative information (i.e., bank-specific soft information codifiable as data); v) availability of guarantees; and vi) first-hand information (i.e., branch-specific soft information).

ii) Bank B is willing to participate if and only if its general signal $g^B = H$:

$$q_g \alpha_u q_s \overline{r} > q_g \alpha_u (1 - q_s) + (1 - q_g)(1 - \alpha_d); \tag{7}$$

Assumption 1 says that the general signal is sufficiently informative and serves as pre-screening of loan applications for both lenders; in other words, general signals are decisive for both lenders. Under Condition (6), the loan is negative NPV to Bank A upon a general signal L, even when the specialized signal is most favorable $s = 1$. This condition implies that Bank B, which only has the general signal and is uncertain about the realization of the specialized fundamental, also rejects the loan upon receiving $g^B = L$. Condition (7) states that upon $g^B = H$, Bank B is willing to lend at $\overline{r}$ if it is the monopolist lender. This condition also implies that Bank A, with an additional specialized signal, is willing to lend at the (exogenous) interest cap if it is the monopolist lender upon $g^A = H$ if it also observes high enough realizations of its specialized signal.

2.3 Credit Market Equilibrium Definition

We now formally define the credit market equilibrium with specialized lending. Before doing so, we define the banks' strategies and their associated profits.

Bank strategies. In equilibrium, each lender makes a potential offer only upon receiving a positive general signal H—recall Assumption 1 guarantees that the general signals are "desicive" for both lenders in making the loan offer or not. Conditional on making offers, we define the space of interest rate offers to be $\mathcal{R} \equiv [0, \overline{r}] \cup \{\infty\}$. Here, $\overline{r}$ is the exogenous maximum interest rate imposed in Section 2.1 and ∞ captures the strategy of not making an offer. We will soon show that the endogenous support of the equilibrium interest rates offered when making an offer is a sub-interval of $[0, \overline{r}]$. Therefore, with a slight abuse of terminology, we refer to that sub-interval as the "support" of the interest rate distribution even though loan rejection ($r = \infty$) could also occur along the equilibrium path.

We denote Bank A's pure strategy by $r^A(s) : \mathcal{S} \to \mathcal{R}$, which induces a distribution of its interest offerings denoted by $F^A(r) \equiv \Pr\left(r^A \leq r\right)$ according to the underlying distribution of the specialized signal. For now, we take as given that Bank A uses pure strategy only, though we formally prove this result in Proposition 1.

Bank B randomizes its interest rate offerings conditional on a positive general signal in equilibrium. In this case, we use $F^B(r) \equiv \Pr\left(r^B \leq r\right)$ to denote the cumulative distribution of its interest rate offerings. Note that since the domain of offers includes $r = \infty$ which captures rejection, it is possible that $F^j(\overline{r}) = \mathbb{P}\left(r^j < \infty | g^j = H\right) \leq 1$ for $j \in \{A, B\}$.

The borrower picks the lowest interest rate possible if multiple loan offers are ever available. For instance, conditional on both banks receiving positive general signals (HH), if Bank B quotes r^B, then its winning probability $1-F^A\left(r^B\right)$ equals the probability that Bank A with specialized signal s offers a rate that is higher than r^B, which includes the event that Bank A rejects the borrower ($r^A(s)=\infty$), presumably because of an unfavorable specialized signal. Upon ties, which occurs when $r^A=r^B<\infty$, borrowers randomly choose the lender with probability one half, although the details of the tie-breaking rule do not matter as ties occur as zero-measure events in equilibrium. When $r^A=r^B=\infty$, no bank wins the competition as they both reject the borrower.

Definition 1. (Credit market equilibrium) A competitive equilibrium in the credit market (with decisive general signals) consists of the following lending strategies and borrower choice:

1. A lender j rejects the borrower or $r^j=\infty$ upon $g^j=L$ for $j\in\{A,B\}$; upon $g^j=H$,
   i) Bank A offers $r^A(s):[0,1]\to\mathcal{R}\equiv[0,\overline{r}]\cup\{\infty\}$ to maximize its expected lending profits given $g^A=H$ and s, which induces a distribution function $F^A(r):\mathcal{R}\to[0,1]$;
   ii) Bank B offers $r^B\in\mathcal{R}$ to maximize its expected lending profits given $g^B=H$, which induces a distribution function $F^B(r):\mathcal{R}\to[0,1]$;
2. Borrower chooses the lower offer $\min\{r^A,r^B\}$.

The following lemma shows that the resulting equilibrium strategies in our setting are still well-behaved as established in the literature (Engelbrecht-Wiggans, Milgrom, and Weber (1983); Broecker (1990)). The key steps of the proof are standard, though we make certain adjustments due to the presence of both discrete and continuous signals.

Lemma 1. ***(Equilibrium Structure)*** *In any credit market equilibrium, there exists an endogenous lower bound of interest rate $\underline{r}>0$, so that the two distributions $F^j(\cdot)$, $j\in\{A,B\}$ share a common support $[\underline{r},\overline{r}]$ (besides ∞ as rejection). Over $[\underline{r},\overline{r})$ both distributions are smooth, i.e. no gap and atomless, so that they admit well-defined density functions. At most only one lender can have a mass point at $\overline{r}$.*

Bank profits and optimal strategies. We use $g^Ag^B\in\{HH,HL,LH,LL\}$ to denote the event of the corresponding general signal realizations, where HL represents Bank A's general signal being H and Bank B's general signal being L. Moreover, we denote by $p_{g^Ag^B}$ the joint probability of any collection of realizations of general signals. For instance, $p_{HH}\equiv\mathbb{P}\left(g^A=H,g^B=H\right)=q_g\alpha_u^2+(1-q_g)(1-\alpha_d)^2$. Similarly, denote by $\mu_{g^Ag^B}\equiv\mathbb{P}\left(\theta_g=1\,\middle|\,g^A,g^B\right)$ the posterior probability of the general state being one conditional on g^Ag^B. For example,

$$\mu_{HH}=\frac{q_g\alpha_u^2}{q_g\alpha_u^2+(1-q_g)(1-\alpha_d)^2}.$$

Under the multiplicative structure with independent states $\{\theta_g, \theta_s\}$, the posterior probability of project success given $\{HH, s\}$ is

$$\mathbb{P}\left(\theta = 1 | \, g^A = H, g^B = H, s\right) = \mu_{HH} \cdot s \tag{8}$$

For Bank A who receives a positive hard signal and a soft signal s, its profit $\pi^A(r\,|s)$, when competing with its opponent lender B by quoting $r \in [\underline{r}, \overline{r}]$, equals

$$\pi^A(r\,|s) \equiv \underbrace{p_{HH}}_{g^A=H, g^B=H} \underbrace{\left[1 - F^B(r)\right]}_{A \text{ wins}} [\mu_{HH} s (1+r) - 1] + \underbrace{p_{HL}}_{g^A=H, g^B=L} [\mu_{HL} s (1+r) - 1], \text{ for } r \in [\underline{r}, \overline{r}]. \tag{9}$$

Bank A can also choose to exit by quoting $r = \infty$, in which case $\pi^A(\infty\,|s) = 0$. We then denote Bank A's optimal interest rate offering by $r^A(s) \equiv \arg\max_{r \in \mathcal{R}} \pi^A(r\,|s)$.

Recall that Bank A cannot observe the realization of Bank B's general signal when making an offer. With probability p_{HH}, both banks get favorable general signals H and Bank A wins with probability $1 - F^B(r)$ if it offers r, whereas with probability p_{HL}, Bank A faces no competition for the borrower since Bank B who receives a low general signal withdraws itself from the credit market. Moreover, whether Bank B participates in the loan market affects Bank A's expected quality of the borrower, which is captured by $\mu_{HH}s$ and $\mu_{HL}s$. Importantly, since Bank B randomizes its strategy upon $g^B = H$, from the perspective of Bank A winning the price competition against Bank B is not informative about borrower quality.

This last observation is in sharp contrast with the problem of the non-specialized Bank B. A standard winner's curse ensues because the outcome of competition against the specialized Bank A is informative about θ_s. More specifically, besides the possibility of the competitor's unfavorable general information as mentioned above, the non-specialized lender B who wins the price competition also infers $r^A(s) > r^B$, which reflects Bank A's specialized information being unfavorable. Taking these inferences into account, Bank B's lending profit when quoting r are

$$\pi^B(r) \equiv \underbrace{p_{HH}}_{g^A=H, g^B=H} \underbrace{\left[1 - F^A(r)\right]}_{B \text{ wins}} \mathbb{E}\left[\mu_{HH}\theta_s(1+r) - 1 | \, r \leq r^A(s)\right] + \underbrace{p_{LH}}_{g^A=L, g^B=H} [\mu_{LH} q_s (1+r) - 1]. \tag{10}$$

Bank B's optimal strategy $F^B(\cdot)$ maximizes its expected payoff

$$\max_{F^B(\cdot)} \int_{\mathcal{R}} \pi^B(r) \, dF^B(r). \tag{11}$$

As it is standard in equilibria in mixed strategies, the profit-maximizing Bank B is indifferent between any action on its support.

3 Credit Market Equilibrium Characterization

To characterize the credit market equilibrium, we first take the equilibrium non-specialized Bank B's profit π^B as given and solve for the other equilibrium objects. Lemma 2 then solves for π^B, which completes the construction.

3.1 Solving for Pricing Strategies of Lenders

Solving for $r^A(s)$. Following Milgrom and Weber (1982), we start by solving for Bank A's equilibrium strategy $r^A(s)$. Suppose that $r^A(s)$ is decreasing, which will be verified later. Because Bank B plays mixed strategies, we know that it makes a constant profit π^B from any interest rate quotes, and when Bank B chooses to reject the borrower upon H with some probability, i.e., when $F^B(\overline{r}) < 1$, we must have $\pi^B = 0$. Moreover, when Bank B quotes $r = r^A(s)$, conditional on $g^A = H$ Bank B understands that it only wins the customer when A's specialized signal is below s. Bank B, therefore, updates the belief about the borrower's quality accordingly—its posterior for the specialized state is $\int_0^s t\phi(t)\,dt$.

On the other hand, conditional on $g^A = L$, Bank B wins the borrower for sure. Plugging $r^B = r^A(s)$ in Bank B's lending profits in Eq. (10), we have the following indifference condition:

$$\pi^B = \underbrace{\left[p_{HH}\mu_{HH}\int_0^s t\phi(t)\,dt + p_{LH}\mu_{LH}q_s\right]}_{B\text{'s expected loan quality (lending benefit)}}\left(1 + r^A(s)\right) - \underbrace{\left(p_{HH}\Phi(s) + p_{LH}\right)}_{B\text{'s expected loan size (lending cost)}}, \tag{12}$$

which holds for any $r^B = r^A(s) \in [\underline{r}, \overline{r})$. It immediately follows that

$$r^A(s) = \frac{\pi^B + p_{HH}\Phi(s) + p_{LH}}{p_{HH}\mu_{HH}\int_0^s t\phi(t)\,dt + p_{LH}\mu_{LH}q_s} - 1, \quad \text{when} \quad s \in [\hat{s}, 1], \tag{13}$$

where $\hat{s}$ is the highest realization of the specialized signal such that Bank A quotes $\overline{r}$, that is, [14]

$$\hat{s} \equiv \sup\left\{s \mid r^A(s) = \overline{r}\right\}. \tag{14}$$

We further define $x \leq \hat{s}$ as the threshold such that $\pi^A(\overline{r} \mid x) = 0$. It is worth highlighting that $x = \hat{s}$ could occur along the equilibrium path. Then it is straightforward to show that $r^A(s) = \overline{r}$ for $s \in [x, \hat{s})$, and $r^A(s) = \infty$ for $s \in [0, x)$.

As shown in Proposition 1 below, the conjectured strategy $r^A(s)$, which is strictly decreasing,

[14] Recall the convention that $\sup\{\emptyset\} = \inf \mathcal{S} = 0$.

gives the unique equilibrium given π^B. Define its inverse function (correspondence) of $r^A(s)$ to be

$$s^A(r) \equiv \begin{cases} r^{A(-1)}(r), & \text{when } r \in [\underline{r}, \overline{r}), \\ [x, \hat{s}), & \text{when } r = \overline{r}, \\ [0, x), & \text{when } r = \infty. \end{cases} \tag{15}$$

Noting that $\hat{s}$ may coincide with x, and we take the convention that $r^A(x) = \overline{r}$. And, the two relevant cutoffs for Bank A's strategy can be written as $\hat{s} = \sup s^A(\overline{r})$, i.e., the highest signal that Bank A quotes $\overline{r}$, and $x = \sup s^A(\infty)$, i.e, the highest signal that Bank A rejects the borrower.

Solving for $F^B(\cdot)$. We now turn to Bank B's strategy. In equilibrium, B's strategy needs to support $r^A(\cdot)$ in (13) to be Bank A's optimal strategy. The first-order-condition (FOC) that maximizes Bank A's objective in (9), which balances the lower probability of winning against the higher payoff from served borrowers, is

$$p_{HH}\left(-\frac{dF^B(r)}{dr}\right)[\mu_{HH}s(1+r)-1] + \left\{p_{HH}\left[1-F^B(r)\right]\mu_{HH}s + p_{HL}\mu_{HL}s\right\} = 0. \tag{16}$$

We then use Bank A's equilibrium strategy $r^A(s)$ that satisfies (16) for all $s \in [\hat{s}, 1]$ to pin down $F^B(\cdot)$.

From Bank B's perspective, by quoting $r = r^A(s)$, the corresponding marginal borrower type (with a specialized signal) is $s^A(r)$. Writing everything in terms of r; when Bank B marginally cuts its quote by dr, it gets $\phi\left(s^A(r)\right)(-s^{A\prime}(r))dr$ additional borrower type with quality $\mu_{HH}s^A(r)$ if there is competition, which occurs with probability p_{HH}. This gain is exactly offset by the marginal lower payoff from the borrower types who are already served. Therefore, Bank B' FOC is

$$\underbrace{p_{HH}\left[\phi\left(s^A(r)\right)\cdot(-s^{A\prime}(r))\right]}_{\text{additional borrower type}}\left[\mu_{HH}s^A(r)(1+r)-1\right] = \underbrace{p_{HH}\mu_{HH}\int_0^{s^A(r)} t\phi(t)\,dt + p_{LH}\mu_{LH}q_s}_{\text{existing borrower types}}. \tag{17}$$

Using the expression for $\mu_{HH}s^A(r)(1+r)-1$ in Bank B's FOC (17) in Eq. (16) which captures Bank A's FOC, we have

$$\frac{dF^B(r)}{dr}\left[\frac{p_{HH}\mu_{HH}\int_0^{s^A(r)} t\phi(t)\,dt + p_{LH}\mu_{LH}q_s}{\phi(s^A(r))\,s^{A\prime}(r)}\right] + p_{HH}\left[1-F^B(r)\right]\mu_{HH}s^A(r) + p_{HL}\mu_{HL}s^A(r) = 0.$$

One can show that the above equation yields the following ODE, which pins down $F^B(\cdot)$:

$$\frac{d}{dr}\left\{\frac{p_{HH}\mu_{HH}\left[1-F^B\left(r\right)\right]+p_{HL}\mu_{HL}}{p_{HH}\mu_{HH}\int_0^{s^A(r)}t\phi\left(t\right)dt+p_{LH}\mu_{LH}q_s}\right\}=0. \tag{18}$$

Here is the intuition behind the differential equation (18). At any interest rate r, both lenders are competing for the same marginal borrower type with (expected) quality $\mu_{HH}\cdot s^A(r)$, which yields an expected profit of $\mu_{HH}\cdot s^A(r)\cdot(1+r)-1$. This term shows up in both lenders' optimization conditions, i.e., (16) for Bank A and (17) for Bank B. We denote by $Q^j(r)$ the total *q*uality of borrowers of Bank $j\in\{A,B\}$ when it offers interest rate r. Then,

$$\begin{aligned}Q^A(r)&=p_{HH}\mu_{HH}s^A(r)\left[1-F^B\left(r\right)\right]+p_{HL}\mu_{HL}s^A(r),\\ Q^B(r)&=p_{HH}\mu_{HH}\int_0^{s^A(r)}t\phi\left(t\right)dt+p_{LH}\mu_{LH}q_s.\end{aligned}$$

Q^A and Q^B differ in that Bank A observes s while Bank B only knows that it gets borrower types with expected specialized fundamental state $s<s^A(r)$ (if $g^A=H$) or q_s (if $g^A=L$). For Bank A, the marginal effect of price cutting on borrower quality is $\frac{1}{\mu_{HH}}\left[\frac{Q^A(r)}{s^A(r)}\right]'$, where the division inside the bracket adjusts for the quality of the specialized fundamental of the marginal borrower type. Then, Bank A's optimal pricing strategy will equate the above marginal benefit to the associated marginal cost of price cutting, which is dr multiplying by the expected borrower quality $Q^A(r)dr$. Therefore we must have

$$\underbrace{\left[\frac{Q^A(r)}{\mu_{HH}s^A(r)}\right]'dr\cdot\left[\mu_{HH}s^A(r)\left(1+r\right)-1\right]}_{\text{MB on marginal borrower type}}=\underbrace{Q^A(r)dr}_{\text{MC on existing borrower types}} \tag{19}$$

$$\Leftrightarrow\frac{\mu_{HH}s^A(r)}{\mu_{HH}s^A(r)\left(1+r\right)-1}=\frac{\left[\frac{Q^A(r)}{s^A(r)}\right]'}{\frac{Q^A(r)}{s^A(r)}}, \tag{20}$$

which is equivalent to Eq. (16). On the other hand, for Bank B, which does not observe s, the marginal effect on customer size is $\frac{1}{\mu_{HH}}\frac{Q^{B\prime}(r)}{s^A(r)}$, implying an optimality condition of

$$\underbrace{\frac{Q^{B\prime}(r)}{\mu_{HH}s^A(r)}dr\cdot\left[\mu_{HH}s^A(r)\left(1+r\right)-1\right]}_{\text{MB on marginal borrower type}}=\underbrace{Q^B(r)dr}_{\text{MC on existing borrower types}}\Leftrightarrow\frac{\mu_{HH}s^A(r)}{\mu_{HH}s^A(r)\left(1+r\right)-1}=\frac{Q'^B(r)}{Q^B(r)}, \tag{21}$$

which is exactly Eq. (17).[15] Combining (20) and (21), we have:

$$\frac{\left[\frac{Q^A(r)}{s^A(r)}\right]'}{\frac{Q^A(r)}{s^A(r)}} = \frac{Q'^B(r)}{Q^B(r)} \Leftrightarrow \frac{d}{dr}\left[\frac{Q^A(r)/s^A(r)}{Q^B(r)}\right] = 0, \tag{22}$$

which is exactly our key ODE in Eq. (18).

The boundary condition $F^B(\underline{r}) = 0$ defines the lower-end support of the offered interest rate. Combining this bound with the ODE in Eq. (18) one can readily derive

$$1 - F^B(r) = \frac{\int_0^{s^A(r)} t\phi(t)\,dt}{q_s}, \text{ for } r \in (\underline{r}, \overline{r}) \tag{23}$$

as we have focused on the interior of the strategy space.[16] It is clear that $F^B(r) < 1$ for $r \in [\underline{r}, \overline{r})$, because $F^B(\overline{r}^-) = \frac{1}{q_s}\int_{s^A(\overline{r}^-)=\hat{s}}^1 t\phi(t)dt < 1$; and Bank B's strategy on the boundary $\overline{r}$ depends on whether it is profitable in equilibrium: it either places a mass of $1 - F^B(\overline{r}^-) = \frac{1}{q_s}\int_0^{\hat{s}} t\phi(t)\,dt > 0$ on $\overline{r}$ if $\pi^B > 0$, or quotes $r = \infty$ (i.e., withdraws) if $\pi^B = 0$. Finally, we observe that parameters on the general signals do not enter $F^B(\cdot)$ in (23) directly, but as shown below they do affect $F^B(\cdot)$ indirectly via the endogenous lower bound of the support $\underline{r}$.

Illustration of lenders' pricing strategies. Figure 2 illustrates the equilibrium strategies for both lenders for two cases, $\pi^B > 0$ and $\pi^B = 0$ indicated by the subscripts "+" and "0," respectively. The exogenous parameter that drives the different profits for Bank B is the interest rate cap $\overline{r}$, which we denote by $\overline{r}_+ > \overline{r}_0$ depending on the equilibrium type. As one would expect, the greater the borrower surplus (implied by a higher interest rate cap) the higher the lender's profits. Panel A (left) depicts Bank A's pricing strategy $r^A(s)$, which is decreasing, while the right panel plots $F^B(r)$ which is Bank B's CDF for its interest rate offerings. We also plot the corresponding cutoff signals $\hat{s}$, at which Bank A's strategy hits $\overline{r}$, and x, at which Bank A exits.

While we discuss the equilibrium strategies in more detail after providing a full characterization of the equilibrium, Figure 2 highlights a key difference between the two types of equilibrium that can arise, one with $\pi^B = 0$—the zero-weak equilibrium as the weak bank earns no profits—and the other with $\pi^B > 0$—the positive-weak equilibrium as the weak bank earns positive profits. As shown in Figure 2, in the case in which $\pi^B = 0$, Bank A has a point mass at $\overline{r}_0$ (corresponding to

[15] Readers might notice the important difference between the two lenders' marginal effects of cutting their prices on the quantity. For Bank A which observes the specialized signal realization directly, its pricing decision should not affect its quality; this is why we scale Q^A first by s and then take derivative, i.e., $\left[\frac{Q^A(r)}{s^A(r)}\right]'$. In contrast, without observing s directly, Bank B's price cutting affects its inferred quality of the borrower type (that it wins over Bank A). Therefore we take the derivative of $Q^B(r)$, which includes the quality of its borrowers, and then scale by the quality of marginal borrower type to avoid double counting.

[16] In deriving (23) we have used the fact that the two lenders share the same general information technology. This implies that the identity of the lender who receives high/low general signal is irrelevant and hence $p_{LH}\mu_{LH} = p_{HL}\mu_{HL}$.

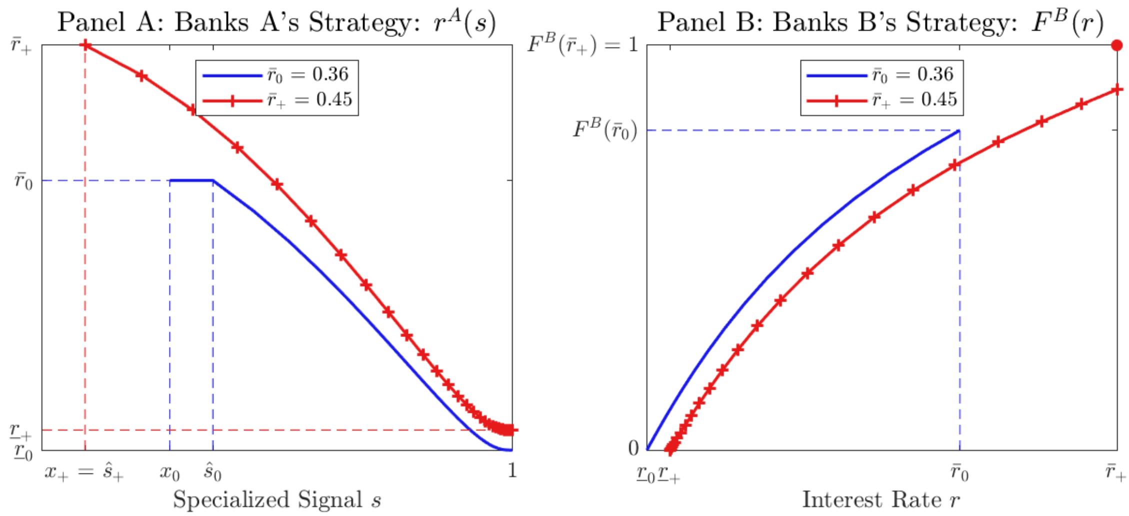


Figure 2: **Equilibrium strategies $r^A(s)$ for Bank A (left) and $F^B(r)$ for Bank B (right).** In both panels, strategies under $\bar{r}_+$ (i.e., positive-weak equilibrium) are depicted in red with "+" markers while strategies with $\bar{r}_0$ (i.e., zero-weak equilibrium) are depicted in blue. In the zero-weak equilibrium, Bank A (but not Bank B) has a point mass at $\bar{r}_0$ while in the positive-weak equilibrium, Bank B (but not Bank A) has a point mass at $\bar{r}_+$. Parameters: $q_g = 0.8$, $q_s = 0.9$, $\alpha_u = \alpha_d = \alpha = 0.8$, and $\tau = 1$.

$s \in (x_0, \hat{s}_0)$) but Bank B does not, while in the case of $\pi^B = 0$ the opposite holds. This reflects the competition at the interest rate cap and it is the exact manifestation of point c) in Lemma 1 (otherwise, lenders will undercut each other at this point).

3.2 Solving for the Equilibrium Profit of Bank B

In the last step, we solve for the equilibrium profit for Bank B, π^B, which then pins down the entire equilibrium. Define s_A^{be} as the specialized signal realization under which Bank A quotes $\bar{r}$ and breaks even (therefore the superscript "be"). Formally, using $\pi^A(\cdot)$ given in (9) and using the strategic response of Bank B in Eq. (23),[17] s_A^{be} is the unique solution to the following equation

$$\pi^A\left(\bar{r}\,\middle|\,s_A^{be}\right) = p_{HH}\frac{\int_0^{s_A^{be}} t\phi(t)\,dt}{q_s}\cdot\left[\mu_{HH}s_A^{be}(1+\bar{r})-1\right] + p_{HL}\left[\mu_{HL}s_A^{be}(1+\bar{r})-1\right] = 0, \tag{24}$$

which admits a unique solution inside the interval $(0,1)$.[18] We define s_B^{be} following a similar logic as follows. Consider the case in which Bank B quotes the maximum rate $\bar{r}$. Then, the potential winner's curse implies that Bank B only wins the borrower when either Bank A's general signal is

[17] Technically speaking Bank A quotes $\bar{r}^-$ so that $1 - F^B\left(\bar{r}^-\right) = \frac{1}{q_s}\int_0^{s_A^{be}} t\phi(t)\,dt$, as (23) requires $r \in [\underline{r}, \bar{r})$.

[18] Note $\pi^A\left(\bar{r}\,\middle|\,s_A^{be}\right)$ as a function of s_A^{be} is strictly increasing. Moreover, we have $\pi^A\left(\bar{r}\,\middle|\,s_A^{be}=0\right) < 0$ and $\pi^A\left(\bar{r}\,\middle|\,s_A^{be}=1\right) = p_{HH}\left[\mu_{HH}(1+\bar{r})-1\right] + p_{HL}\left[\mu_{HL}(1+\bar{r})-1\right] > 0$; the latter is implied by that Bank A is willing to make an offer given $g^A = H$.

$g^A = L$ or its specialized signal is sufficiently unfavorable, i.e., $s < s_B^{be}$. The break-even condition for Bank B uniquely defines s_B^{be}, as follows.

$$0 = \pi^B(\overline{r}) = p_{HH}\left[\mu_{HH}\left(\int_0^{s_B^{be}} t\phi(t)dt\right)(1+\overline{r}) - \Phi\left(s_B^{be}\right)\right] + p_{LH}\left[\mu_{LH}q_s(1+\overline{r}) - 1\right]. \tag{25}$$

Lemma 2 below shows that the relative ranking between s_B^{be} and s_A^{be} fully determines π^B and $\hat{s}$ in equilibrium, with both being fully characterized explicitly. Intuitively, the equilibrium crucially depends on which lender quoting $\overline{r}$ hits zero profits first when the specialized signal goes down from the top. If $s_A^{be} < s_B^{be}$ then Bank B hits zero profit first, and this supports the equilibrium of $\pi^B = 0$ with $\hat{s} = s_B^{be}$; otherwise we have $\pi^B > 0$ with $\hat{s} = s_A^{be}$.

Lemma 2. *Given s_A^{be} defined in (24), the equilibrium Bank B profit is*

$$\pi^B = \max\left\{\left[p_{HH}\mu_{HH}\int_0^{s_A^{be}} t\phi(t)\,dt + p_{LH}\mu_{LH}q_s\right](1+\overline{r}) - \left(p_{HH}\Phi\left(s_A^{be}\right) + p_{LH}\right), 0\right\}.$$

When $s_B^{be} < s_A^{be}$ we are in the positive-weak equilibrium in which the weak Bank B makes a positive profit, and $x = \hat{s} = s_A^{be}$. Otherwise, when $s_B^{be} \geq s_A^{be}$ we are in the zero-weak equilibrium where Bank B earns zero profits, with $x < \hat{s} = s_B^{be}$.

To understand the result, note that s_B^{be} is the highest specialized signal under which Bank A's offer hits $\overline{r}$, given $\pi^B = 0$.[19] Moreover, recall that s_A^{be} is the level of specialized signal under which Bank A just breaks even when quoting $\overline{r}$. Then if $s_B^{be} < s_A^{be}$, Bank A hits zero profit first, implying that it will lose money upon receiving a specialized signal $s = s_B^{be} < s_A^{be}$. Combining these two pieces, we know that quoting $\overline{r}$ at s_B^{be}, under the assumption of $\pi^B = 0$, must be off-equilibrium for Bank A. Therefore in equilibrium $\pi^B > 0$ and Bank A withdraws itself upon $s < x = \hat{s} = s_A^{be}$. If on the other hand $s_B^{be} \geq s_A^{be}$, we are in the alternative scenario where $\hat{s} = s_B^{be}$ and $\pi^B = 0$; Bank A who is making a positive profit at s_B^{be} will keep quoting $\overline{r}$ for $s < s_B^{be}$, until $s < x$ upon which it exits.

3.3 Credit Market Equilibrium

We now present the main result of our paper. The credit market equilibrium, which is fully characterized analytically, not only helps us understand the observed pattern of interest rates when some lenders are specialized but also allows us to study the implications of the evolution of information technologies.

[19] Note that (25) can be rewritten as $s_B^{be} = \arg_{s\in\mathcal{S}}\sup\left\{r^A\left(s;\pi^B=0\right) = \frac{p_{HH}\Phi(s)+p_{LH}}{p_{HH}\mu_{HH}\left(\int_0^s t\phi(t)dt\right)+p_{LH}\mu_{LH}q_s} - 1 \geq \overline{r}\right\}$. (Recall we take the convention that $\arg\sup\emptyset = \inf\mathcal{S} = 0$.)

Credit market equilibrium characterization. The next proposition summarizes the credit market equilibrium with specialized lending.

Proposition 1. ***(Credit Market Equilibrium)*** *In the unique equilibrium, Bank A follows a pure strategy as in Definition 1. In this equilibrium, lenders reject borrowers upon a low general signal realization $h^j = L$ for $j \in \{A, B\}$. Otherwise (i.e., when $h^j = H$), their strategies are characterized as follows, with the equilibrium π^B given in Lemma 2.*

1. *Bank A with specialized signal s offers*

$$r^A(s) = \begin{cases} \min\left\{\frac{\pi^B + p_{HH}\Phi(s) + p_{LH}}{p_{HH}\mu_{HH}\int_0^s t\phi(t)dt + p_{LH}\mu_{LH}q_s} - 1, \overline{r}\right\} & \text{for} \quad s \in [x, 1], \\ \infty, & \text{for} \quad s \in [0, x). \end{cases} \tag{26}$$

   The equation pins down $\underline{r} = r^A(1)$. If $s \in (\hat{s}, 1]$ where $\hat{s} = \sup s^A(\overline{r})$, $r^A(\cdot)$ is strictly decreasing with its inverse function $s^A(\cdot) = r^{A(-1)}(\cdot)$.

2. *Bank B makes an offer with cumulative probability given by ($\mathbf{1}_{\{X\}} = 1$ if X holds)*

$$F^B(r) = \begin{cases} 1 - \frac{\int_0^{s^A(r)} t\phi(t)dt}{q_s}, & \text{for} \quad r \in [\underline{r}, \overline{r}), \\ 1 - \mathbf{1}_{\{\pi^B = 0\}} \cdot \frac{\int_0^{\hat{s}} t\phi(t)dt}{q_s}, & \text{for} \quad r = \overline{r}. \end{cases} \tag{27}$$

   When $\pi^B = 0$, $F^B(\overline{r}) = F^B(\overline{r}^-)$ is the probability that Bank B makes the offer (and with probability $\frac{1}{q_s}\int_0^{\hat{s}} t\phi(t)\,dt$ it withdraws by quoting $r^B = \infty$); when $\pi^B > 0$, $F^B(\overline{r}) = 1$ and there is a point mass of $\frac{1}{q_s}\int_0^{\hat{s}} t\phi(t)\,dt$ at $\overline{r}$.

The proof for Proposition 1 mainly covers three issues. First, we show that the specialized lender always adopts pure strategy in any equilibrium; in other words, Bank A's pure strategy, which is implicitly taken as given in Definition 1, is a result rather than an assumption. Second, we prove that the FOC conditions used in the equilibrium construction detailed in Section 3 are sufficient to ensure global optimality. Third, somewhat surprisingly, thanks to the endogenous adjustment of π^B and $\underline{r}$, we never need to "iron" a la Myerson (1981) at the interior part of the range for equilibrium interest rates. In fact, in our model, Bank A never bunches its quotes—except at $\overline{r}$ when the zero-weak equilibrium ensues. (This is consistent with point 3 in Lemma 1 that states that Bank B will undercut if Bank A bunches at some interior interest rate.)

Properties of credit market equilibrium. Figure 3 illustrates the main properties of the credit market equilibrium with specialized lenders. For the purpose of exposition, we assume that Bank A's specialized signal s is obtained from observing a noisy version of θ_s, i.e., $\theta_s + \epsilon$, so that

$$s = \mathbb{E}[\theta_s | \theta_s + \epsilon]. \tag{28}$$

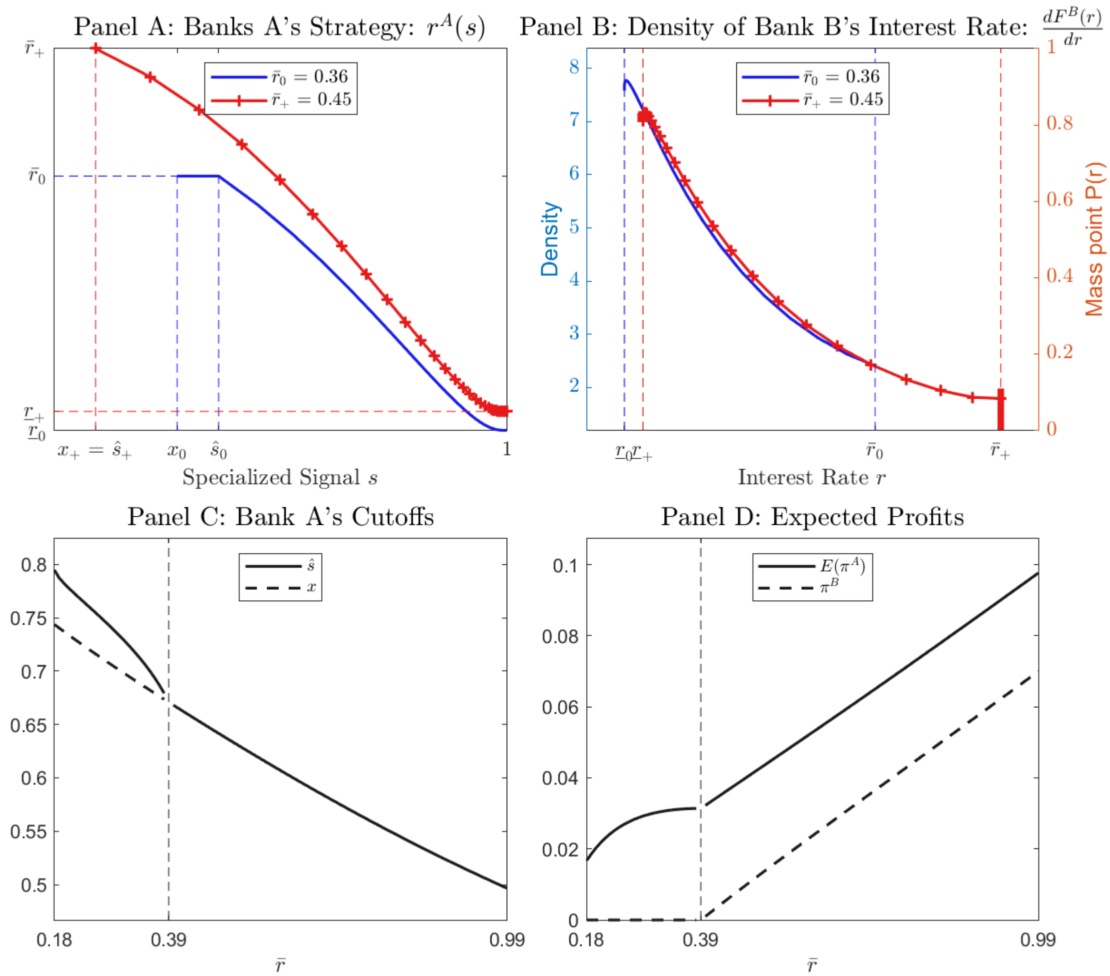


Figure 3: **Equilibrium strategies and profit.** In the top two panels, we plot equilibrium strategies for both lenders. Panel A depicts $r^A(s)$ as a function of s and Panel B plots $dF^B(r)/dr$ for as a function r; strategies with $\bar{r}_+$ are depicted in red with markers while strategies with $\bar{r}_0$ are depicted in blue. Panel C depicts Bank A's thresholds $\hat{s} = \sup s^A(\bar{r})$ and $x = \sup s^A(\infty)$, and Panel D depicts the expected profits for two lenders. Parameters: $q_g = 0.8$, $q_s = 0.9$, $\alpha_u = \alpha_d = \alpha = 0.8$, and $\tau = 1$.

Here, $\epsilon \sim \mathcal{N}(0, 1/\tau)$ indicates a white noise, with the precision parameter τ capturing the signal-to-noise ratio of Bank A's specialized information technology.

The top two panels in Figure 3 plot both lenders' pricing strategies conditional on making an offer; Panel A is the same as that in Figure 2 for convenience while Panel B plots the density dF^B/dr for Bank B. We observe that $r^A(s)$ decreases in s—when the specialized Bank A receives a more favorable specialized signal about credit quality, it bids more aggressively with a lower rate to win the borrower over the competitor Bank B. This strategic response to exploit the competitor bank is weakened when the private assessment of credit quality is low, leading Bank A to scale back. In fact, Bank A rejects the borrower when $s < x$. In contrast, as shown in Panel B, the competitor Bank B randomizes as it only observes the general signal.

Panel C plots the two specialized signal cut-offs for Bank A, i.e., $\hat{s}$ at which it starts quoting $\overline{r}$ and x at which it starts rejecting the borrower. Panel D plots the expected profits—$\mathbb{E}(\pi^A)$ and π^B—for two lenders. Both panels are plotted against the exogenous interest rate cap $\overline{r}$.

Recall that $\overline{r}$, which is the return of the good project, captures the surplus in competition. Thus, a higher total surplus gives rise to less fierce competition, and as a result, both lenders—including the weak lender B—are making profits upon a favorable general signal H. This immediately explains Panel D, which shows that π^B turns strictly positive for sufficiently high $\overline{r}$. Put differently, the model features a positive-(zero-) weak equilibrium when $\overline{r}$ is relatively high (low).

For a better illustration, consider the competition at interest rate $\overline{r}$. In the positive-weak equilibrium (high $\overline{r}$'s), the non-specialized Bank B places a point mass on this interest rate, enjoying some "local monopoly power" in competition as it is the only lender when Bank A rejects the borrower upon $s < \hat{s} = x$. This is possible because when the project's surplus (captured by $\overline{r}$) is sufficiently large, the nonspecialized Bank B is still profitable by quoting $\overline{r}$ despite the winner's curse.[20] In contrast, in the positive-weak equilibrium (low $\overline{r}$'s), the specialized Bank A is the monopolistic lender who places a point mass on this interest rate (when $s \in (x, \hat{s})$, as shown in Panel C) while the nonspecialized Bank B withdraws.

4 Model Implications and Discussion

In this section, we discuss the economic implications of our model. First, we study the interest rate wedge, which is the rate difference between the loans made by specialized and non-specialized lenders, respectively. We highlight the difference between bids vs winning bids on granted loans, and explain how the private-information-based pricing featured in our model helps generate the negative interest rate wedge observed in the data. Second, we endogenize the asymmetric information structure, i.e., Bank A specializes with an additional specialized signal compared to Bank B, by introducing information acquisition in an ex-ante symmetric framework. Finally, we extend our baseline multiplicative setting with two states to one with many states and study 'the role of 'correlated" general signals, an application that is practically relevant given the recent "open data" initiative.

4.1 Specialized Lending: Interest Rate Wedge

Consistent with Figure 1 in the Introduction, Blickle, Parlatore, and Saunders (2023) document two robust empirical patterns: the loans on the balance sheet of specialized lenders tend to have higher quality and lower interest rates. In our setting specialized lenders are extending higher quality loans

[20] Bank B who quotes $\overline{r}$ gets the borrower too if Bank A receives an unfavorable general signal $g^A = L$. Despite this winner's curse, the surplus is sufficiently high so that the nonspecialized Bank B is still profitable by quoting $\overline{r}$.

thanks to their informational advantage; this is a robust prediction of any information-based model, including those canonical ones a la Broecker (1990) and Marquez (2002). This section therefore focuses on the interest rate wedge between the specialized and nonspecialized lender.

Interest rate wedge: bids vs. winning bids

An econometrician observes the granted bank loans that are accepted by borrowers. Put differently, the loans that we use to calculate loan quality and interest rates are already on the book of the lender who won the bidding competition.

In our credit market competition setting, when Bank A makes a loan offer ($r^A < \infty$), it is accepted by the borrower if $r^A < r^B \leq \infty$, i.e., either if there is no offer from Bank B (when $h^B = L$ so $r^B = \infty$), or Bank A's rate is lower than that offered by Bank B. Therefore, the theoretical counterpart of a negative rate differential in Blickle, Parlatore, and Saunders (2023) is that specialized lenders charge lower interest rates on their granted loans relative to the nonspecialized lender:

$$\Delta r \equiv \underbrace{\mathbb{E}\left[r^A \middle| r^A < r^B \leq \infty\right]}_{\text{interest rate of } A\text{'s granted loan}} - \underbrace{\mathbb{E}\left[r^B \middle| r^B < r^A \leq \infty\right]}_{\text{interest rate of } B\text{'s granted loan}} < 0. \tag{29}$$

Here, we use $r^i < r^j \leq \infty$ to denote the event that Bank i wins the competition, which necessarily requires a non-infinity offer.

There is a crucial difference in the (expected) interest rate wedge calculated from "bids," i.e., banks' offered interest rates, and the one calculated from "winning bids," i.e., banks' rates on their granted loans. First of all, the winning bid is a first-order statistic (i.e., the smaller one given two quotes). Second, and conceptually more important in the context of credit market competition, banks often protect themselves from the winner's curse by simply rejecting loan applications, which we model as a quoting ∞ for mathematical convenience. This explains $r^j < \infty$ for Bank j in the conditioning of Eq. (29).

An example from He, Huang, and Zhou (2023) illustrates this point in a stark way. There, banks are endowed with general signals only, it assumes a bad news structure (i.e., $\alpha_u^j = 1$ and $\alpha_d^j < 1$ so that only false positives can occur), and banks differ in the precision of their signals. Suppose that $\alpha_d^A > \alpha_d^B$ which captures the idea that Bank A is relatively more informed, just like in our model. As shown in He, Huang, and Zhou (2023), following a favorable general signal the equilibrium CDF of offered interest rates for both banks, denoted by $\hat{F}(\cdot)$, coincide in the interior of the common support. More precisely, when $r \in [\underline{r}, \overline{r})$ we have:[21]

$$\hat{F}(r) \equiv \mathbb{P}(\tilde{r}_A < r) = \mathbb{P}(\tilde{r}_B < r) = \frac{r - \frac{1-q}{q}\left(1 - \alpha_d^B\right)}{r - \frac{1-q}{q}\left(1 - \alpha_d^B\right)\left(1 - \alpha_d^A\right)}.$$

[21] At the common endogenous lower bound $\underline{r} = \frac{(1-q)\left(1-\alpha_d^B\right)}{q}$, we have $\hat{F}(\underline{r}) = 0$.

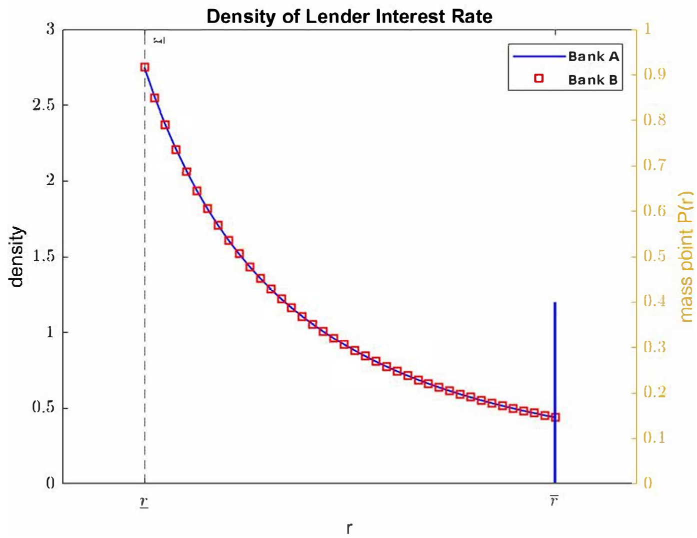


Figure 4: **Example of Lender Bidding Rates in Canonical Models.** We plot the density functions (left scale) and probability mass points (right scale) of lenders' interest rate offering upon favorable signals in He, Huang, and Zhou (2023). Our framework nests He, Huang, and Zhou (2023) by setting $q_s = 1$ (degenerate specialized information), $\alpha_A^u = \alpha_B^u = 1$ (bad-news information structure) and $\alpha_A^d > \alpha_B^d$ (Bank A has better information technology).

Therefore both densities in the interior of $[\underline{r}, \overline{r})$ also coincide, as shown in Figure 4 which plots the bidding strategies of both lenders in He, Huang, and Zhou (2023). The only difference between the bank's strategies is at the upper boundary $\overline{r}$: Bank A quotes the monopolistic rate $r = \overline{r}$ with a positive mass $1 - \hat{F}\left(\overline{r}^-\right) > 0$ while Bank B rejects the borrower by quoting $r = \infty$ with the same probability.

One can immediately see that, consistent with the intuition of the winner's curse, the bidding rates from the less informed Bank B are higher (i.e., first-order stochastic dominance) than those from the more informed Bank A. However, one can formally show that the implied interest rate wedge of winning bids (granted loans), conditional on both making offers (upon H signals), goes the opposite way because Bank A earns a monopolistic profit of $\overline{r}$ exactly when Bank B rejects (quoting $r^B = \infty$). As we will elaborate shortly, we label this force the "information rent" effect. The smaller the interest rate cap $\overline{r}$, the stronger the winner's curse faced by the less informed Bank B, and therefore the stronger the information rent effect. The next section formalizes this statement.[22]

[22]Recall that this discussion only concerns the interest rate wedge conditional on participation from both lenders, in this case one can formally prove that $\Delta r > 0$. However, from an unconditional perspective one also needs to take into account the possibility of an unfavorable general signal under which each lender rejects by quoting $r = \infty$. Given a bad news structure which can only generates false positive, the stronger Bank A is more likely to receive an unfavorable general signal (which is truth revealing) and therefore reject. This force complicates the analysis and we show in Proposition 2 that $\Delta r > 0$ when $\overline{r}$ is sufficiently small (i.e., when loan rejection occurs often in equilibrium).

Economic mechanisms of interest rate wedge

We now discuss the underlying economics of a negative interest rate wedge, i.e., lower winning bids from specialized lenders, through the lens of information-based credit market competition models, both canonical ones and ours with additional specialized signals.

Canonical models: the effect of information rent. In canonical credit competition models, the information technology is parameterized by the signal precision, which captures the lenders' ability to screen out uncreditworthy borrowers. The most natural way to capture "specialized lending" in this canonical setting is by imposing asymmetric screening abilities (on general signals and assume away specialized signal) along the line of Marquez (2002); He, Huang, and Zhou (2023), as illustrated in Figure 4.

Recall the notation of the general signal information structure given by Eq. (3) in Section 2.2; the literature has primarily focused on the following two parameterizations. The first is the bad news structure adopted in He, Huang, and Zhou (2023) discussed above assuming that $\alpha_d^A > \alpha_d^B$ to capture Bank A being specialized. Alternatively, Marquez (2002) adopts a symmetric information structure, so that $\alpha_u^A = \alpha_d^A > \alpha_u^B = \alpha_d^B$.[23] For ease of exposition, in both cases we use $\alpha^A > \alpha^B$ to denote Bank A having a more informative (binary) signal.

As emphasized before, in these canonical models only quantity decisions (i.e., whether to lend or not) are based on the signal realizations while pricing decisions (offered interest rates) are randomized. We have the following proposition.

Proposition 2. ***(Counterfactual Prediction in Canonical Models.)***

1. *Under a bad news structure, there exists a threshold* $\hat{\bar{r}}$ *such that* $\Delta r > 0$ *for* $\bar{r} < \hat{\bar{r}}$*;*

2. *Under a symmetric information structure, when* $\alpha = \alpha^A$ *and* $\alpha^B \uparrow \alpha$*,* $\Delta r > 0$ *for* $\bar{r} \leq \frac{1}{q} - 1$ *or* $\frac{1}{q} < \frac{1}{1-\alpha+\alpha^2}$*.*

Note that the canonical models can generate $\Delta r < 0$ for sufficiently high $\bar{r}$. However, as shown in Appendix A.3, the empirically relevant parameters violate these conditions. We therefore conclude that canonical models generate $\Delta r > 0$ as a model prediction, which is counterfactual to the empirical finding in Blickle, Parlatore, and Saunders (2023).

In general, as Bank A's private signal is more precise, the weak lender B is more concerned about the winner's curse, i.e., picking up a "lemon" whom the competitor lender assessed as L and rejected. As a result, the weak Bank B randomly withdraws even after receiving a favorable signal $g^B = H$,[24] yielding Bank A to become a monopolist. This economic force, which we refer to as

[23] That is to say, in the bad news structure, Bank A makes less false positive mistakes than Bank B, while in the symmetric information structure, Bank A makes fewer false positive and false negative mistakes than Bank B.

[24] In the context of Proposition 2, probabilistic withdrawal of the weak bank given $g^B = H$ holds always under the bad news structure, while holds when $\alpha^A - \alpha^B > 0$ is sufficiently large under a symmetric information structure.

information rent, drives the specialized Bank A to have higher expected winning bids (i.e., rates on granted loans) than Bank B, opposite to the empirical regularity.

In the bad news structure, the information rent effect intensifies if the weak lender rejects borrowers more often in the equilibrium. When the exogenous interest rate cap $\bar{r}$ gets smaller, the weak lender who faces a more severe winner's curse rejects its loan applications more often, which explains the first part of Proposition 2.

The analysis for the symmetric information case is more involved. The second part of Proposition 2 considers the limiting case of $\alpha^B \uparrow \alpha^A$, and shows that under empirically relevant primitives calibrated in Appendix A.3,[25] we would have the counterfactual prediction $\Delta r > 0$ even when $\alpha^B \uparrow \alpha^A$. Presumably, the information rent effect is stronger when the gap in information technology, i.e., $\alpha^A - \alpha^B > 0$ is larger.[26] The formal theoretical result in Proposition 2 therefore allows us to argue that canonical models generate counterfactual empirical implications on rates.

Our model: private-information-based pricing. By introducing Bank A's informed rate offers, our model naturally generates the empirical regularity that "loans made by specialized banks have lower rates" documented by Blickle, Parlatore, and Saunders (2023). As illustrated by Panel A in Figure 2, the specialized Bank A who receives a more favorable specialized signal about credit quality bids more aggressively (i.e., offers a lower rate) to win the borrower over the competitor Bank B. In fact, Bank A rejects the borrower when its specialized signal falls below a certain threshold (i.e., $s < x$).

The early discussion regarding "bids versus winning bids" right after inequality (29) suggests that whether Bank B rejects (by quoting $r^B = \infty$) plays a role. As we discuss above, the counterfactual prediction $\Delta r > 0$ is more likely to occur if Bank B rejects more often (so Bank A enjoys more information rent). The same economic mechanism is present in our model. Indeed, the force of private-information-based pricing is more likely to prevail in a positive-weak equilibrium where Bank B never rejects along the equilibrium path; it even enjoys some "local monopoly power" by having a point mass at $\bar{r}$, so that Bank B is the only lender when Bank A rejects the customer upon $s < x$. When Bank B never withdraws from the competition upon receiving a high signal, the better informed Bank A undercuts to win higher quality borrowers while leaving those lemons to Bank B (who then make loans with higher wining bids).[27]

[25] We calibrate q and α based on two empirical moments in the U.S. banking industry. First, according to this Federal Reserve report the non-performing loan (NPL) ratio is about 2%; second, Yates (2020) reports that that the approval rate for business C&I loans ranges from 55% (small firms) to 80% (large firms). Matching to these two moments in Appendix A.3 we show that the implied parameters violate $q < 1-\alpha+\alpha^2$ in Proposition 2. For instance, taking an approval rate of 70%, we obtain $q = 0.9629$ and $\alpha = 0.716$, which violate $q < 1 - \alpha + \alpha^2$. Note that our conclusion is independent of the parameter value of $\bar{r}$, which is harder to gauge. (One could set $\bar{r} = 36\%$ according to the usury law in many states that caps interest rates, but it only applies to consumer loans.)

[26] Although we have not been able to prove this claim formally, it is confirmed in all of our numerical exercises.

[27] Otherwise, Bank B who actively withdraws from the competition is less likely to make loans to lemons to start with, which is the force favoring the information rent in canonical models. Note, in canonical models, even if the

Is $\pi^B > 0$ a necessary condition? A special case. The above discussion seems to suggest that a positive profit for Bank B ($\pi^B > 0$) is a necessary condition for a negative interest rate wedge. When $\overline{r} = \infty$, Bank B could make zero profit or be profitable, but it never withdraws in equilibrium; and as illustrated in the discussion of bids vs winning bids, it is the endogenous withdrawal from the weaker bank makes the interest rate wedge conceptually interesting. The following analytical result on a special case with $\overline{r} = \infty$, together with a uniformly distributed specialized signal and a degenerate general fundamental, clarifies this point.

Proposition 3. ***(A Special Case of Uniform Distribution)*** *Suppose that $\overline{r} = \infty$ so that rejection is off equilibrium, general signals are degenerate ($q_g = 1$), and the specialized signal's distribution follows $\phi(s) = 1 + \epsilon\,[2 \cdot \mathbf{1}_{s \le 0.5} - 1]$. Then we have $\pi^B = 0$, $\Delta r = 0$ when $\epsilon = 0$ (i.e., $s \sim \mathbb{U}[0,1]$), and $\Delta r > 0$ ($\Delta r < 0$) when $\epsilon > 0$ ($\epsilon < 0$) for infinitesimal ϵ.*

There are several important implications of this proposition. First, when the specialized signal follows a uniform distribution (together with a degenerate general signal and $\overline{r} = \infty$), the two aforementioned effects—information rent and private-information-based pricing—equalize, and lenders have the same realized interest rates on their granted loans. Second, starting from this benchmark, any tilting toward private-information-based pricing—e.g., tilting more probability mass toward favorable specialized signals and therefore lower rates—would generate a negative interest rate wedge observed in the data. Third, $\pi^B > 0$ is not necessary for $\Delta r < 0$. In Proposition 3 we have $\pi^B = 0$ always for the uninformed Bank B given the degenerate general information. The last point is also intuitive: in the special case above we have $\overline{r} = \infty$ so Bank B never withdraws from the competition, while the previous discussion suggests that it is Bank B's rejection along the equilibrium path—not profitability per se—that favors the information rent effect.

Model comparative statics on interest rate wedge

Figure 5 plots the comparative statics of interest rate wedge Δr with respect to several key model parameters. For ease of discussion, we also indicate the regions of zero-weak and positive equilibria. The top two panels (A and B) concern information technology parameters α (precision of general signals) and τ (precision of the specialized signal). In the bottom panels, Panel C focuses on the interest rate cap $\overline{r}$ which also captures the total surplus in this economy. Panel D plots the comparative statics with respect to $1/q_g$, which relates to the relative importance of general and specialized information in this model (to be explained shortly).

Let us focus on the information technology parameters first. The general pattern is that when information technology improves—either the general signal precision α (Panel A) or the specialized signal precision τ (Panel B)—the credit market competition is more likely to be in the zero-weak

weak bank may earn profits given a high borrower surplus (say large q and $\overline{r}$), it never enjoys the "local" monopoly power—the strong bank never withdraws upon H while the weak bank never has a point mass at $\overline{r}$.

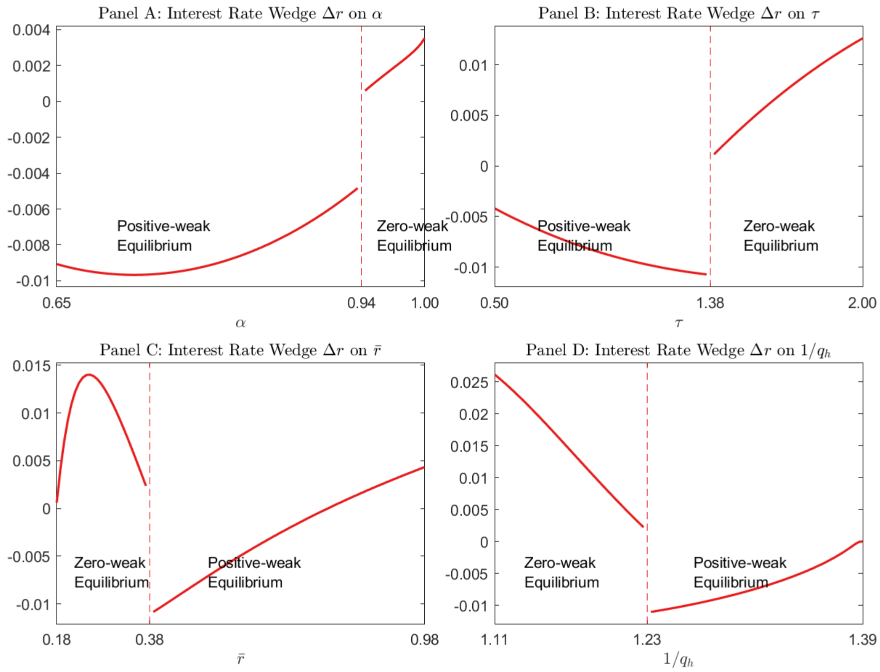


Figure 5: **Interest rate wedge.** Panel A to Panel D depict $\Delta r = \mathbb{E}\left[r^A \middle| r^A < r^B \leq \infty\right] - \mathbb{E}\left[r^B \middle| r^B < r^A \leq \infty\right]$ as a function of α, τ, $1/q_h$ and $\bar{r}$. The positive-weak equilibrium arises when α or τ lies below a certain value and $1/q_h$ and $\bar{r}$ exceed a certain value. Initial Parameters: $\bar{r} = 0.45$, $q_h = 0.8$, $q_s = 0.9$, $\tau = 1$ and $\alpha_u = \alpha_d = \alpha = 0.8$.

equilibrium where the nonspecialized Bank B is sufficiently "weak" and hence makes zero profits. This is intuitive because i) a higher general signal precision α levels the playing field on general information and hence effectively enlarges the specialized information advantage of the specialized bank, and ii) a higher specialized signal precision τ directly boosts the specialized bank's specialized information advantage. Since the effect of private-information-based pricing tends to dominate in a positive-weak equilibrium, a sufficiently low information technology helps deliver a negative interest rate wedge; this is depicted in the two top panels in Figure 5. Note that the interest rate wedge is discontinuous when the economy enters the region of a zero-weak equilibrium. In that case, Bank B reallocates the probability mass of $1 - F^B(\bar{r}^-) > 0$ from $\bar{r}$ to ∞ (see Panel B in Figure 3).

Moving on to the interest rate cap $\bar{r}$, it is intuitive that the credit market equilibrium moves to the positive-weak region when the total surplus increases. In Panel C, the interest rate wedge jumps down to be negative first, but then increases and turns positive when $\bar{r}$ is sufficiently high. This is consistent with the spirit of Proposition 3 that the sign of Δr does not depend on whether $\pi_B > 0$, highlighting the robustness of the economic mechanism of private-information-based pricing.

Panel D conducts another comparative static which points to the relative importance of general versus specialized information. More specifically, consider varying $1/q_g$ but fixing the project success probability q, which implies that $q_s = q/q_g$. In He, Huang, and Parlartore (2024) we explain that this comparative static exercise corresponds to the scenario in which general signals increase their scope so that they cover more fundamental states that are critical to the success of the funded project.[28] Interestingly, this exercise yields the opposite comparative statics to the standard information technology parameters (α and τ in top two panels) modeled as signal precision. Intuitively, now Bank B, equipped with general information that covers more fundamental states, becomes relatively stronger (rather than weaker when α and/or τ increase), so the credit market equilibrium is more likely to be in the region of positive-weak (and delivers a negative interest rate wedge). Motivated by the recent advancements in big data technology, He, Huang, and Parlartore (2024) employ this framework to study the concept of "hardening soft information."

Connection to the IO literature on imperfect competition and adverse selection

The empirical pattern and our theoretical analyses on the negative interest rate wedge between asymmetrically informed lenders are connected to the industrial organization (IO) literature on imperfect competition and adverse selection (Mahoney and Weyl, 2017; Crawford, Pavanini, and Schivardi, 2018). Within that body of literature, market power (of lenders) and adverse selection (of borrowers) are considered distinct market frictions conceptually. Market power pertains to the situation where the demand for the firm's (differentiated) products remains relatively inelastic with respect to its price, whereas adverse selection is characterized by the observation that the effective revenue of marginal consumers decreases as the firm raises its price.[29] Piecing these two forces together, the key takeaway is an interaction effect: while firms with greater market power should charge higher prices, this standard force should be attenuated by adverse selection, which hurts marginal revenue when firms raise their prices.

We would like to highlight two points. First, different from the IO literature, which takes market power and adverse selection as two independent market frictions, our theory starts from a more basic primitive of "information asymmetry," with winner's curse faced by asymmetrically informed lenders as the only underlying economic force. Although one could broadly link the above-mentioned market power and adverse selection to unobservable borrower types, strictly speaking,

[28] As explained in Remark 3 where we introduce multi-dimensional fundamental states, He, Huang, and Parlartore (2024) study hard and soft information in credit market competition. We interpret $\theta_g \equiv \prod_{n=1}^{\hat{N}} \theta_n$ ($\theta_s \equiv \prod_{n=\hat{N}+1}^{N} \theta_n$) as the borrower's "hard" ("soft") fundamental state, and model the expansion of the scope of "hard" information by an increase in $\hat{N}$ (so θ_g covers more fundamental states that are critical to the project's success). In the short-run, this expansion of $\hat{N}$ does not alter the information scope of the soft signal so that θ_g and θ_s overlap, but in the long-run the coverage of θ_s also shrinks so that θ_g and θ_s do not overlap. Our exercise in Panel D corresponds to the long-run scenario. For the short-run scenario, in spite that the expansion of $\hat{N}$ induces a correlation between θ_s and θ_g, we are still able to analytically characterize the credit market equilibrium. See more details in He, Huang, and Parlartore (2024).

[29] In the insurance market example used in Mahoney and Weyl (2017), a higher insurance premium is associated with lower-quality insurance buyers and hence a higher service cost. In Crawford, Pavanini, and Schivardi (2018) which studies the enterprise loan market, a higher interest rate may attract worse borrowers or induce riskier projects, leading to lower interest revenues.

there is no "market power" enjoyed by the specialized lender. In our model, money from any funding source is perfectly fungible just like in Huang (2023). Moreover, there is no "adverse selection" from borrowers either, because both types of borrowers will take loans at any interest rate.[30]

Second, note that prices in the above-mentioned IO literature are "bids" as opposed to "winning bids." As we have emphasized earlier, conditional on bidding the standard winner's curse effect in any information-based credit market competition models would induce a more informed lender (loosely interpreted as lender with a stronger market power) to bid a lower price, but "winning bids" could reverse once we take into account of loan rejection by the less informed lender. The same issue applies to Crawford, Pavanini, and Schivardi (2018) who only consider bidding prices. Future research should study whether this difference and endogenous rejection reverse the conclusions from the IO literature.

4.2 Generalized Information Structure

We have assumed a multiplicative setting with two independent fundamental states—the general and specialized states—as explained in Section 2.2. We first explain the two important features of this assumption which are the key to the tractability of our model. We then consider the generalized information structure that maintains these two desirable features, and further characterize the resulting credit market equilibrium.

Two key properties for model tractability

Decisive general signal. First, in many credit markets, the computer-based general information signal is usually used as pre-screening and decisive for loan granting, while the specialized information collected by the specialized bank tailors interest rate terms (see Remark 2). To capture the above commonly observed lending practice, the multiplicative structure makes the "general" state decisive in project success, leading such lending strategies more likely to arise in equilibrium.

Conditional independence. Second, as we will show shortly, what brings the tractability of our common value auction setting with asymmetrically informed bidders is conditional independence of all signals, i.e.,

$$\tilde{g}^A \perp\!\!\!\perp \tilde{g}^B \perp\!\!\!\perp \tilde{s} \mid \theta = 1 \ . \tag{30}$$

That is to say, conditional on project success (which is the event that the project pays off anyway), all signals (including the specialized one by lender A and two general ones by both lenders) are independent with each other. Our setting in Section 2.2 with independent general and specialized states, clearly satisfies (30).

[30] As typical in corporate finance literature say Tirole (2010), we are implicitly assuming that both types of borrower receive nonpledgeable private benefits from the project, so they strictly prefer to take the loan even if $r = \bar{r}$.

However, tractability does not rely on unconditional independence. Consider a generalized multi-dimensional information setting under (5), but θ_g and θ_s overlap with each other. For instance, $\theta = \theta_1\theta_2\theta_3$, with $\theta_g = \theta_1\theta_2$ and $\theta_s = \theta_2\theta_3$. Under this setting, we note that conditional independence (30) always holds, but unconditionally all three signals $\{\tilde{g}^A, \tilde{g}^B, s\}$ are correlated because the general state θ_g and specialized state θ_s are correlated.[31]

Equilibrium characterization under generalized information structure

We now solve for the credit market equilibrium under a general information structure, with two major assumptions as outlined above. First, lenders only participate when the general signal realization is H, with parameter restrictions in the same spirit as Assumption 1 but tailored for the general information structure. Second, conditional on the project's state $\theta = 1$, signals are independent across general and specialized and across lenders; this is implied by the multiplicative structure (5). Since the major derivation is also available in He, Huang, and Parlartore (2024), we keep the presentation minimal here (but detailed analysis is available in the Appendix A.7).

Consider a specialized signal $z \sim \phi_z(z)$ for $z \in [\underline{z}, \overline{z}]$ where both $\underline{z}$ and $\overline{z}$ can be unbounded. Denote by $\mu_{g^A g^B}(z) \equiv \mathbb{P}\left(\theta = 1 \middle| g^A, g^B, z\right)$ the posterior probability density for $\theta = 1$, i.e., the state of the project being successful. Without loss of generality, we assume that $\mu_{HH}(z)$ strictly increases in z (as we can always use $\mu_{HH}(z)$ as a signal; recall the posterior s serves as the signal in the baseline model given in Section 2). This implies that just as in the baseline, there exists $\hat{z}$ at which Bank A starts to quote $\overline{r}$, and z_x below which it starts rejecting borrowers. Let $\overline{\mu}_{g^A g^B} \equiv \mathbb{P}\left(\theta = 1 | g^A, g^B\right)$ denote the posterior probability of $\theta = 1$ based on general signals.

Denote further by $p_{g^A g^B}(z) \equiv \mathbb{P}\left(g^A, g^B, z\right)$ and $\overline{p}_{g^A g^B} \equiv \mathbb{P}\left(g^A, g^B\right)$, and let $\alpha_u^j \equiv \mathbb{P}\left(g^j = H | \theta = 1\right)$ for $j \in \{A, B\}$ (so two lenders can differ in their signal precisions in the general information), and $\phi_z(z|\theta = 1)$ be the density of z conditional on $\theta = 1$. The following proposition summarizes the credit market competition equilibrium with specialized lenders under this generalized information structure.

Proposition 4. ***(Credit Market Equilibrium under General Information Structure)*** *Lender $j \in \{A, B\}$ rejects the borrower (by quoting $r = \infty$) upon $g^j = L$; when $g^j = H$, lender j may make offers from a common support $[\underline{r}, \overline{r}]$ (or reject) with the following properties.*

1. *Bank A who observes a specialized signal z offers*

$$r^A(z) = \begin{cases} \min\left\{ \frac{\pi^B + \int_{\underline{z}}^{z} p_{HH}(t)dt + \overline{p}_{LH}}{\int_{\underline{z}}^{z} p_{HH}(t) \cdot \mu_{HH}(t)dt + \overline{p}_{LH}\overline{\mu}_{LH}} - 1, \overline{r} \right\}, & \textit{for} \quad z \in [z_x, \overline{z}] \\ \infty, & \textit{for} \quad z \in [\underline{z}, z_x). \end{cases} \tag{31}$$

[31]The multiplicative structure in (5) is the key: $\theta = 1$ implies that all fundamental states $\{\theta_n, n \in 1, ..., N\}$ take the value of one.

This equation pins down $\underline{r} = r^A(\overline{z})$*,* $\hat{z} = \sup\left\{z : r^A(z) = \overline{r}\right\}$*, and* $z_x = \sup\left\{z : r^A(z) = \infty\right\}$*.*

2. *Bank B makes an offer by randomizing its rate according to:*

$$F^B(r) = \begin{cases} \frac{\alpha_u^A}{\alpha_u^B}\left[1 - \int_{\underline{z}}^{z^A(r)} \phi_z\left(t|\,\theta = 1\right) dt\right], & for \quad r \in [\underline{r}, \overline{r}), \\ 1 - \mathbf{1}_{\{\pi^B = 0\}} \cdot \left\{1 - \frac{\alpha_u^A}{\alpha_u^B}\left[1 - \int_{\underline{z}}^{\hat{z}} \phi_z\left(t|\,\theta = 1\right) dt\right]\right\}, & for \quad r = \overline{r}. \end{cases} \tag{32}$$

3. *The endogenous non-specialized Bank B's profit* π^B *is determined similarly as Lemma 2, with detailed expression provided in Appendix A.7.*

Proposition 4 shows that the simple equilibrium structure survives under the more generalized information structure. Following the same logic as in the baseline model, lenders' customer quantities change proportionately with interest rates in equilibrium. To see this, when cutting interest rates at $r \in [\underline{r}, \overline{r})$, both lenders are competing for the same marginal borrowers, whose revenue should equal a unit loss from each lender's existing customer base so that lenders are indifferent and use a mixed strategy. For the existing customers, only the good type of customers who repay the loan matter. As a result, as long as specialized and general signals are independent conditional on the project being successful, their effects on equilibrium strategies are separable, and a simple characterization as in Proposition 4 ensues.

4.3 Information Acquisition and Endogenous Specialization

Although the information structure is likely to be fixed in the short run, in the long run, banks choose what type of information they want to have about borrowers. For example, banks can invest in equipment that allows them to analyze their existing transaction data more efficiently (general information), or spend resources gathering information about specific borrowers (specialized information). In this section, we look at the lender's information acquisition problem and derive conditions under which the information structure we study in the previous sections is an equilibrium outcome.

Setting and information acquisition technologies. We introduce another borrower firm—which we call b—in addition to the borrower firm a in our baseline model. We may equally interpret a and b as different industries.

There are two types of technologies that respectively relate to "general" information and "specialized" information. For the "general" information technology, a lender j invests once in equipment at a cost of κ_g, which allows the lender to process data (say financial and operating data) and produce a *general information* based private signal $g_i^j \in \{H, L\}$ for each firm $i = a, b$, independently (across two lenders and two firms). This captures the idea that general information is collected via standardized and transferable data such as credit reports and income statements, so once the IT equipment, software, and APIs are installed, credit analysis is easy to implement on multiple firms and the information generated is also standardized and coarse.

For the "specialized" information technology, a lender needs to collect specialized information on firms one by one. Lender i specializes in firm j if it spends κ_s to acquire a *specialized information* based private signal s_i^j, whose smooth distribution is characterized by the CDF $\Phi(s)$ and pdf $\phi(s)$ for $s \in \mathcal{S} \equiv [0,1]$. If a bank wants to acquire specialized information about both firms, it needs to pay $2\kappa_s$.

We are interested in the following equilibrium: Bank A (B) endogenously specializes in firm a (b)—i.e., acquires both general and specialized signals on firm a (b)—and competes with the other non-specialized Bank B (A) who only acquires general signal on firm a (b). Given this equilibrium structure, we omit the indexation for firm i from now on when referring to the specialized signals. The baseline model analyzed in Section 2.3 is the subgame for either firm following the equilibrium information acquisition strategies.

Incentive compatibility conditions. Banks make their information acquisition decisions simultaneously. Moreover, we assume that information acquisition is observable when banks enter the credit market competition game. This implies that a lender's deviation from the proposed equilibrium information acquisition will lead to a different information structure in the credit market competition. Hence, to examine the incentives of banks to acquire each type of information, we need to define the bank's lending profits in all possible subgames following a deviation.

Denote by $\Pi_j^i\left(I_A^g, I_A^s, I_B^s, I_B^s\right)$ the expected lending profits of bank j in industry i when the information structure in industry i is given by $(I_A^g, I_A^s, I_B^g, I_B^s,)$, where I_j^g and I_j^s take value of one if bank j acquired general information and specialized information in industry i, respectively, and zero otherwise. The symmetry on industries implies that a bank's expected lending profits in industry i only depend on the information structure in that industry and not on the industry itself, i.e.,

$$\Pi_j^a\left(I_A^g, I_A^s, I_B^s, I_B^g\right) = \Pi_j^b\left(I_A^g, I_A^s, I_B^g, I_B^s\right). \tag{33}$$

Therefore, we drop index i from the expected lending profits. What is more, we focus on Bank A's incentives in what follows since the no deviation conditions for banks A and B are symmetric.

Bank A can deviate along three dimensions: it can choose not to acquire general information, it can choose not to acquire specialized information about industry a, and it can choose to acquire specialized information in industry b. Bank A's incentives to deviate along these dimensions will depend on the costs of acquiring information. As one would expect, the lower the cost of acquiring general information, the more likely Bank A has incentives to acquire general information and not deviate along this dimension. For deviations along the specialized information dimension, the cost of acquiring specialized information has to be low enough such that it is worth acquiring specialized information in industry a and having an informational advantage over Bank B in this industry but high enough such that it is not worth acquiring specialized information in industry b to stop being the less informed lender. This intuition can be formally stated in the following incentive compatibility constraints. Bank A does not want to deviate by

1. not acquiring general information
$$\Pi_A\left(I_A^g=1, I_A^s=1, I_B^g=1, I_B^s=0\right)-\Pi_A\left(I_A^g=0, I_A^s=1, I_B^g=1, I_B^s=0\right)+$$
$$\Pi_A\left(I_A^g=1, I_A^s=0, I_B^g=1, I_B^s=1\right)-\Pi_A\left(I_A^g=0, I_A^s=0, I_B^g=1, I_B^s=1\right) \geq \kappa_g; \quad \text{(G)}$$
2. not acquiring general information nor specialized information in industry a
$$\Pi_A\left(I_A^g=1, I_A^s=1, I_B^g=1, I_B^s=0\right)-\Pi_A\left(I_A^g=0, I_A^s=0, I_B^g=1, I_B^s=0\right)+$$
$$\Pi_A\left(I_A^g=1, I_A^s=0, I_B^g=1, I_B^s=1\right)-\Pi_A\left(I_A^g=0, I_A^s=0, I_B^g=1, I_B^s=1\right) \geq \kappa_g+\kappa_s; \quad \text{(NI)}$$
3. not acquiring specialized information in industry a
$$\Pi_A\left(I_A^g=1, I_A^s=1, I_B^g=1, I_B^s=0\right)-\Pi_A\left(I_A^g=1, I_A^s=0, I_B^g=1, I_B^s=0\right) \geq \kappa_s; \quad \text{(S}a\text{)}$$
4. and, acquiring specialized information in industry b
$$\Pi_A\left(I_A^g=1, I_A^s=1, I_B^g=1, I_B^s=1\right)-\Pi_A\left(I_A^g=1, I_A^s=0, I_B^g=1, I_B^s=1\right) \leq \kappa_s. \quad \text{(NSb)}$$

Consistent with the intuition above, Constraints (G) and (NI) impose an upper bound on κ_g so that Bank A has incentives to acquire general information. Analogously, Constraints (NI) and (Sa) impose an upper bound on κ_s so that Bank A wants to acquire specialized information in industry a, while Constraint (NSb) imposes a lower bound on κ_s to assure Bank A does not want to acquire specialized information in industry b.

Deviation payoffs. Our goal is to show that there exist costs κ_g and κ_s such that the conditions above hold for some parameterization. To do so, we need to characterize the deviation payoffs. We provide the expressions for $\Pi_A\left(I_A^g, I_A^s, I_B^g, I_B^s\right)$ in Appendix A.6.

Note that an uninformed bank will make zero profits (Milgrom and Weber, 1982; Engelbrecht-Wiggans, Milgrom, and Weber, 1983), i.e,

$$\Pi_A\left(I_A^g=0, I_A^s=0, I_B^g=1, I_B^s=0\right)=\Pi_A\left(I_A^g=0, I_A^s=0, I_B^g=1, I_B^s=1\right)=0.$$

Then it follows that Constraint (NI) is equivalent to the participation constraint of Bank A. Moreover, this condition implies that for any cost of acquiring specialized information κ_s such that (Sa) is satisfied, we can always find a cost of general information κ_g small enough to satisfy (G) and (NI). Therefore, there will be an equilibrium with specialized lenders as long as κ_s satisfies the bounds imposed by (Sa) and (NSb). For this to be the case, it is enough to find parameters such that the benefits from acquiring specialized information to become the more informed lender are greater than the benefits from acquiring specialized information to stop being the less informed lender. This is confirmed in Figure 6 in Appendix A.6, which depicts the range of information acquisition costs κ_g and κ_s so that the conjectured information structure with a specialized lender and the ensuring lending competition indeed form an equilibrium.

5 Concluding Remarks

One of banks' main roles in the economy is producing information to allocate credit. In this paper, we show that the nature of information produced by banks affects the credit market equilibrium and the degree of competition among banks. More specifically, we explore how multi-dimensional information determines credit market outcomes in the presence of specialized lenders.

By considering soft and hard information, we can explain empirical patterns in bank lending specialization unexplained by canonical models where information technology is solely characterized by signal precision (one-dimensional). Moreover, our model with multiple sources of uncertainty and information allows us to differentiate between the quality and breadth of information. This distinction is crucial in understanding the changing landscape in the credit market due to technological advances related to data gathering and processing that lead to the hardening of soft information.

From a modeling perspective, including a continuously distributed signal within a credit market equilibrium enables us to examine private-information-based pricing, a practically pertinent aspect with crucial importance for the banking sector. Furthermore, by incorporating both soft and hard information—which reflects potentially many more underlying states—among asymmetric lenders, our paper markedly advances the field of auction literature involving such lenders in which each lender possesses private information (in contrast to Milgrom and Weber (1982) where one bidder knows strictly more than the other). We fully characterize the equilibrium in closed form and anticipate broader applications based on our framework and the solution methodology.

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A Technical Appendices

A.1 Credit Competition Equilibrium

Proof of Lemma 1

Proof. Note that the property of no gap implies common support $[\underline{r}, \overline{r}]$, because if a bank's interest rate offering has a larger lower bound or a smaller upper bound interest rate than its competitor's, this is one example of gaps in the first bank's support.

To show that the distributions have no gap, suppose that, say, the support of F^B has a gap $(r_1, r_2) \subset [\underline{r}, \overline{r}]$.[32] Then F^A should have no weight in this interval either, as any $r^A(s) \in (r_1, r_2)$ will lead to the same demand for Bank A and so a higher r will be more profitable. At least one lender does not have a mass point at r_1 (it is impossible that both distributions have a mass point at $\overline{r}_1$), under which its competitor has a profitable deviation by revising r_1 to $r \in (r_1, r_2)$ instead. Contradiction.

Regarding point mass, suppose that one distribution, say F^B has a mass point at $\tilde{r} \in [\underline{r}, \overline{r})$. Then Bank A would not quote any $r^A(s) \in [\tilde{r}, \tilde{r} + \epsilon]$ and it would strictly prefer quoting $r^A = \tilde{r} - \epsilon$ instead. In other words, the support of F^A must have a gap in the interval $[\tilde{r}, \tilde{r} + \epsilon]$. This contradicts the property of no gaps which we have shown. Finally, it is impossible that both distributions have a mass point at $\overline{r}$.

□

A.2 Proof of Proposition 1

Proof. This part proves that Bank A's equilibrium interest rate quoting strategy as a function of specialized signal $r^A(s)$ is always decreasing; this implies that the FOC that helps us derive Bank A's strategy also ensures the global optimality.

Write Bank A's value $\Pi^A(r, s)$ as a function of its interest rate quote and specialized signal, in the event of $g^A = H$ and s. (We use π to denote the equilibrium profit but Π for any strategy.) Recall that Bank A solves the following problem:

$$\max_r \Pi^A(r, s) = \underbrace{p_{HH}}_{g^A=H, g^B=H} \underbrace{\left[1 - F^B(r)\right]}_{A \text{ wins}} \left[\mu_{HH} s (1 + r) - 1\right] + \underbrace{p_{HL}}_{g^A=H, g^B=L} \left[\mu_{HL} s (1 + r) - 1\right] \quad (34)$$

[32] The same argument follows if the support of F^A has a gap in the conjectured equilibrium, and then for Bank B, any quotes within the gap lead to the same demand of the same posterior quality of customers, where Bank B updates its belief from Bank A's strategy.

with the following FOC:

$$0=\Pi_r^A\left(r\left(s\right),s\right)=\underbrace{p_{HH}\left[-\frac{dF^B\left(r\right)}{dr}\right]}_{\text{lost customer}}\left[\underbrace{\left[\mu_{HH}s\left(1+r\right)-1\right]}_{\text{customer return}}\right]+\underbrace{p_{HH}\left[1-F^B\left(r\right)\right]}_{\text{customer}}\underbrace{\mu_{HH}s}_{\text{MB of customer}}+p_{HL}\mu_{HL}s. \tag{35}$$

One useful observation is that on the support, it must hold that $\mu_{HH}s\left(1+r\right)-1>0$; otherwise, $\mu_{HL}s\left(1+r\right)-1<\mu_{HH}s\left(1+r\right)-1\leq 0$, implying that Bank A's profit is negative (so it will exit).

Lemma 3. *Consider s_1,s_2 in the interior domain with corresponding interest rate quote r_1 and r_2. The marginal value of quoting r_2 for type $s=s_1$ is*

$$\Pi_r^A\left(r_2,s_1\right)=\frac{s_2-s_1}{\mu_{HH}s_2\left(1+r_2\right)-1}\left\{p_{HH}\left[1-F^B\left(r_2\right)\right]\mu_{HH}+p_{HL}\mu_{HL}\right\}$$

and its sign depends on the sign of s_2-s_1.

Proof. Evaluating the FOC of type s_1 but quoting r_2:

$$\Pi_r^A\left(r_2,s_1\right)=p_{HH}\left[-\frac{dF^B\left(r_2\right)}{dr}\right]\left[\mu_{HH}s_1\left(1+r_2\right)-1\right]+p_{HH}\left[1-F^B\left(r_2\right)\right]\mu_{HH}s_1+p_{HL}\mu_{HL}s_1. \tag{36}$$

FOC at type s_2 yields

$$\Pi_r^A\left(r_2,s_2\right)=p_{HH}\left[-\frac{dF^B\left(r_2\right)}{dr}\right]\left[\mu_{HH}s_2\left(1+r_2\right)-1\right]+p_{HH}\left[1-F^B\left(r_2\right)\right]\mu_{HH}s_2+p_{HL}\mu_{HL}s_2=0,$$

or

$$\frac{dF^B\left(r_2\right)}{dr}=\frac{p_{HH}\left[1-F^B\left(r_2\right)\right]\mu_{HH}s_2+p_{HL}\mu_{HL}s_2}{p_{HH}\left[\mu_{HH}s_2\left(1+r_2\right)-1\right]}. \tag{37}$$

Plugging in this term to (36), $\Pi_r^A\left(r_2,s_1\right)$ becomes

$$\begin{aligned}
&-\frac{\mu_{HH}s_1\left(1+r_2\right)-1}{\mu_{HH}s_2\left(1+r_2\right)-1}\left\{p_{HH}\left[1-F^B\left(r_2\right)\right]\mu_{HH}s_2+p_{HL}\mu_{HL}s_2\right\}+p_{HH}\left[1-F^B\left(r_2\right)\right]\mu_{HH}s_1+p_{HL}\mu_{HL}s_1\\
&=\left[s_1-\frac{\mu_{HH}s_1\left(1+r_2\right)-1}{\mu_{HH}s_2\left(1+r_2\right)-1}\cdot s_2\right]\left\{p_{HH}\left[1-F^B\left(r_2\right)\right]\mu_{HH}+p_{HL}\mu_{HL}\right\}\\
&=\left(s_2-s_1\right)\cdot\frac{p_{HH}\left[1-F^B\left(r_2\right)\right]\mu_{HH}+p_{HL}\mu_{HL}}{\mu_{HH}s_2\left(1+r_2\right)-1},
\end{aligned}$$

which is the claimed marginal benefit of quoting r_2 for type s_1. Its sign depends on s_2-s_1 because the denominator is positive as we noted right after Eq. (35). □

Lemma 3 has three implications. First, as long as $r^A(\cdot)$ is (strictly) increasing in some segment, then Bank A would like to deviate in this segment. To see this, suppose that $r_1 > r_2$ when $s_1 > s_2$ for s_1, s_2 arbitrarily close. Because Lemma 1 has shown that Bank A's strategy is smooth, r_2 is arbitrarily close to r_1. Then $\Pi_r^A(r_2, s_1) < 0$, implying that the value is convex and the Bank A at s_1 (who in equilibrium is supposed to quote r_1) would like to deviate further.

Second, the monotonicity implied by Lemma 3 helps us show that Bank A uses a pure strategy. To see this, for any $s_1 > s_2$ that induce interior quotes $r_1, r_2 \in [\underline{r}, \overline{r})$, however close, in equilibrium we must have $\sup r^A(s_1) < \inf r^A(s_2)$ by monotonicity. Combining this with Part 3 of Lemma 1, i.e., the induced distribution $F^A(\cdot)$ is atomless except for at $\overline{r}$ and has no gaps, we know that Bank A must adopt a pure strategy in the interior of $[\underline{r}, \overline{r})$, or for $s \leq \hat{s}$. Finally, the following argument shows pure strategy for $s < \hat{s}$: i) randomize over $s = 0$ is a zero-measure set; and ii) on $s > \hat{s}$ Bank A can either quote $\overline{r}$ or ∞, which, generically, gives different values (and hence rules out randomization).

Third, if $r^A(\cdot)$ is decreasing globally over $\mathcal{S}$, then the FOC is sufficient to ensure global optimality. Consider a type s_1 who would like to deviate to $\check{r} > r_1$; then

$$\Pi^A(\check{r}, s_1) - \Pi^A(r_1, s_1) = \int_{r_1}^{\check{r}} V_r^A(r, s_1)\, dr.$$

Given the monotonicity of $r(s)$, we can find the corresponding type $s(r)$ for $r \in [r_1, \check{r}]$. From Lemma 3 we know that

$$\Pi_r^A(r, s_1) = (s(r) - s_1) \frac{p_{HH}\left[1 - F^B(r)\right]\mu_{HH} + p_{HL}\mu_{HL}}{\mu_{HH} s(r)(1+r) - 1}$$

which is negative given $s(r) < s_1$. Therefore the deviation gain is negative. Similarly, we can show a negative deviation gain for any $\check{r} < r_1$.

Next we show that $r^A(\cdot)$ is single-peaked over the space of $\mathcal{S} = [0, 1]$.

Lemma 4. *Given any exogenous $\pi^B \geq 0$, $r^A(\cdot)$ single-peaked over $\mathcal{S} = [0, 1]$ with a maximum point.*

Proof. When $r \in [\underline{r}, \overline{r})$, the derivative of $r^A(s)$ with respect to s is

$$\frac{dr^A(s)}{ds} = \frac{p_{HH}\phi(s)\left(\overbrace{p_{HH}\mu_{HH}\left[\int_0^s t\phi(t)\,dt - s\Phi(s)\right]}^{M_1(s)<0,\text{and} M_1'(s)<0} + \overbrace{p_{LH}\mu_{LH}q_s - \left(\pi^B + p_{LH}\right)\mu_{HH}s}^{M_2(s)?0,\ \text{but} M_2'(s)<0}\right)}{\left(p_{HH}\mu_{HH}\int_0^s t\phi(t)\,dt + p_{LH}\mu_{LH}q_s\right)^2}.$$

As $\int_0^s t\phi(t)\,dt < s\Phi(s)$, the first term in the bracket $M_1(s) < 0$, and

$$M_1'(s) = -p_{HH}\mu_{HH}\Phi(s) < 0.$$

For $M_2(s) = p_{LH}\mu_{LH}q_s - \left(\pi^B + p_{LH}\right)\mu_{HH}s$, it has an ambiguous sign, but is decreasing in s. This implies that $M_1(s) + M_2(s)$ decreases with s. It is easy to verify that $M_1(0) + M_2(0) > 0$ and $M_1(1) + M_2(1) < 0$. Therefore $r^A(s)$ first increases and then decreases, therefore single-peaked. □

Suppose that the peak point is $\tilde{s}$; then Bank A should simply charge $r(s) = \tilde{r}$ for $s < \tilde{s}$ for better profit. This is the standard "ironing" technique and we therefore define the following ironed strategy formally (here, we also take care of the capping $r \le \overline{r}$):

$$r^A_{ironed}(s) \equiv \sup_{t\in[s,1]} \min\left(r^A(t), \overline{r}\right).$$

By definition $r^A_{ironed}(s)$ is monotonely decreasing.

We now argue that in equilibrium, π^B and $\underline{r}$ adjust so that $r^A(\cdot)$ is always monotonely decreasing over $[x, 1]$. (Since we define $r^A(s) = \infty$ for $s < x$, monotonicity over the entire signal space $[0, 1]$ immediately follows.) There are two subcases to consider.

1. Suppose that $\tilde{r} = \overline{r}$. In this case, $r^A(s)$ in Eq. (13) used in Lemma 3 and 4 does not apply to $s < \tilde{s}$ because the equation is defined only over the left-closed-right-open interval $[\underline{r}, \overline{r})$. Instead, $r^A(s)$ in this region is determined by Bank A's optimality condition: as r^A does not affect the competition from Bank B (which equals $F^B(\overline{r}^-)$), Bank A simply sets the maximum possible rate $r^A(r) = \overline{r}$ unless it becomes unprofitable (for $s < x$). (This is our zero-weak equilibrium with $\pi^B = 0$, and there is no "ironing" in this case.)

2. Suppose that $\tilde{r} < \overline{r}$; then bank A quotes $\tilde{r}$ for all $s < \hat{s}$. But this is not an equilibrium—Bank A now leaves a gap in the interval $[\tilde{r}, \overline{r}]$, contradicting with point 3) in Lemma 1 (there, we rule out gaps in equilibrium). Intuitively, Bank B always would like to raise its quotes inside $[\tilde{r}, \overline{r}]$ to $\overline{r}$; there is no "ironing" in this case. (This is our positive-weak equilibrium with $\pi^B > 0$.)

□

Proof of Lemma 2

Proof. First, we argue that equilibrium $\hat{s} \equiv \arg\sup_s \left\{s : r^A(s) \ge \overline{r}\right\}$ either equals s^{be}_A or s^{be}_B. To see this, if $\pi^B = 0$, we have $\hat{s} = s^{be}_B$ by construction. If $\pi^B > 0$, then Bank B always makes an offer upon H, i.e., $F^B(\overline{r}) = 1$. We also know that $F^B(\overline{r}^-) = 1 - \frac{\int_0^{s^A(r)=\hat{s}^+} t\phi(t)dt}{q_s} < 1$, because Bank A must reject the borrower when s realizes as close to 0 and $\hat{s} > 0$. Hence, $F^B(r)$ has a point mass at

$\overline{r}$. It follows that $F^A(r)$ is open at $\overline{r}$: $\hat{s} = x$ and $\pi^A\left(r^A(\hat{s})\,|\hat{s}\right) = 0$, which is exactly the definition of s_A^{be} and so $\hat{s} = s_A^{be}$.

Now we prove the claim in this lemma. In the first case of $s_B^{be} < s_A^{be}$, we have $\hat{s} \leq s_A^{be}$ and thus Bank A's probability of winning when quoting $r^A = \overline{r}$ is at most $\frac{\int_0^{s_A^{be}} t\phi(t)dt}{q_s} \geq \frac{\int_0^{\hat{s}} t\phi(t)dt}{q_s} = 1 - F^B(\overline{r}^-)$. The definition of s_A^{be} says that Bank A upon s_A^{be} breaks even when quoting $r^A\left(s_A^{be}\right) = \overline{r}$ and facing this most favorable winning probability $\frac{\int_0^{s_A^{be}} t\phi(t)dt}{q_s}$. Then upon a worse specialized signal $s_B^{be} < s_A^{be}$, Bank A must reject the borrower because offering $\overline{r}$ leads to losses, which rules out $\hat{s} = s_B^{be}$. According to our earlier observation of $\hat{s} = s_B^{be}$ or s_A^{be}, we have $\hat{s} = s_A^{be}$ and $\pi^B > 0$ in this case, where π^B could be characterized from Eq. (12) at $r = \overline{r}$.

In the second case of $s_B^{be} \geq s_A^{be}$, we have $\hat{s} \leq s_B^{be}$ and thus Bank B's probability of winning when quoting $r^B = \overline{r}$ is at most $\Phi\left(s_B^{be}\right) \geq \Phi(s) = 1 - F^A(\overline{r}^-)$. The definition of s_B^{be} says that Bank B breaks even when quoting $r^B = \overline{r}$ and facing this most favorable winning probability $\Phi\left(s_B^{be}\right)$. Then if the competition from A were more aggressive, say $1 - F^A(\overline{r}^-) = \Phi\left(s_A^{be}\right)$, Bank B would make a loss when quoting $\overline{r}$, so $\hat{s} = s_A^{be}$ cannot support an equilibrium. Hence, in this case, $\hat{s} = s_B^{be}$ and $\pi^B = 0$. From the definition of s_A^{be}, Bank A's equilibrium break-even condition $0 = \pi^A(\overline{r}|x)$, and the fact that $s_B^{be} \geq s_A^{be}$ in this case, we have

$$\begin{aligned}
0 =& \frac{p_{HH} \int_0^{s_A^{be}} t\phi(t)\,dt}{q_s} \left[\mu_{HH} s_A^{be}(1+\overline{r}) - 1\right] + p_{HL}\left[\mu_{HL} s_A^{be}(1+\overline{r}) - 1\right] \\
=& \frac{p_{HH} \int_0^{s_B^{be}} t\phi(t)\,dt}{q_s} \left[\mu_{HH} x(1+\overline{r}) - 1\right] + p_{HL}\left[\mu_{HL} x(1+\overline{r}) - 1\right] \\
\geq& \frac{p_{HH} \int_0^{s_A^{be}} t\phi(t)\,dt}{q_s} \left[\mu_{HH} x(1+\overline{r}) - 1\right] + p_{HL}\left[\mu_{HL} x(1+\overline{r}) - 1\right].
\end{aligned}$$

Hence, $x \leq s_A^{be} \leq s_B^{be} = \hat{s}$. □

A.3 Proof of Proposition 2 and Calibration

Lemma 5. *For any $r \in [\underline{r}, \overline{r})$, we have*

$$\frac{F^B(r)}{F^A(r)} = \frac{\alpha_u^A}{\alpha_u^B}, \ \frac{dF^B(r)/dr}{dF^A(r)/dr} = \frac{\alpha_u^A}{\alpha_u^B}$$

i.e., Bank A offers higher interest rates than Bank B in the sense of F.O.S.D..

Proof. For any $r \in [\underline{r}, \overline{r})$, lenders' profit functions are

$$\pi^A = \underbrace{p_{HH}}_{B \text{ gets H}} \underbrace{\left(1 - F^B(r)\right)}_{\text{wins}} [\mu_{HH}(r+1) - 1] + \underbrace{p_{HL}}_{B \text{ gets L}} [\mu_{HL}(r+1) - 1], \tag{38}$$

$$\pi^B = \underbrace{p_{HH}}_{A \text{ gets H}} \underbrace{\left(1 - F^A(r)\right)}_{\text{wins}} [\mu_{HH}(r+1) - 1] + \underbrace{p_{LH}}_{A \text{ gets L}} [\mu_{LH}(r+1) - 1]. \tag{39}$$

These two equations imply that

$$\frac{F^B(r)}{F^A(r)} = \frac{p_{HH}[\mu_{HH}(r+1) - 1] + p_{HL}[\mu_{HL}(r+1) - 1] - \pi^A}{p_{HH}[\mu_{HH}(r+1) - 1] + p_{LH}[\mu_{LH}(r+1) - 1] - \pi^B}. \tag{40}$$

And, evaluating Eq. (38), (39) at $r = \underline{r}$ and using $F^A(\underline{r}) = F^B(\underline{r}) = 1$ gives lenders' profits:

$$\pi^A(\underline{r}) = p_{HH}[\mu_{HH}(\underline{r}+1) - 1] + p_{HL}[\mu_{HL}(\underline{r}+1) - 1],$$
$$\pi^B(\underline{r}) = p_{HH}[\mu_{HH}(\underline{r}+1) - 1] + p_{LH}[\mu_{LH}(\underline{r}+1) - 1].$$

Using these in Eq. (40), we have

$$\frac{F^B(r)}{F^A(r)} = \frac{(p_{HH}\mu_{HH} + p_{HL}\mu_{HL})(r - \underline{r})}{(p_{HH}\mu_{HH} + p_{LH}\mu_{LH})(r - \underline{r})} = \frac{\mathbb{P}\left(g^A = H, \theta = g\right)}{\mathbb{P}(g^B = H, \theta = g)} = \frac{\alpha_u^A}{\alpha_u^B}.$$

Here, $F^B(r) = \frac{\alpha_u^A}{\alpha_u^B} F^A(r)$ immediately implies that $\frac{dF^B(r)/dr}{dF^A(r)/dr} = \frac{\alpha_u^A}{\alpha_u^B}$. □

Proof of Proposition 2

Part 1: Bad-news information structure. This structure corresponds to

$$\alpha_u^A = \alpha_u^B = 1, \quad 1 > \alpha_d^A > \alpha_d^B > 0;$$

i.e., lenders only make Type II mistakes. In this part, we use $\alpha^j \equiv \alpha_d^j$ as a lender's signal precision, which captures the probability that bad-type borrowers are correctly identified as L, and $\alpha^A > \alpha^B$.

Proof. From Lemma 5, lender bidding strategies $F^A(\cdot), F^B(\cdot)$ over $[0, \overline{r}] \cup \{\infty\}$ satisfy

$$F^B(r) = \begin{cases} F^A(r), & r \in [0, \overline{r}), \\ F^A(r^-), & r = \overline{r}. \end{cases}$$

We use this result to express Δr as a function of $F^B(r)$. Specifically,

$$\begin{aligned}
\mathbb{E}\left[r^A \middle| r^A < r^B \le \infty\right] &= \frac{p_{HH}\int_{\underline{r}}^{\overline{r}}\left[1-F^B(r)\right] r dF^A(r) + p_{HL}\int_{\underline{r}}^{\overline{r}} r dF^A(r)}{p_{HH}\int_{\underline{r}}^{\overline{r}}\left[1-F^B(r)\right] dF^A(r) + p_{HL}} \\
&= \frac{p_{HH}\int_{\underline{r}}^{\overline{r}}\left[1-F^B(r)\right] r dF^B(r) + p_{HH}\overline{r}\left[1-F^B(\overline{r})\right]^2 + p_{HL}\left[\overline{r}-\int_{\underline{r}}^{\overline{r}} F^B(r)\, dr\right]}{p_{HH}\int_{\underline{r}}^{\overline{r}}\left[1-F^B(r)\right] dF^B(r) + p_{HH}\left[1-F^B(\overline{r})\right]^2 + p_{HL}} \\
&= \overline{r} - \frac{p_{HH}\int_{\underline{r}}^{\overline{r}}\left\{\frac{1}{2}-\frac{\left[1-F^B(r)\right]^2}{2}\right\} dr + p_{HL}\int_{\underline{r}}^{\overline{r}} F^B(r)\, dr}{p_{HH}\left\{-\frac{\left[1-F^B(\overline{r})\right]^2}{2}+\frac{1}{2}\right\} + p_{HH}\left[1-F^B(\overline{r})\right]^2 + p_{HL}},
\end{aligned}$$

and

$$\begin{aligned}
\mathbb{E}\left[r^B \middle| r^B < r^A \le \infty\right] &= \frac{p_{HH}\int_{\underline{r}}^{\overline{r}}\left[1-F^A(r)\right] r dF^B(r) + p_{LH}\int_{\underline{r}}^{\overline{r}} r dF^B(r)}{p_{HH}\int_{\underline{r}}^{\overline{r}}\left[1-F^B(r)\right] dF^B(r) + p_{LH}F^B(\overline{r})} \\
&= \frac{p_{HH}\int_{\underline{r}}^{\overline{r}}\left[1-F^B(r)\right] r dF^B(r) + p_{LH}\left[\overline{r}F^B(\overline{r})-\int_{\underline{r}}^{\overline{r}} F^B(r)\, dr\right]}{p_{HH}\int_{\underline{r}}^{\overline{r}}\left[1-F^B(r)\right] dF^B(r) + p_{LH}F^B(\overline{r})} \\
&\overline{r} - \frac{p_{HH}\int_{\underline{r}}^{\overline{r}}\left\{\frac{1}{2}-\frac{\left[1-F^B(r)\right]^2}{2}\right\} dr + p_{LH}\int_{\underline{r}}^{\overline{r}} F^B(r)\, dr}{p_{HH}\left\{\frac{1}{2}-\frac{\left[1-F^B(r)\right]^2}{2}\right\} + p_{LH}F^B(\overline{r})}.
\end{aligned}$$

Hence,

$$\begin{aligned}
\Delta r &\equiv \mathbb{E}\left[r^A \middle| r^A < r^B \le \infty\right] - \mathbb{E}\left[r^B \middle| r^B < r^A \le \infty\right] \\
&= \frac{p_{HH}\int_{\underline{r}}^{\overline{r}}\left\{\frac{1}{2}-\frac{\left[1-F^B(r)\right]^2}{2}\right\} dr + p_{LH}\int_{\underline{r}}^{\overline{r}} F^B(r)\, dr}{p_{HH}\left\{\frac{1}{2}-\frac{\left[1-F^B(\overline{r})\right]^2}{2}\right\} + p_{LH}F^B(\overline{r})} - \frac{p_{HH}\int_{\underline{r}}^{\overline{r}}\left\{\frac{1}{2}-\frac{\left[1-F^B(r)\right]^2}{2}\right\} dr + p_{HL}\int_{\underline{r}}^{\overline{r}} F^B(r)\, dr}{p_{HH}\left\{-\frac{\left[1-F^B(\overline{r})\right]^2}{2}+\frac{1}{2}\right\} + p_{HH}\left[1-F^B(\overline{r})\right]^2 + p_{HL}}.
\end{aligned} \tag{41}$$

Now we plug in the expressions of $F^B(r)$ to show that the canonical model leads to counterfactual predictions when $\overline{r}$ is relatively small. From He, Huang, and Zhou (2023),

$$F^B(r) = \frac{\frac{r}{\overline{r}}-1}{\frac{r}{\overline{r}}-\left(1-\alpha^A\right)},$$

and the key terms are accordingly

$$\int_{\underline{r}}^{\overline{r}} F^B(r)\,dr = \overline{r} - \underline{r} - \alpha^A \underline{r} \ln\left(\frac{\overline{r}}{\underline{r}} - 1 + \alpha^A\right) + \alpha^A \underline{r} \ln \alpha^A,$$

$$\int_{\underline{r}}^{\overline{r}} \left\{\frac{1}{2} - \frac{\left[1 - F^B(r)\right]^2}{2}\right\} dr = \frac{\underline{r}}{2} \cdot \frac{\left(\frac{\overline{r}}{\underline{r}} - 1\right)^2}{\frac{\overline{r}}{\underline{r}} - 1 + \alpha^A}.$$

Let $M(\overline{r}) \equiv \frac{\overline{r}}{\underline{r}} - \left(1 - \alpha^A\right)$ and then

$$\begin{aligned}\Delta r =& p_{HH} \cdot \frac{\underline{r}\alpha^A}{2} \cdot \left(\frac{M - \alpha^A}{M}\right)^2 \left(\frac{p_{HH}\alpha^A}{M} + p_{LH}\right) + \frac{p_{HH}}{2}\left[\int_{\underline{r}}^{\overline{r}} F^B(r)\right](p_{LH} + p_{HL})\left(\frac{\alpha^A}{M}\right)^2 \\ &+ p_{LH}p_{HL}\frac{\alpha^A}{M}\left[\int_{\underline{r}}^{\overline{r}} F^B(r)\right] + (p_{HL} - p_{LH})\frac{p_{HH}}{2} \cdot \underline{r} \cdot \frac{\left(M - \alpha^A\right)^2}{M} - (p_{HL} - p_{LH})\frac{p_{HH}}{2}\left[\int_{\underline{r}}^{\overline{r}} F^B(r)\,dr\right].\end{aligned}$$

Note that only the last term $-(p_{HL} - p_{LH})\frac{p_{HH}}{2}\left[\int_{\underline{r}}^{\overline{r}} F^B(r)\,dr\right]$ is negative. In addition, this term approaches zero as $\overline{r} \to \underline{r} = \frac{(1-q)\left(1-\alpha^B\right)}{q}$, and

$$\frac{\partial\left[\int_{\underline{r}}^{\overline{r}} F^B(r)\,dr\right]}{\partial \overline{r}} = 1 - \frac{\alpha^A}{M} > 0.$$

Therefore, there exists some threshold $\hat{\overline{r}}$ such that when $\overline{r} \leq \hat{\overline{r}}$, the canonical model has counterfactual prediction $\Delta r > 0$.

□

Part 2: Symmetric information structure. This structure corresponds to

$$\alpha^j \equiv \alpha_u^j = \alpha_d^j \in \left(\frac{1}{2}, 1\right], \quad \text{for} \quad j \in \{A, B\}.$$

In this context, the specialized lender Bank A's signal is more precise, $\alpha^A > \alpha^B$.

Lemma 6. $\mathbb{E}\left[r^A \middle| r^A < r^B \leq \infty\right] \geq \mathbb{E}\left[r^B \middle| r^B < r^A \leq \infty\right]$ *is equivalent to the following inequality*

$$\begin{aligned}&\frac{\mathbb{P}\left(x^A = H\right)\frac{\alpha^B}{\alpha^A}\int_{\underline{r}}^{\overline{r}} F^B(r)\,dr + p_{HH}\int_{\underline{r}}^{\overline{r}} F^A(r)\,r dF^B(r) - p_{HH}\overline{r}\frac{\alpha^B}{2\alpha^A}\left(F^B(\overline{r})\right)^2}{p_{HH}\left[1 - F^B(\overline{r}) + \frac{\alpha^B}{2\alpha^A}\left(F^B(\overline{r})\right)^2\right] + p_{HL}} \\ \leq& \frac{\mathbb{P}\left(x^B = H\right)\int_{\underline{r}}^{\overline{r}} F^B(r)\,dr + p_{HH}\int_{\underline{r}}^{\overline{r}} F^A(r)\,r dF^B(r) - p_{HH}\frac{\alpha^B}{2\alpha^A}\left(F^B(\overline{r})\right)^2\overline{r}}{p_{HH}\left[F^B(\overline{r}) - \frac{\alpha^B}{2\alpha^A}\left(F^B(\overline{r})\right)^2\right] + p_{LH}F^B(\overline{r})}.\end{aligned}$$

Proof. The expected rate of a lender's loan is

$$\mathbb{E}\left[r^A \middle| r^A < r^B \leq \infty\right] \triangleq \frac{\underbrace{p_{HH}}_{\text{B gets H}} \int_{\underline{r}}^{\overline{r}} \underbrace{\left[1 - F^B(r)\right]}_{\text{A wins}} r dF^A(r) + \underbrace{p_{HL}}_{\text{B gets L}} \int_{\underline{r}}^{\overline{r}} r dF^A(r)}{p_{HH} \int_{\underline{r}}^{\overline{r}} \left[1 - F^B(r)\right] dF^A(r) + p_{HL}}, \tag{42}$$

$$\mathbb{E}\left[r^B \middle| r^B < r^A \leq \infty\right] \triangleq \frac{\underbrace{p_{HH}}_{\text{A gets H}} \int_{\underline{r}}^{\overline{r}} \underbrace{\left[1 - F^A(r)\right]}_{\text{B wins}} r dF^B(r) + \underbrace{p_{LH}}_{\text{A gets L}} \int_{\underline{r}}^{\overline{r}} r dF^B(r)}{p_{HH} \int_{\underline{r}}^{\overline{r}} \left[1 - F^A(r)\right] dF^B(r) + p_{LH} F^B(\overline{r})}. \tag{43}$$

In the first step, we rewrite the equations as functions of $dF^B(r)$ and dr which are continuous at $\overline{r}$. Using integration by parts and Lemma 5, we have

$$\int_{\underline{r}}^{\overline{r}} r dF^A(r) = r F^A(r)\Big|_{\underline{r}}^{\overline{r}} - \int_{\underline{r}}^{\overline{r}} F^A(r)\, dr = \overline{r} - \int_{\underline{r}}^{\overline{r}} F^A(r)\, dr = \overline{r} - \frac{\alpha^B}{\alpha^A} \int_{\underline{r}}^{\overline{r}} F^B(r)\, dr.$$

In the last step, although Lemma 5 does not apply at $r = \overline{r}$, it is of zero measure. Similarly, the probability of Bank A winning in competition is

$$\begin{aligned}
\int_{\underline{r}}^{\overline{r}} \left[1 - F^B(r)\right] dF^A(r) &= \int_{\underline{r}}^{\overline{r}} dF^A(r) - \int_{\underline{r}}^{\overline{r}} F^B(r)\, dF^A(r) \\
&\underbrace{=}_{\text{integration by parts}} 1 - \left[F^B(\overline{r}) - \int_{\underline{r}}^{\overline{r}} F^A(r)\, dF^B(r)\right] \\
&\underbrace{=}_{F^A = \frac{\alpha^B}{\alpha^A} F^B} 1 - F^B(\overline{r}) + \int_{\underline{r}}^{\overline{r}} \frac{\alpha^B}{\alpha^A} F^B(r)\, dF^B(r) \\
&= 1 - F^B(\overline{r}) + \frac{\alpha^B}{2\alpha^A}\left(F^B(\overline{r})\right)^2,
\end{aligned}$$

and thus the probability of Bank B winning is the residual

$$\int_{\underline{r}}^{\overline{r}} \left[1 - F^A(r)\right] dF^B(r) = F^B(\overline{r}) - \frac{\alpha^B}{2\alpha^A}\left(F^B(\overline{r})\right)^2.$$

Similarly,

$$\begin{aligned}
\int_{\underline{r}}^{\overline{r}} F^B(r)\, r dF^A(r) &= \int_{\underline{r}}^{\overline{r}-} F^B(r)\, r dF^A(r) + F^B(\overline{r})\, \overline{r} \left[1 - F^A(\overline{r}^-)\right] \\
&\underbrace{=}_{F^A = \frac{\alpha^B}{\alpha^A} F^B,\, F^B(\overline{r}^-) = F^B(\overline{r})} \int_{\underline{r}}^{\overline{r}} F^A(r)\, r dF^B(r) + F^B(\overline{r})\, \overline{r} \left(1 - \frac{\alpha^B}{\alpha^A} F^B(\overline{r})\right)
\end{aligned}$$

Plug these terms into Eq. (42) and (43), and we have

$$\begin{aligned}\mathbb{E}\left[r^A \middle| r^A < r^B \leq \infty\right] &= \frac{\mathbb{P}\left(g^A = H\right) \int_{\underline{r}}^{\overline{r}} r dF^A\left(r\right) - p_{HH} \int_{\underline{r}}^{\overline{r}} F^B\left(r\right) r dF^A\left(r\right)}{p_{HH}\left[1 - F^B\left(\overline{r}\right) + \frac{\alpha^B}{2\alpha^A}\left(F^B\left(\overline{r}\right)\right)^2\right] + p_{HL}} \\ &= \overline{r} - \frac{\mathbb{P}\left(g^A = H\right) \frac{\alpha^B}{\alpha^A} \int_{\underline{r}}^{\overline{r}} F^B\left(r\right) dr + p_{HH} \int_{\underline{r}}^{\overline{r}} F^A\left(r\right) r dF^B\left(r\right) - p_{HH} \overline{r} \frac{\alpha^B}{2\alpha^A}\left(F^B\left(\overline{r}\right)\right)^2}{p_{HH}\left[1 - F^B\left(\overline{r}\right) + \frac{\alpha^B}{2\alpha^A}\left(F^B\left(\overline{r}\right)\right)^2\right] + p_{HL}};\end{aligned}$$

for Bank B,

$$\begin{aligned}\mathbb{E}\left[r^B \middle| r^B < r^A \leq \infty\right] &= \frac{\mathbb{P}\left(g^B = H\right) \int_{\underline{r}}^{\overline{r}} r dF^B\left(r\right) - p_{HH} \int_{\underline{r}}^{\overline{r}} F^A\left(r\right) r dF^B\left(r\right)}{p_{HH}\left[F^B\left(\overline{r}\right) - \frac{\alpha^B}{2\alpha^A}\left(F^B\left(\overline{r}\right)\right)^2\right] + p_{LH} F^B\left(\overline{r}\right)} \\ &= \overline{r} - \frac{\mathbb{P}\left(g^B = H\right) \int_{\underline{r}}^{\overline{r}} F^B\left(r\right) dr + p_{HH} \int_{\underline{r}}^{\overline{r}} F^A\left(r\right) r dF^B\left(r\right) - p_{HH} \frac{\alpha^B}{2\alpha^A}\left(F^B\left(\overline{r}\right)\right)^2 \overline{r}}{p_{HH}\left[F^B\left(\overline{r}\right) - \frac{\alpha^B}{2\alpha^A}\left(F^B\left(\overline{r}\right)\right)^2\right] + p_{LH} F^B\left(\overline{r}\right)}.\end{aligned}$$

Therefore, $\mathbb{E}\left[r^A \middle| r^A < r^B \leq \infty\right] \geq \mathbb{E}\left[r^B \middle| r^B < r^A \leq \infty\right]$ is equivalent to the stated inequality. □

Lemma 7. *In the case of* $q > \frac{1}{1+\overline{r}}$*, when* $\alpha^B \uparrow \alpha^A$*, there exists a threshold* $\hat{\alpha}\left(\alpha^A\right) < \alpha^A$ *so that when* $\alpha^B > \hat{\alpha}\left(\alpha^A\right)$ *we have* $F^B\left(\overline{r}\right) = 1$*.*

Proof. Let $\alpha^B = \alpha^A - \epsilon$. Bank B's profit could be pinned down by setting $r = \overline{r}^-$,

$$\begin{aligned}\pi^B =& p_{HH}\left[1 - F^A\left(\overline{r}^-\right)\right]\left[\mu_{HH}\left(\overline{r}+1\right) - 1\right] + p_{LH}\left[\mu_{LH}\left(\overline{r}+1\right) - 1\right] \\ \underbrace{\geq}_{F^A\left(\overline{r}^-\right) \leq 1} & \; p_{LH}\left(\mu_{LH}\left(\overline{r}+1\right) - 1\right) \\ \underbrace{=}_{\alpha^B = \alpha^A - \epsilon} & \; q\left(1 - \alpha^A\right)\left(\alpha^A - \epsilon\right)\overline{r} - \left(1-q\right)\alpha^A\left(1 - \left(\alpha^A - \epsilon\right)\right) \\ =& \left(1 - \alpha^A\right)\alpha^A\left[q\overline{r} - \left(1-q\right)\right] - \epsilon\left[q\left(1 - \alpha^A\right)\overline{r} + \left(1-q\right)\alpha^A\right].\end{aligned}$$

Hence, when $\epsilon < \frac{\left(1-\alpha^A\right)\alpha^A\left[q\overline{r} - (1-q)\right]}{q\left(1-\alpha^A\right)\overline{r} + (1-q)\alpha^A}$, or equivalently, when

$$\alpha^B > \hat{\alpha}\left(\alpha^A\right) = \alpha^A - \frac{\left(1 - \alpha^A\right)\alpha^A\left[q\overline{r} - \left(1-q\right)\right]}{q\left(1 - \alpha^A\right)\overline{r} + \left(1-q\right)\alpha^A},$$

we have $\pi^B > 0$ and $F^B\left(\overline{r}\right) = 1$. □

Proof of Proposition 2 Part 2

Proof. There are two cases depending on whether $q < \frac{1}{1+\overline{r}}$, i.e., whether the project has a negative NPV prior.

The first case of $q < \frac{1}{1+\overline{r}}$ is easier. When $\alpha^B \uparrow \alpha^A$ and $\alpha^A - \alpha^B = o\left(q - \frac{1}{1+\overline{r}}\right)$, Bank B's signal distributions and strategies approach that of Bank A except at $r = \overline{r}$ (Lemma 5):

$$F^B(r) \uparrow F^A(r) \quad \text{for any} \quad r \in [\underline{r}, \overline{r}), \quad \text{and} \quad F^B(\overline{r}) < 1 = F^A(\overline{r}).$$

Then from Lemma 6,

$$\begin{aligned}\frac{\overline{r} - \mathbb{E}\left[r^A \middle| r^A < r^B \leq \infty\right]}{\overline{r} - \mathbb{E}\left[r^B \middle| r^B < r^A \leq \infty\right]} &= \frac{p_{HH}\left[F^B(\overline{r}) - \frac{1}{2}\left(F^B(\overline{r})\right)^2\right] + p_{LH}F^B(\overline{r})}{p_{HH}\left[1 - F^B(\overline{r}) + \frac{1}{2}\left(F^B(\overline{r})\right)^2\right] + p_{HL}} \\ &\underbrace{\leq}_{\text{RHS set } F^B(\overline{r})=1} \frac{\frac{1}{2}p_{HH} + p_{LH}}{\frac{1}{2}p_{HH} + p_{HL}} = 1,\end{aligned}$$

where the last inequality holds because the ratio is increasing in $F^B(\overline{r})$. Hence, $\mathbb{E}\left[r^A \middle| r^A < r^B \leq \infty\right] \geq \mathbb{E}\left[r^B \middle| r^B < r^A \leq \infty\right]$ always holds in this case.

Now consider the second case $q \geq \frac{1}{1+\overline{r}}$. When $\alpha^B \to \alpha^A$, since $\mathbb{E}\left[r^A \middle| r^A < r^B \leq \infty\right]$ decreases while $\mathbb{E}\left[r^B \middle| r^B < r^A \leq \infty\right]$ increase in $F^B(\overline{r})$, it is sufficient to show that the equivalent inequality in Lemma 6 holds under $F^B(\overline{r}) = 1$, i.e.,

$$\begin{aligned}&\frac{\mathbb{P}\left(g^A = H\right)\frac{\alpha^B}{\alpha^A}\int_{\underline{r}}^{\overline{r}} F^B(r)\,dr + p_{HH}\int_{\underline{r}}^{\overline{r}} F^A(r)\,r dF^B(r) - p_{HH}\overline{r}\frac{\alpha^B}{2\alpha^A}}{p_{HH}\frac{\alpha^B}{2\alpha^A} + p_{HL}} \\ \leq &\frac{\mathbb{P}\left(g^B = H\right)\int_{\underline{r}}^{\overline{r}} F^B(r)\,dr + p_{HH}\int_{\underline{r}}^{\overline{r}} F^A(r)\,r dF^B(r) - p_{HH}\frac{\alpha^B}{2\alpha^A}\overline{r}}{p_{HH}\left(1 - \frac{\alpha^B}{2\alpha^A}\right) + p_{LH}},\end{aligned} \tag{44}$$

where both the LHS and RHS are positive. When $q > \frac{1}{1+\overline{r}}$, recall that Lemma 7 shows $F^B(\overline{r}) = 1$ as $\alpha^B \to \alpha^A$ under $q > \frac{1}{1+\overline{r}}$ and so the inequality is also necessary.

Denote by $N \triangleq \int_{\underline{r}}^{\overline{r}} F^B(r)\,dr > 0$, and $M \triangleq \overline{r}\frac{\alpha^B}{2\alpha^A} - \int_{\underline{r}}^{\overline{r}} F^A(r)\,r dF^B(r)$. $M > 0$ because

$$\int_{\underline{r}}^{\overline{r}} F^A(r)\,r dF^B(r) < \overline{r}\int_{\underline{r}}^{\overline{r}} F^A(r)\,dF^B(r) = \overline{r}\int_{\underline{r}}^{\overline{r}} \frac{\alpha^B}{\alpha^A}F^B(r)\,dF^B(r) = \overline{r}\frac{\alpha^B}{\alpha^A}\int_{\underline{r}}^{\overline{r}} d\left(\frac{F^B(r)^2}{2}\right) = \overline{r}\frac{\alpha^B}{2\alpha^A}.$$

Collect terms in the key inequality (44), we have

$$\begin{aligned}
&\left\{\left[p_{HH}\left(1-\frac{\alpha^B}{2\alpha^A}\right)+p_{LH}\right](p_{HH}+p_{HL})\frac{\alpha^B}{\alpha^A}-\left(p_{HH}\frac{\alpha^B}{2\alpha^A}+p_{HL}\right)(p_{HH}+p_{LH})\right\}N \\
\leq & p_{HH}\left[p_{HH}\left(1-\frac{\alpha^B}{2\alpha^A}\right)+p_{LH}-\left(p_{HH}\frac{\alpha^B}{2\alpha^A}+p_{HL}\right)\right]M
\end{aligned} \tag{45}$$

Let $\alpha^B=\alpha^A-\epsilon$ and calculate the coefficients. Note that as $\alpha^B=\alpha^A-\epsilon$, we have $p_{HL}-p_{LH}=(2q-1)\,\epsilon$.[33] The coefficient on the LHS of (45):

$$\begin{aligned}
&\left[p_{HH}\left(1-\frac{\alpha^B}{2\alpha^A}\right)+p_{LH}\right](p_{HH}+p_{HL})\frac{\alpha^B}{\alpha^A}-\left(p_{HH}\frac{\alpha^B}{2\alpha^A}+p_{HL}\right)(p_{HH}+p_{LH}) \\
=&\left(\frac{p_{HH}}{2}+\frac{\epsilon}{2\alpha^A}p_{HH}+p_{LH}\right)(p_{HH}+p_{HL})\left(1-\frac{\epsilon}{\alpha^A}\right)-\left(\frac{p_{HH}}{2}-\frac{\epsilon}{2\alpha^A}p_{HH}+p_{HL}\right)(p_{HH}+p_{LH}) \\
=&-\frac{p_{HH}}{2}(2q-1)\,\epsilon+\frac{\epsilon}{2\alpha^A}p_{HH}^2-\frac{\epsilon}{2\alpha^A}p_{LH}p_{HH}-\frac{\epsilon}{\alpha^A}p_{LH}p_{HL}
\end{aligned}$$

The coefficient on the RHS of (45):

$$\begin{aligned}
p_{HH}\left[p_{HH}\left(1-\frac{\alpha^B}{2\alpha^A}\right)+p_{LH}-\left(p_{HH}\frac{\alpha^B}{2\alpha^A}+p_{HL}\right)\right]&=\frac{\epsilon}{\alpha^A}p_{HH}^2-p_{HH}(p_{HL}-p_{LH}) \\
&=\frac{\epsilon}{\alpha^A}p_{HH}^2-p_{HH}(2q-1)\,\epsilon
\end{aligned}$$

Plug the coefficients back into the inequality (45), so we need to show that

$$\begin{aligned}
0\leq&\left\{\frac{\epsilon}{\alpha^A}p_{HH}^2-p_{HH}(2q-1)\,\epsilon\right\}M-\left\{-\frac{p_{HH}}{2}(2q-1)\,\epsilon+\frac{\epsilon}{2\alpha^A}p_{HH}^2-\frac{\epsilon}{2\alpha^A}p_{LH}p_{HH}-\frac{\epsilon}{\alpha^A}p_{LH}p_{HL}\right\}N \\
=&\left[(2q-1)-\frac{p_{HH}}{\alpha}\right]\frac{p_{HH}(N-2M)}{2}\epsilon+\left(\frac{1}{2}p_{LH}p_{HH}+p_{LH}p_{HL}\right)\frac{N}{\alpha}\epsilon.
\end{aligned}$$

[33] We have $p_{HL}=q\alpha^A\left(1-\alpha^B\right)+(1-q)\,\alpha^B\left(1-\alpha^A\right)$ and $p_{LH}=q\left(1-\alpha^A\right)\alpha^B+(1-q)\,\alpha^A\left(1-\alpha^B\right)$ and then therefore $p_{HL}-p_{LH}=q\left(\alpha^A-\alpha^B\right)+(1-q)\left(\alpha^B-\alpha^A\right)=(2q-1)\,\epsilon$.

Note that

$$\begin{aligned}N-2M &= \int_{\underline{r}}^{\overline{r}} F^B(r)\,dr - 2\left(\overline{r}\frac{\alpha^B}{2\alpha^A} - \int_{\underline{r}}^{\overline{r}} F^A(r)\,r dF^B(r)\right)\\ &= \int_{\underline{r}}^{\overline{r}} F^B(r)\,dr - 2\left(\overline{r}\frac{\alpha^B}{2\alpha^A} - \frac{\alpha^B}{\alpha^A}\int_{\underline{r}}^{\overline{r}} F^B(r)\,r dF^B(r)\right)\\ &= \int_{\underline{r}}^{\overline{r}} F^B(r)\,dr - 2\left(\overline{r}\frac{\alpha^B}{2\alpha^A} - \frac{\alpha^B}{2\alpha^A}\overline{r} + \frac{\alpha^B}{\alpha^A}\int_{\underline{r}}^{\overline{r}} \frac{\left(F^B(r)\right)^2}{2}dr\right)\\ &= \int_{\underline{r}}^{\overline{r}} F^B(r)\,dr - \frac{\alpha^B}{\alpha^A}\int_{\underline{r}}^{\overline{r}} \left(F^B(r)\right)^2 dr > 0.\end{aligned}$$

Therefore, one sufficient condition is

$$2q-1 \geq \frac{p_{HH}}{\alpha} = \frac{q\alpha^2 + (1-q)(1-\alpha)^2}{\alpha}$$

collecting terms, it requires $q \geq 1-\alpha+\alpha^2$. Since $1-\alpha+\alpha^2$ increases in α for $\alpha \in \left(\frac{1}{2}, 1\right)$, this imposes a simple condition that prior needs to be sufficiently good and information technology α cannot be too high. □

Calibration For calibration, we rely on two empirical moments in the U.S. banking industry to gauge the magnitudes of q and α. First, this website on Federal Reserve reports the NPL ratio to be about 2%; second, Yates (2020) shows that the approval rate for business C&I loans is from 55% (small) to 80% (large).

We gauge q and α from the limiting case where Bank B's information technology α^B approaches that of Bank A, i.e., $\alpha^B \to \alpha^A = \alpha$. Depending on the primitives, Bank B may either make zero or positive profit in the unique equilibrium, which we call zero-weak or positive weak in analogous to our main equilibrium characterization with multi-dimensional information.

Recall that at the beginning of the proof of Proposition 2 we have shown that condition (29) fails in the zero-weak case (i.e., if and only if $q < \frac{1}{1+\overline{r}}$ where Bank B makes zero profit). Therefore we only need to consider the positive-weak case.

In this case, lenders are symmetric: upon H each lender makes interest rate with randomized strategy, with a winning probability of 0.5. Therefore we can write down the NPL ratio and approval rate of, say Bank A,

$$2\% = \frac{\mathbb{P}\left(\theta = 0 \middle| r^A < r^B < \infty\right)}{\mathbb{P}\left(r^A < r^B < \infty\right)} = \frac{(1-q)\left[\frac{(1-\alpha)^2}{2} + \alpha(1-\alpha)\right]}{(1-q)\left[\frac{(1-\alpha)^2}{2} + \alpha(1-\alpha)\right] + q\left[\frac{\alpha^2}{2} + \alpha(1-\alpha)\right]},$$
$$y = \mathbb{P}\left(g^A = H\right) = q\alpha + (1-q)(1-\alpha), \text{for } y \in [0.55, 0.80].$$

which allows us to solve for the pair (q, α). For instance, when $y = 0.7$ one can solve for $q = 0.9629$ and $\alpha = 0.716$, which satisfies the proposed sufficient condition $q > 1 - \alpha + \alpha^2$. The same result holds for $y = 0.55$ (so that $q = 0.9771$ and $\alpha = 0.5524$) or $y = 0.8$ (so that $q = 0.9349$ and $\alpha = 0.8449$).

A.4 Proof of Proposition 3

Proof. Based on the credit competition equilibrium in Proposition 1, the expected rates of a lender's issued loan are:

$$\mathbb{E}\left[r^A \middle| r^A < r^B \leq \infty\right] = \frac{\underbrace{p_{HH}}_{g^B=H} \int_x^1 \underbrace{\left[1 - F^B\left(r^A(t)^-\right)\right]}_{A \text{ wins}} r^A(t)\,\phi(t)\,dt + \underbrace{p_{HL}}_{g^B=L} \int_x^1 r^A(t)\,\phi(t)\,dt}{p_{HH}\int_x^1 \left[1 - F^B\left(r^A(t)^-\right)\right]\phi(t)\,dt + p_{HL}\int_x^1 \phi(t)\,dt},$$

$$\mathbb{E}\left[r^B \middle| r^B < r^A \leq \infty\right] = \frac{\underbrace{p_{HH}}_{g^A=H} \int_{\hat{s}}^1 \underbrace{\Phi(t)}_{B \text{ wins}} r(t)\,d\left[-F^B(r(t))\right] + \underbrace{p_{LH}}_{g^A=L} \int_x^1 r(t)\,dF^B(r(t))}{p_{HH}\int_{\hat{s}}^1 \Phi(t)\,d\left[-F^B(r(t))\right] + p_{LH}F^B(\overline{r})}.$$

Note that when the positive weak equilibrium arises, $F^B(r(s))$ has a point mass of size $1 - F^B(\overline{r}^-)$ at $\overline{r}$ or $r^A(\hat{s})$.

We first impose the following conditions

a) Each lender receives a perfect general signal, $g^j = \theta_h$ for $j \in \{A, B\}$,

b) $\overline{r} \to \infty$,

and then

$$\mathbb{E}\left[r^A + 1 \middle| r^A < r^B \leq \infty\right] = \frac{\int_0^1 \Phi(t)\,\phi(t)\,dt}{\int_0^1 t\Phi(t)\left[\frac{\int_0^t \nu\phi(\nu)d\nu}{t\Phi(t)}\right]\phi(t)\,dt},$$

$$\mathbb{E}\left[r^B + 1 \middle| r^B < r^A \leq \infty\right] = \frac{\int_0^1 \Phi(t)\left[\frac{t\Phi(t)}{\int_0^t \nu\phi(\nu)d\nu}\right]\phi(t)\,dt}{\int_0^1 \Phi(t)\,t\phi(t)\,dt}.$$

Additionally, c) the specialized signal distribution is $\phi(s) = 1 + \epsilon\left[2 \cdot \mathbf{1}_{s \leq 0.5} - 1\right]$. Then

$$\mathbb{E}\left[r^A + 1 \middle| r^A < r^B \leq \infty\right] = 2 \cdot \frac{\frac{1}{8}(1+\epsilon)^2 + \frac{\epsilon(1-\epsilon)}{2} + \frac{3}{8}(1-\epsilon)^2}{\frac{1}{24}(1+\epsilon)^2 + \frac{\epsilon(1-\epsilon)}{4} + \frac{7}{24}(1-\epsilon)^2},$$

$$\mathbb{E}\left[r^B + 1 \middle| r^B < r^A \leq \infty\right] = 2 \cdot \frac{\frac{1}{8}(1+\epsilon)^2 + \frac{\epsilon(1-\epsilon)}{2} + \frac{3}{8}(1-\epsilon)^2 + \epsilon^2(1-\epsilon)\int_{0.5}^{1} \frac{\left(t-\frac{1}{2}\right)}{\frac{\epsilon}{2}+(1-\epsilon)t^2} dt + \epsilon(1-\epsilon)^2 \int_{0.5}^{1} \frac{t\left(t-\frac{1}{2}\right)}{\frac{\epsilon}{2}+(1-\epsilon)t^2}}{\frac{1}{24}(1+\epsilon)^2 + \frac{3\epsilon(1-\epsilon)}{8} + \frac{7}{24}(1-\epsilon)^2}$$

Note that when $\epsilon = 0$, $\Delta r = 0$. When $\epsilon \to 0$, we have (ignoring higher order terms of ϵ)

$$\begin{aligned}
\frac{\partial \Delta r}{\partial \epsilon} &= \lim_{\epsilon \to 0} \frac{\Delta r(\epsilon)}{\epsilon} \\
&= \frac{1}{\epsilon}\left(\frac{1}{\frac{1}{3} - \frac{1}{4}\epsilon} - \frac{1 + \epsilon - \epsilon \ln 2}{\frac{1}{3} - \frac{1}{8}\epsilon}\right) \\
&= 3 \ln 2 - \frac{15}{8} > 0.
\end{aligned}$$

Hence, when $\epsilon > 0$ ($\epsilon < 0$), i.e., $\phi(s)$ tilts toward more (less) favorable realizations, we have $\Delta r > 0$ ($\Delta r < 0$). □

A.5 Derivation of Correlated General Signals

Another aspect of information technology advancement is that the lenders' general information signals become more correlated. Formally, with probability ρ_h, lenders receive the same signal realization $g^c \in \{H, L\}$ and

$$\mathbb{P}\left(g^c = H \,|\theta_h = 1\right) = \mathbb{P}\left(g^c = L \,|\theta_h = 0\right) = \alpha;$$

with probability $1 - \rho_h$, each receives an independent general signal according to Eq. (3).

With more correlated general signals or a higher ρ_h, lenders are more likely to agree on the customer quality and so more likely to compete (the event of HH). In terms of inference, the posterior upon disagreement (that comes from the uncorrelated part of the assessment) is still the prior q_h.[34] Taken together, competition becomes fiercer, because lenders are more likely to compete but not more concerned about the winner's curse.

A.6 Information Acquisition

In this part, we first characterize lending profits and then provide a numerical illustration in which the specialization equilibrium arises.

[34] Upon competition (HH), lenders are less sure about a good quality borrower, i.e., $\mu_{HH}(\rho_h)$ decreases in ρ_h.

A.6.1 Lending Profits

We characterize lending profits as a function of information acquisition, $\Pi_A\left(I_A^g, I_A^s, I_B^g, I_B^s\right)$ (we focus on Bank A due to symmetry.) We omit the case where there is an uninformed lender.

$\mathbf{I_A^g = 1, I_A^s = 1, I_B^g = 1, I_B^s = 0}$ **(Specialization).** This is the equilibrium that we focus on—each lender has a general information signal and only Bank A has a specialized signal s. Bank A's expected lending profit before signal realizations is thus

$$\Pi_A\left(I_A^g = 1, I_A^s = 1, I_B^g = 1, I_B^s = 0\right) = \int_x^1 \pi^A\left(r^A(s)\Big|\, s\right)\phi(s)\,ds,$$

where $\pi^A\left(r^A(s)\Big|\, s\right)$ is the profits for given signal realizations H and s and is given in Eq. (9). Using the equilibrium strategies in Proposition 1, we have

$$\pi^A\left(r^A(s)\Big|\, s\right) = p_{HH}\cdot\frac{\int_0^{\min\{s,\hat{s}\}}(s-t)\,\phi(t)\,dt}{q_s} + \left(\pi^B + p_{LH}\right)\cdot\frac{s}{q_s} - p_{HL}, \text{ for } s \geq x.$$

The expression shows that Bank A earns the information rent from the specialized signal. Bank A observes s, while Bank B may only negatively update the prior q_s when winning the competition that $s^A \leq s(r)$; this is reflected in the terms $\frac{s}{q_s}$ and $\frac{\int_0^{\min\{s,\hat{s}\}}(s-t)\phi(t)dt}{q_s}$.

In this case, Bank B's profit $\Pi_B\left(I_A^g = 1, I_A^s = 1, I_B^g = 1, I_B^s = 0\right) = \pi^B$ is given in Lemma 2. By symmetry, $\Pi_A\left(I_A^g = 1, I_A^s = 0, I_B^g = 1, I_B^s = 1\right) = \Pi_B\left(I_A^g = 1, I_A^s = 1, I_B^g = 1, I_B^s = 0\right) = \pi^B$.

$\mathbf{I_A^g = 0, I_A^s = 1, I_B^g = 1, I_B^s = 0}$ **(Asymmetric technology).** In this case, Bank A only collects specialized information while Bank B only collects general information in industry a. This case is nested in the previous case of specialization ($I_A^g = 1, I_A^s = 1, I_B^g = 1, I_B^s = 0$), by reformulating Bank A to have an uninformative general signal, e.g.,

$$\mathbb{P}\left(g^A = H\Big|\,\theta_h = 1\right) = \mathbb{P}\left(g^A = H\Big|\,\theta_h = 0\right) = 1.$$

$\mathbf{I_A^g = 1, I_A^s = 0, I_B^g = 1, I_B^s = 0}$ **(General information only).** In this case, both lenders only acquire general information, i.e., investing in IT and data processing that apply to both industries. The credit competition corresponds to Broecker (1990) with two lenders. Lenders are symmetric and the lending profit of, say Bank A, is

$$\Pi_A\left(I_A^g = 1, I_A^s = 0, I_B^g = 1, I_B^s = 0\right) = \max\left\{p_{LH}\left(\mu_{HH} q_s \overline{r} - 1\right), 0\right\}.$$

The "max" operator arises because either both lenders withdraw with positive probability (zero profits), or both lenders make profits and neither has a point mass at $\overline{r}$, i.e., $F^j\left(\overline{r}^-\right) = 1$.

$\mathbf{I_A^g = 1, I_A^s = 1, I_B^g = 1, I_B^s = 1}$ **(Acquire all information).** In this symmetric case, each lender invests in both information technologies and receives both the general and specialized signals. We characterize the credit market equilibrium based on Riordan (1993) which considers the competition between two lenders each with a continuous private signal. Here, each lender additionally has a binary signal that represents the general information. Following the modeling of Riordan (1993), we work with the direct specialized signal z. Specifically, let z and Z denote the realization and the random variable of the specialized signal respectively, and let

$$\tilde{F}(z) \equiv \mathbb{P}(Z \leq z | \theta_s = 1), \; \tilde{G}(z) \equiv \mathbb{P}(Z \leq z | \theta_s = 0)$$

denote the CDFs of Z conditional on the underlying state θ_s, with the corresponding PDFs denoted by $\tilde{f}$ and $\tilde{g}$. Introduce $\mu(z) \equiv \mathbb{P}(\theta_s = g | S)$ as the posterior belief, which is s in our baseline model.

A lender only bids when the general signal is H and the specialized signal $z \geq x$. Let $R(z) \equiv r(z) + 1$ denote the equilibrium gross rate quote. Given competitor's strategy $R(z)$, the lending profits from any R is then

$$\begin{aligned} \pi(R | z) = & \left[p_{HH}\mu_{HH}\mu(z)\tilde{F}(t(R)) + p_{HL}\mu_{HL}\mu(z) \right] R \\ & - p_{HH}\left[(1-\mu(z))\tilde{G}(t(R)) + \mu(z)\tilde{F}(t(R)) \right] - p_{HL}, \end{aligned} \tag{46}$$

where $t(R)$ the signal such that the other bank offers R. The first order condition w.r.t. R is

$$\begin{aligned} \frac{\partial \pi(R(t) | z)}{\partial R} = & \left[p_{HH}\mu_{HH}\mu(z)\tilde{F}(t) + p_{HL}\mu_{HL}\mu(z) \right] \\ & + \left\{ p_{HH}\mu_{HH}\mu(z)\tilde{f}(t)R(t) - p_{HH}\left[(1-\mu(z))\tilde{g}(t) + \mu(z)\tilde{f}(t) \right] \right\} \frac{dt}{dR}. \end{aligned}$$

The equilibrium strategy satisfies

$$\left. \frac{\partial \pi(R(t) | z)}{\partial t} \right|_{t=z} = 0$$

By symmetry, we have

$$\frac{dt}{dR} = \frac{1}{R'(t)}.$$

These two conditions imply

$$p_{HH}\mu_{HH}\tilde{f}(z)R(z) + \left(p_{HH}\mu_{HH}\tilde{F}(z) + p_{HL}\mu_{HL} \right) R'(z) = \frac{p_{HH}(1-\mu(z))\tilde{g}(z) + p_{HH}\mu(z)\tilde{f}(z)}{\mu(z)}, \tag{47}$$

or equivalently,

$$\frac{d\left\{ \left[p_{HH}\mu_{HH}\tilde{F}(z) + p_{HL}\mu_{HL} \right] R(z) \right\}}{dz} = \frac{p_{HH}(1-\mu(z))\tilde{g}(z) + p_{HH}\mu(z)\tilde{f}(z)}{\mu(z)}.$$

Integrating over z, we have

$$R(z) = \frac{\int_{\underline{z}}^{z} \frac{p_{HH}(1-\mu(t))\tilde{g}(t)+p_{HH}\mu(t)\tilde{f}(t)}{\mu(t)} dt + constant}{p_{HH}\mu_{HH}\tilde{F}(z) + p_{HL}\mu_{HL}}. \tag{48}$$

The unknown constant is pinned down by the boundary condition $\pi(\bar{r}+1 \,|x) = 0$: Upon the threshold signal x, a lender quotes the maximum interest rate $\bar{r}+1$ and makes zero profit,

$$0 = \left[p_{HH}\mu_{HH}\mu(x)\tilde{F}(x) + p_{HL}\mu_{HL}\mu(x)\right](\bar{r}+1) - p_{HH}\left[(1-\mu(x))\tilde{G}(x) + \mu(x)\tilde{F}(x)\right] - p_{HL}. \tag{49}$$

Then a lender's lending profit is

$$\Pi_A\left(I_A^g = 1, I_A^s = 1, I_B^g = 1, I_B^s = 1\right) = \int_x^{\bar{z}} \pi\left(R(z)|\, z\right)\left[q_s\tilde{f}(z) + (1-q_s)\tilde{g}(z)\right] dz,$$

where $R(z)$ is given by Eq. (48) and (49), profit $\pi(R(z), z)$ is given by Eq. .

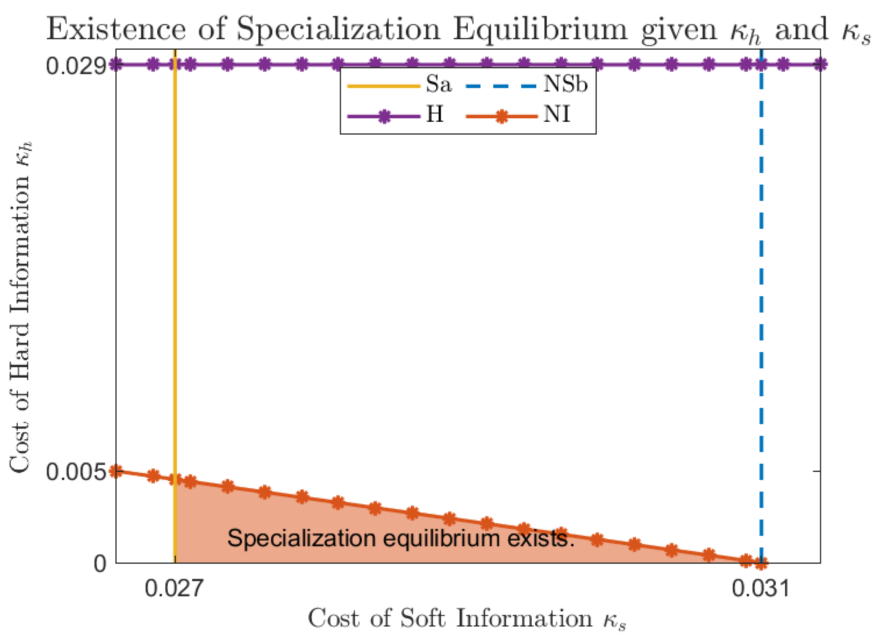

Figure 6: **Specialization Equilibrium** This plot depicts the incentive compatibility constraints where Bank A does not want to deviate from the specialization equilibrium. Parameters: $\bar{r} = 0.36$, $\rho_h = 0$, $q_h = 0.8$, $q_s = 0.9$, $\alpha_u = \alpha_d = \alpha = 0.7$, and $\tau = 1$.

A.6.2 Specialization Equilibrium

Figure 6 shows the region of information acquisition costs κ_h and κ_s to support the specialization equilibrium so that one of the banks endogenously becomes the specialized bank in one industry by acquiring both specialized and general information while the other is non-specialized by acquiring the general information only. In sum, we need κ_h to be sufficiently small while κ_s to lie in an intermediate range.

A.7 General Information Structure

In this extension, we focus on the well-behaved structure (i.e., smooth distribution of interest rates over $[\underline{r}, \overline{r})$ and decreasing $r^A(z)$) and show that the lender strategies in Proposition 4 correspond to an equilibrium.

Proof of Proposition 4

Proof. **Bank A's strategy**

In the region of $z \in (\hat{z}, 1]$ that corresponds to $r^A(z) \in [\underline{r}, \overline{r})$, $r^A(\cdot)$ is strictly decreasing so the inverse function $z^A(\cdot) \equiv r^{A(-1)}(\cdot)$ is properly defined. Bank B's lending profit when quoting $r \in [\underline{r}, \overline{r})$ is

$$\begin{aligned}\pi^B(r) &= \underbrace{\overline{p}_{HH}}_{g^A=H} \cdot \underbrace{\int_{\underline{z}}^{z^A(r)}}_{B\,\text{wins}} \left[\underbrace{\mu_{HH}(t)}_{\text{repay}}(1+r) - 1\right] \phi_z(t|\,HH)\,dt + \underbrace{\overline{p}_{LH}}_{g^A=L} \left[\underbrace{\overline{\mu}_{LH}}_{\text{repay}}(1+r) - 1\right] \\ &= (1+r)\left[\int_{\underline{z}}^{z^A(r)} p_{HH}(t)\,\mu_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}\right] - \int_{\underline{z}}^{z^A(r)} p_{HH}(t)\,dt - \overline{p}_{LH} \end{aligned} \tag{50}$$

Bank A's equilibrium strategy $r^A(z)$ for $z \in [\hat{z}, 1]$ is such that Bank B is indifferent across $r \in [\underline{r}, \overline{r})$. Hence,

$$r^A(z) = \frac{\overbrace{\pi^B + \int_{\underline{z}}^{z} p_{HH}(t)\,dt + \overline{p}_{LH}}^{B\text{'s lending amount}}}{\underbrace{\int_{\underline{z}}^{z} p_{HH}(t)\cdot\mu_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}}_{B\text{'s customers who repay}}} - 1, \quad \text{where} \quad \hat{z} \leq s \leq \overline{z}. \tag{51}$$

In addition, $r^A(z) = \overline{r}$ for $z \in [z_x, \hat{z})$ and Bank A rejects the borrower when $z \in [\underline{z}, z_x)$, where z_x satisfies

$$\pi^A\left(r^A(z_x) = \overline{r} \middle|\, z_x\right) = 0.$$

This completes the proof of Bank A's strategy in Proposition 4.

Bank B's strategy

Bank A's offered interest rate $r^A(z)$ upon $z \in [\hat{z}, \overline{z}]$ maximizes

$$\pi^A\left(r^A(z)\middle| z\right) = \underbrace{p_{HH}(z)}_{g^B=H}\underbrace{\left[1-F^B(r)\right]}_{A \text{ wins}}\left[\underbrace{\mu_{HH}(z)}_{\text{repay}}(1+r)-1\right] + \underbrace{p_{HL}(z)}_{g^B=L}\left[\underbrace{\mu_{HL}(z)}_{\text{repay}}(1+r)-1\right]$$

The FOC w.r.t. r is

$$\underbrace{\left[-\frac{d\left[F^B(r)\right]}{dr}\right]}_{\Delta\text{winning prob}}\underbrace{p_{HH}(z)\left[\mu_{HH}(z)(1+r)-1\right]}_{\text{profit upon winning}} + \underbrace{p_{HH}(z)\left[1-F^B(r)\right]\mu_{HH}(z) + p_{HL}(z)\mu_{HL}(z)}_{\text{existing customer}} = 0.$$

Bank A's optimal strategy $r^A(z)$ satisfies this first order condition,

$$\begin{aligned} 0 = & -\frac{d\left[F^B\left(r^A(z)\right)\right]}{dr}p_{HH}(z)\left[\mu_{HH}(z)\left(1+r^A(z)\right)-1\right] \\ & + p_{HH}(z)\left[1-F^B\left(r^A(z)\right)\right]\mu_{HH}(z) + p_{HL}(z)\mu_{HL}(z). \end{aligned} \tag{52}$$

From Eq. (51) about $r^A(z)$, we derive the following key equation by taking derivatives w.r.t. z,

$$\underbrace{\frac{dr^A(z)}{dz}\left[\int_{\underline{z}}^{z} p_{HH}(t)\mu_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}\right]}_{\text{B: }\uparrow\text{marginal customer return}} + \underbrace{p_{HH}(z)\left[\left(r^A(z)+1\right)\mu_{HH}(z)-1\right]}_{\text{B: }\uparrow\text{existing customer revenue}} = 0.$$

Plug this equation into the FOC (52), and we have

$$-\frac{d\left[F^B\left(r^A(z)\right)\right]}{dz}\left[\int_{\underline{z}}^{z} p_{HH}(t)\mu_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}\right] = p_{HH}(z)\left[1-F^B(r)\right]\mu_{HH}(z) + p_{HL}(z)\mu_{HL}(z),$$

which is equivalent to

$$\frac{d}{dz}\left\{\frac{1-F^B\left(r^A(z)\right)}{\int_{\underline{z}}^{z}\mu_{HH}(t)p_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}}\right\} = \frac{p_{HL}(z)\mu_{HL}(z)}{\left[\int_{\underline{z}}^{z} p_{HH}(t)\mu_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}\right]^2}. \tag{53}$$

Since signals are independent conditional on the state being $\theta = 1$, the right-hand-side equals

$$\begin{aligned} & \frac{q\mathbb{P}(HL|\theta=1)\phi_z(z|\theta=1)}{\left[\int_{\underline{z}}^{z} q\mathbb{P}(HH|\theta=1)\phi_z(t|\theta=1)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}\right]^2} \\ = & -\frac{\mathbb{P}\left(g^B=L\middle|\theta=1\right)}{\mathbb{P}(g^B=H|\theta=1)}\frac{d}{dz}\left[\frac{1}{\int_{\underline{z}}^{z} q\mathbb{P}(HH|\theta=1)\phi_z(t|\theta=1)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}}\right]. \end{aligned}$$

Then the solution $F^B\left(r^A(z)\right)$ to the ODE (53) satisfies

$$\frac{1-F^B\left(r^A(z)\right)}{\int_{\underline{z}}^{z}\mu_{HH}(t)\,p_{HH}(t)\,dt+\overline{p}_{LH}\overline{\mu}_{LH}}=-\frac{\mathbb{P}\left(g^B=L\middle|\theta=1\right)}{\mathbb{P}\left(g^B=H\middle|\theta=1\right)}\left[\frac{1}{\int_{\underline{z}}^{z}\mu_{HH}(t)\,p_{HH}(t)\,dt+\overline{p}_{LH}\overline{\mu}_{LH}}\right]+Const.$$

Using the boundary condition $F^B\left(r^A(\overline{z})=\underline{r}\right)=0$, we solve for the constant

$$Const=\frac{1}{\mathbb{P}(\theta=1)}\frac{1}{\mathbb{P}\left(g^B=H\middle|\theta=1\right)^2}.$$

Therefore,

$$\begin{aligned}
F^B(r)=&\frac{1}{\mathbb{P}\left(g^B=H\middle|\theta=1\right)}-\frac{\int_{\underline{z}}^{z^A(r)}\mu_{HH}(t)\,p_{HH}(t)\,dt+\overline{p}_{LH}\overline{\mu}_{LH}}{\mathbb{P}(\theta=1)\,\mathbb{P}\left(g^B=H\middle|\theta=1\right)^2}\\
=&\frac{1}{\mathbb{P}\left(g^B=H\middle|\theta=1\right)}-\frac{\mathbb{P}(\theta=1)\,\mathbb{P}\left(HH\middle|\theta=1\right)\int_{\underline{z}}^{z^A(r)}\phi_z\left(t\middle|\theta=1\right)dt+\mathbb{P}(\theta=1)\,\mathbb{P}\left(LH\middle|\theta=1\right)}{\mathbb{P}(\theta=1)\,\mathbb{P}\left(g^B=H\middle|\theta=1\right)^2}\\
=&\frac{\mathbb{P}\left(g^A=H\middle|\theta=1\right)}{\mathbb{P}\left(g^B=H\middle|\theta=1\right)}\left[1-\int_{\underline{z}}^{z^A(r)}\phi_z\left(t\middle|\theta=1\right)dt\right].
\end{aligned}$$

Bank B's profit π^B

Now we are left with one unknown variable π^B in Eq. (51). Similar to the baseline model, the equilibrium could be positive-weak or zero-weak, depending on whether Bank A upon specialized signal realization z_A^{be} or Bank B breaks even when quoting $\overline{r}$. We define z_A^{be} and z_B^{be} as

$$\begin{aligned}
0=\pi^A\left(\overline{r}\middle|z_A^{be}\right)=&p_{HH}\left(z_A^{be}\right)\frac{\mathbb{P}\left(g^A=H\middle|\theta=1\right)}{\mathbb{P}\left(g^B=H\middle|\theta=1\right)}\left[1-\int_{\underline{z}}^{z_A^{be}}\phi_z\left(t\middle|\theta=1\right)dt\right]\cdot\left[\mu_{HH}\left(z_A^{be}\right)(1+\overline{r})-1\right]\\
&+p_{HL}\left(z_A^{be}\right)\left[\mu_{HL}\left(z_A^{be}\right)(1+\overline{r})-1\right],\\
0=\pi^B\left(\overline{r};z_B^{be}\right)=&\int_{\underline{z}}^{z_B^{be}}p_{HH}(t)\,\mu_{HH}(t)\,(1+\overline{r})\,dt-\int_{\underline{z}}^{z_B^{be}}p_{HH}(t)\,dt+\overline{p}_{HL}\left[\overline{\mu}_{HL}(1+\overline{r})-1\right].
\end{aligned}$$

Equilibrium π^B is then

$$\pi^B=\max\left\{\int_{\underline{z}}^{z_A^{be}}p_{HH}(t)\,\mu_{HH}(t)\,(1+\overline{r})\,dt-\int_{\underline{z}}^{z_A^{be}}p_{HH}(t)\,dt+\overline{p}_{HL}\left[\overline{\mu}_{HL}(1+\overline{r})-1\right],0\right\}.$$

When $z_A^{be}>z_B^{be}$, equilibrium is positive weak with $\pi^B>0$, and $\hat{z}=z_x=z_A^{be}$; when $z_A^{be}\le z_B^{be}$, equilibrium is zero weak with $\pi^B=0$, and $z_B^{be}=\hat{z}>z_x$. □