Dynamic Market Choice *

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Abstract

In practice, many assets are traded in both transparent centralized markets and opaque decentralized markets. To explain traders' market choices, we develop a model of dynamic learning and market selection between the centralized and decentralized markets. With heterogeneous trader value correlations, we find that when asset payoff sensitivity or volatility is sufficiently low, traders prefer the decentralized market; when asset sensitivity or volatility is intermediate, switching between centralized and decentralized markets is the optimal market choice; when asset values are sensitive to volatile fundamentals, assets are traded only in the centralized market. The model's predictions are supported by empirical evidence from the Chinese corporate bond market. Our research uncovers new welfare implications for various market designs with dynamic market choices.

Keywords: endogenous market structure, decentralized market, dynamic learning, transparency

JEL Classification: D47, D83, G10, G14

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1 Introduction

In practice, many assets are traded in both transparent centralized and opaque decentralized markets. For instance, equities are mostly traded on transparent centralized exchanges, but they can also be traded in more opaque and decentralized markets, such as dark pools and over-the-counter (OTC) markets.[1] Bonds are available for trading on exchanges, OTC, or both.[2] Traders' market choices change over time. For example, during periods of high volatility, the volume of equity transactions in dark pools compared to that in exchanges tends to be lower.[3] Traders' active participation in the opaque decentralized markets has attracted policy debates on whether to introduce more transparency or shut down the decentralized markets. However, a fundamental question remains unclear. What determines traders' market choices? Understanding their choices can help policymakers design better markets.

To address this question, we develop a model with endogenous dynamic market choices. The main insight from the analysis is that asset payoff sensitivity and volatility influence price history informativeness, which in turn affects dynamic market choices. To grasp the intuition behind traders' dynamic market choices, let us start with the classic static trade-off discussed in the literature (e.g. Rostek and Weretka, 2012; Yoon, 2017; Rostek and Wu, 2021; Babus and Parlatore, 2022). The centralized market is larger and can be more liquid than the decentralized market. However, traders can trade exclusively with the best counterparty with the most opposite trading needs in the decentralized market, while they face

[1]A dark pool is a type of alternative trading system (ATS) that allows institutional investors to trade securities without publicly revealing their intentions during the search for a buyer or seller. They emerged in the 1980s when the Securities and Exchange Commission (SEC) allowed brokers to transact large blocks of shares. See https://www.investopedia.com/terms/d/dark-pool.asp. In the U.S., the OTC equity markets include OTC QX, OTC QB, and OTC Pink Marketplace with different financial standards and regulations (Ang et al., 2013; Brüggemann et al., 2018).

[2]For example, in China, bonds are available for trading on exchanges, over-the-counter (OTC), or both. Mutual funds, insurance companies, and security firms can trade corporate bonds dual-listed on both exchange and OTC markets. See Section 6 for more institutional details about the Chinese corporate bond markets. In the U.S. and most European countries, bonds are predominantly traded in OTC markets, with a recent rise of more transparent electronic trading (Nagel, 2016; Bessembinder et al., 2020; O'Hara and Zhou, 2021). Before World War II, corporate bonds and municipal bonds were actively traded in centralized exchanges (Biais and Green, 2019).

[3]See Investors Flee Dark Pools As Market Volatility Erupts, The Wall Street Journal, Sept. 2, 2011, and "Dark Pools" Draw More Trading Amid Low Volatility, The Wall Street Journal, May 3, 2019. Traders' trading places are also different for assets with different payoff sensitivities. Securities whose payoffs are designed to be less sensitive to issuers' fundamentals, such as bonds, are primarily traded in over-the-counter markets. Those more sensitive to fundamentals, like equities, are traded both in centralized markets and dark pools. The most sensitive securities,for instance, options are predominantly traded in centralized markets.

competitors in the centralized market.[4] In the dynamic model, this trade-off between liquidity and competition will change as traders not only learn from the current price as in the static models, but also learn from the price history. As traders learn from the price history, the adverse selection in the market is lower. Consequently, market liquidity improves with access to the price history. This improvement is more pronounced in the decentralized market due to its smaller size relative to the centralized market.[5] What is unique to the dynamic market is that the price history and market choices evolve endogenously over time. More informative past prices lead traders to favor the decentralized market over the centralized one. Asset payoff sensitivity and volatility determine the growth and decay rate of the informativeness of past prices, thus shaping dynamic market choices.

The dynamic model features short-lived traders arriving each period to trade a risky asset. The *asset properties* can be summarized with asset payoff sensitivity and volatility. The asset value changes across time with AR(1) shocks. *Asset payoff sensitivity* measures how much asset value changes with shocks. *Asset volatility* measures the innovations in the AR(1) shocks. In each round, traders choose between a centralized market with a double auction for all participants, or a decentralized market where they find the best counterparties for bilateral double auctions that give them the highest expected utility. Traders have private values with varying degrees of correlation, representing different hedging needs or disagreements in asset value. From the traders' perspective, the centralized and decentralized markets differ in market size, correlation with counterparties, and transparency. In the decentralized market, traders exclusively engage with their best counterparties - those with the lowest value correlation. In the centralized market, traders interact not only with their best counterparties but also with competitors who have more correlated values. The decentralized market is opaque; traders cannot see past prices from this market. However, traders can observe and learn about their values from past centralized market prices. After they choose the market, traders receive private signals about the asset, submit demand schedules and the market clears.[6]

[4]Here we focus on the trade-offs when traders are indifferent between the centralized and decentralized markets. When traders in the centralized market have highly correlated values, even the liquidity can be lower in the centralized market than in the decentralized markets due to adverse selection. Decentralized markets will be the obvious optimal market choice. (e.g. Rostek and Weretka, 2012; Yoon, 2017; Rostek and Wu, 2021).

[5]In fact, this intuition applies to any additional public signals. See Rostek and Wu (2024) for more discussions on the equilibrium existence conditions and properties for bilateral double auctions with public signals.

[6]The assumption that traders receive private signals after they choose the market follows Yoon (2017). It ensures that comparative statics are not affected by the realization of the signals. It is not crucial for the results.

Different dynamic market choices can naturally emerge as price history evolves endogenously over time. In this model, the impact of past prices on the current market choice can be summarized by a single sufficient statistic: *price history informativeness*. It measures how much traders can learn about their values from price history. Higher price history informativeness improves liquidity and increases traders' expected utility, with this improvement being more pronounced in the decentralized market.

Two asset properties determine the evolution of *price history informativeness* and therefore traders' market choices. The first is the asset's payoff sensitivity to shocks in fundamentals. When an asset is insensitive to shocks, its value remains relatively stable over time. Consequently, the price history becomes more informative and decays slowly. If the sensitivity is sufficiently low, the first round centralized market price is sufficiently informative for traders to remain in the decentralized market afterwards. For assets more sensitive to value changes across rounds, traders switch between the decentralized and centralized markets. Traders tend to prefer the decentralized market as price history accumulates. However, once they have chosen the decentralized market, its opaque nature causes price history informativeness to decay – the price history gradually becomes stale and uninformative as new shocks change the asset values. As price history informativeness decreases, the decentralized market becomes illiquid. It drives traders back to the centralized market when the price history is sufficiently informative. For the most sensitive assets, price history informativeness is sufficiently low for traders to always stay in the centralized market.

Asset volatility also affects market choices. If volatility is sufficiently low, traders remain in the decentralized market starting from the second round. An intermediate level of volatility makes price history informativeness decay faster, leading traders to switch between centralized and decentralized markets. High volatility diminishes the informativeness of past prices, compelling traders to remain in the centralized market.

Only when traders' value correlations are highly homogeneous will they choose to stay in the centralized market for all rounds. When value correlations are very heterogeneous, traders consistently opt for the decentralized market. This choice stems from the significant benefit each trader gains by matching bilaterally with the counterparty having the lowest correlation, and thus the most diverse values. In this case, the advantage of heterogeneous values outweighs the reduced liquidity of a decentralized market. Conversely, when value correlations are highly homogeneous, traders invariably choose the centralized market. This preference arises because similar correlations across traders diminish the advantage of exclusive trading with a specific counterparty in the decentralized market. The higher liquidity attracts traders to the centralized market.

We test model predictions on asset properties and market choices in the Chinese corporate bond market. In China, two bond markets co-exist: an over-the-counter (OTC) market and a centralized exchange market. Non-bank financial institutions, such as mutual funds or insurance companies, can choose to trade in either market. We focused on traders' market choices for corporate bonds dual-listed in both markets, collecting daily transaction data from January 1 to May 31, 2018. Consistent with the model predictions, we find that bonds with higher sensitivity to shocks to fundamentals are more likely to be traded in the centralized market; and that assets with higher volatility, as measured by greater price volatility in the last 30 days, are more likely to be traded in the centralized market.

We then directly test the key mechanism regarding price history informativeness and dynamic market choices using the same dataset. Our model predicts that as price history accumulates, traders tend to shift from the centralized market to the decentralized market. Empirically, we find that bond traders are more likely to switch from the centralized market to the over-the-counter market when the bond trades more frequently.

Besides evidence from the Chinese corporate bond market, we also find evidence in support of the model prediction on volatility and market choices in the U.S. equity market. In Appendix D, we test the model predictions in the U.S. equity market.[7] Menkveld et al. (2017) and Buti et al. (2022) find evidence with U.S. equity data in support of our model prediction, i.e., the share of equities traded in dark pools decreases when the asset volatility is high. These empirical findings, consistent across various asset classes and market structures, strongly support the model's key implications about what drives traders to choose between centralized and decentralized trading venues.

The dynamic market choice we explore has new policy implications for market designs. In particular, we highlight a novel trade-off between the current round decentralized market efficiency and future traders' utilities. Given this trade-off, designs that improve efficiency in the decentralized market without post-trade transparency may hurt the overall welfare. This trade-off is absent in static market designs. For instance, while a static model might suggest that introducing pre-trade transparency in the decentralized market improves efficiency, it may decrease welfare in a dynamic context where the decentralized

[7] We collect data for equities traded in exchanges, alternative trading systems (ATS), and over-the-counter (OTC) markets during 2019-2022 from the Financial Industry Regulatory Authority (FINRA) and Wharton Research Data Service (WRDS). We classify the centralized exchanges such as Nasdaq and NYSE as centralized markets, ATS and OTC as decentralized markets. We use the standard deviation of prices in the last 100 trading days for each stock as a proxy for volatility. We find a negative correlation between price volatility and the proportion of transaction volume traded in the ATS and OTC.

market lacks post-trade transparency. This occurs because price history informativeness decreases for future traders, as the more efficient pre-trade transparent decentralized market attracts more trades while simultaneously making prices unobservable. In scenarios with sufficient trading rounds and stable asset values due to low sensitivity or volatility, the long-term effect of diminished price history informativeness outweighs the efficiency gains in the decentralized market, ultimately leading to decreased welfare.

The trade-off between current-round decentralized market efficiency and future traders' utilities also informs market structure design. The prevalent coexistence of centralized and decentralized markets has sparked policy debates about mandating all trades in the centralized market. In a static version of this model, we might conclude that such a mandate deprives traders of the ability to trade with the best counterparty when beneficial. However, in the dynamic model, while shutting down the opaque decentralized market may decrease traders' utility in the current round, it can improve overall welfare by increasing price history informativeness for future traders.

Literature: This paper is closely related to the literature studying endogenous market structure. One strand of literature focuses on the endogenous formation of core-periphery trading networks (Chang and Zhang, 2015; Glode and Opp, 2016; Wang, 2016; Farboodi et al., 2018; Babus and Parlatore, 2022; Hugonnier et al., 2022; Sambalaibat, 2022; Farboodi, 2023; Farboodi et al., 2023). This paper is more closely related to the literature on the endogenous formation of coexisting centralized and decentralized markets (Pagano, 1989; Rust and Hall, 2003; Yoon, 2017; Vogel, 2019; Seppi, 1990; Desgranges and Foucault, 2005; Bolton et al., 2016; Lee and Wang, 2018; Huang and Xu, 2021; Dugast et al., 2022). While the literature focuses on static endogenous market choices, this paper studies dynamic market choices. In terms of underlying frictions giving rise to decentralized markets, the papers closest to this paper are Yoon (2017) and Babus and Parlatore (2022), which focus on the trade-off between the larger market size of the centralized market and counterparty with a lower correlation (i.e. more disagreement in the word of Babus and Parlatore (2022)) in the decentralized markets. While Yoon (2017) and Babus and Parlatore (2022) focus on traders' role in determining market structures, e.g. value correlation across traders and private signal precision, this paper highlights the impact of asset properties on traders' market choices.[8] We show that high price history informativeness due to low asset sensitivity and volatility is a new mechanism. That explains why some traders choose to trade

[8]The asset sensitivity and volatility are properties related to the asset's value correlation across rounds, as opposed to the value correlation across traders.

in decentralized markets. By incorporating the impact of price history on traders' market choices, this paper endogenizes the dynamic evolution of price history informativeness, traders' beliefs, price impacts and market choices.

Second, this paper is related to the literature studying the welfare implication of coexisting centralized and decentralized markets dynamically (e.g. Miao, 2006; Antill and Duffie, 2021; Blonien, 2023). Papers using a static approach to analyze the welfare of coexisting centralized and decentralized markets include Zhu (2014), Buti et al. (2017), Malamud and Rostek (2017), Liu et al. (2018), and also the papers mentioned above that endogenize this market structure. Existing papers assume exogenous timing for traders to participate in decentralized markets. This paper contributes to the literature by endogenizing the time for traders to choose between the decentralized and the centralized market.

Finally, this paper is related to papers on transparency designs in financial markets (Duffie et al., 2017; Asriyan et al., 2017; Ollar et al., 2021; Back et al., 2020; Kakhbod and Song, 2020, 2022; Glebkin et al., 2023; Cespa and Vives, 2023; Vairo and Dworczak, 2023; Rostek and Wu, 2024; Rostek et al., 2024). The existing literature usually considers transparency designs without allowing traders to choose the venues. In this paper, we show transparency designs with endogenous dynamic market choices. Among this strand of literature, two existing papers (Rostek and Wu, 2024; Rostek et al., 2024) are closest to this paper. Rostek and Wu (2024) provided the existence condition for bilateral double auction which this paper builds on. Rostek et al. (2024) explores which assets should be traded over the counter by jointly analyzing market structure and transparency design. Instead, this paper examines the dynamic endogenous choice of venues with a new trade-off between the current round decentralized market efficiency and future traders' utilities.

2 Model

Market Structure Consider a market of one divisible risky asset and one risk-free asset as a numéraire. The market has T rounds, and $I \geq 4$ even number of short-lived traders arrive each round. In each round before they trade, and conditional on the history of prices that they observe, traders first choose the market structure $\mathcal{M} = \{CM, DM\}$ that gives them the higher expected utility.[9] The traders can choose the market that can either be one *centralized market* (CM) where all traders participate in the same exchange, or *decentralized markets* (DM) where traders are matched with a counterparty according to an algorithm a la Irving (1985). The matching is pairwise stable in the sense that no trader wants to leave the current counterparty and form a new pair. We assume that trades only choose the DM if they strictly prefer it to CM. This prevents our results from being driven by the indeterminacy of a tie-breaking rule in the case that the DM and CM lead to the same utility.

Information structure Each trader i's value of the risky asset is given $\theta_{i,t} \equiv d_t + e_{i,t}$. The common value part is given by $d_t = u + \xi f_t$, where f_t are shocks to the asset fundamentals, u are macro-level risks unrelated to the asset fundamentals such as interest rate risk, and ξ measures the asset's value sensitivity to the asset fundamentals relative to the macro-level risks. The higher ξ, the more sensitive the asset payoff is to shocks. Without loss of generality we normalize d_t to have a standard normal distribution, $d_t \sim \mathcal{N}(0,1)$, $u \sim \mathcal{N}(0, \frac{1}{1+\xi^2})$ and $f_t \sim \mathcal{N}(0, \frac{1}{1+\xi^2})$.[10] f_t is time-varying given the growth of the underlying asset, e.g. firm issuers. It follows an AR(1) process $f_t = \kappa f_{t-1} + y_t$, where $\kappa \in [0,1]$, $y_t \sim \mathcal{N}(0, (1-\kappa^2)\frac{1}{1+\xi^2})$ is the innovation independent of any other random variables. κ measures the autocorrelation of the shocks across rounds. $e_{i,t} \sim \mathcal{N}(0, \epsilon^2)$ captures the heterogeneity of traders' value. $e_{i,t}$ is independent of u and f_t. By assumption the mean of $\theta_{i,t}$ is normalized as $\mathbb{E}[\theta_{i,t}] = 0$. Denote the variance of $\theta_{i,t}$ as $\sigma_\theta^2 \equiv 1 + \epsilon^2$. We allow $e_{i,t}$ to be correlated across traders, such that $\{\theta_{i,t}\}_i$ has the joint correlation matrix at round t,

$$\mathcal{C}_t \equiv \begin{pmatrix} 1 & \rho_{1,2,t} & \dots & \rho_{1,I,t} \\ \rho_{2,1,t} & 1 & \dots & \rho_{2,I,t} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{I,1,t} & \rho_{I,2,t} & \dots & 1 \end{pmatrix}.$$

[9]Please refer to the information structure for the details of the price history. Note that we do not allow traders to split orders, i.e. to submit demand schedules to DM and CM simultaneously. Order splitting is proved to be equivalent to CM in our set-up, given that the price and price impact will equalize between DM and CM and the two markets operate as if they are one centralized market (Malamud and Rostek, 2017; Rostek and Wu, 2021). In practice, order splitting is often imperfect because traders are unable to observe prices in both the DM and CM at the same time and cannot execute their orders based on both prices simultaneously due to various regulations and market frictions. We find empirical evidence that traders choose the market instead of splitting orders in the Chinese corporate bond market (see Section 6).

[10]This normalization ensures that the comparative statics with ξ does not change anything else other than the relative sensitivity to risks. In particular, it does not change the traders' value variances. The normalization ensures the comparative analysis is not affected by the magnitude of risk but the sensitivity, but it is not necessary to generate all the results in the paper. With the normalization, u can also be seen as a numéraire for payoff sensitivity to shocks.

To simplify the analysis, we assume that for any trader i, there is only one trader $j \neq i$ whose value correlates ρ_ℓ with trader i, and any other traders $k \neq j, i$ has value correlation $\rho_{i,k} > \rho_\ell$ with trader i. Later in Section 3, we will see that this assumption ensures unique pairwise matching a la Irving (1985).

Following Rostek and Weretka (2012), the market is equicommonal by assumption, i.e., the average correlation between any trader i and the residual market is the same, $\frac{1}{I-1}\sum_{j\neq i}\rho_{i,j,t} = \bar{\rho}_t$.

Traders are uncertain about the asset value $\theta_{i,t}$ and do not observe u, $\{f_t\}_t$ and $\{e_{i,t}\}_{i,t}$. After they choose the market and before trading, each trader observes a private noisy signal about his true value $s_{i,t} = \theta_{i,t} + \varepsilon_{i,t}$, where $\varepsilon_{i,t} \sim \mathcal{N}(0, \sigma^2\sigma_\theta^2)$ and σ^2 measures the relative importance of noise in the signal. We assume σ is sufficiently large, $\sigma \geq ((\frac{2(I-1)}{I})^{1/3} - 1)^{-1/2}$.[11]

Traders can observe the current market price and submit demand contingent on that. Besides the private signals and the current market price, traders also observe past prices in the CM. Traders cannot observe prices in the DM other than the price in their current pair. We define the observed price history at round t as $\mathcal{H}_t \equiv \{p_s^{CM}\}_{s<t}$. Given the symmetric market assumption, if the optimal market choice of all traders at round t is the CM, then the price history updates as $\mathcal{H}_{t+1} = \mathcal{H}_t \cup \{p_t\}$, otherwise, $\mathcal{H}_{t+1} = \mathcal{H}_t$.

Preferences: The market is a double auction in a linear-normal setting. After the traders choose the market, they submit a demand schedule $q_{i,t}$, to maximize the payoffs conditioning on the history of past round price information $\mathcal{H}_t$, the current round signal $s_{i,t}$, and the current round price p_t,

$$\mathbb{E}[U_{i,t}|\mathcal{H}_t, s_{i,t}, p_t] = \mathbb{E}[\theta_{i,t}q_{i,t} - \frac{\alpha}{2}(q_{i,t})^2 - p_t q_{i,t}|\mathcal{H}_t, s_{i,t}, p_t].$$

The linear-quadratic utility function form follows the literature (Kyle, 1989; Vives, 2011; Rostek and Weretka, 2012; Yoon, 2017), where α is traders' risk aversion.

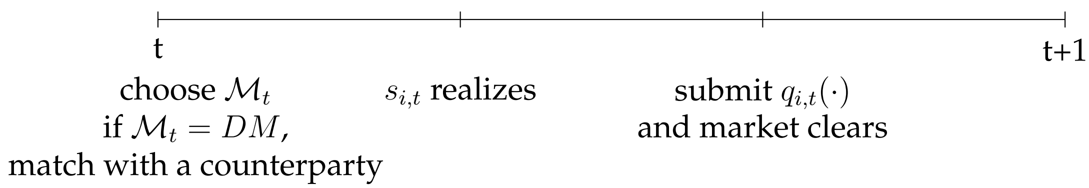
The centralized market clears with p_t when $\sum_i q_{i,t}(p_t) = 0$. In the decentralized market, after the traders are matched bilaterally in $N = \frac{I}{2}$ pairs, each pair $n \in N$ clears independently with $p_{t,n}$ when $\sum_{i\in I(n)} q_{i,t}(p_{t,n}) = 0$. This set-up of a decentralized market has a close mapping to markets in reality. In terms of the bond market, we use this decentralized trading mechanism to model the bilateral trade in the over-the-counter (OTC) market. In terms of the stock market, we use this decentralized market trading mechanism to mo-

[11] This assumption is a sufficient but not necessary condition to generate all the results in the paper. This is to avoid the nonmonotonicity of utility to σ, and to simplify the proof of Lemma 3. See Vives (2011) for a discussion of the nonmonotonic impact of σ.

del the dark pools operating as continuous non-displayed limit order books. This is the type of dark pool with the largest market share (around 70 percent) of total U.S. dark pool volumes in 2011 according to Tabb Group (2011). It includes many dark pools owned by major broker-dealers.[12]

Timing: We summarize the timing of each round with Figure 1.

Figure 1: Timing



3 Equilibrium

As the traders are short-lived, trading is static with a time-varying information set. Therefore, we are subject to solving the model round by round forwardly given price history $\mathcal{H}_t$. In each round, the problem is solved with backward induction. First, we solve the trading strategy given the market structure. Then, we solve each trader's optimal market structure choice, by comparing each trader's expected utility in CM and DM. We apply the tie-breaking rule of choosing CM when CM and DM give the trader the same utility. By symmetry, each trader will have the same market choice and ex-ante expected utility. Given the optimal market choice, we can determine the evolution of the price history. We focus on the Pareto-dominant linear Bayesian Nash equilibrium.

3.1 Second Stage Trading Equilibrium

Denote the chosen market structure as $\mathcal{M}^*$. By symmetry, choosing the market structure is equivalent to choosing the number of traders in the market $I_{t,\mathcal{M}^*}$ and the average correlation across traders $\rho_{t,\mathcal{M}^*}$. It is easy to see that $\mathcal{M}^* = CM$, the number of traders in the exchange is $I_{t,\mathcal{M}^*} = I_t$ with an average correlation between any trader and the residual market $\rho_{t,M^*} = \bar{\rho}$. If $\mathcal{M}^* = DM$, the number of traders in each pair is $I_{t,\mathcal{M}^*} = 2$ and every pair clears independently. Without solving the ex-ante expected utility, we will not be able to know each trader's choice of counterparty. For now, let us assume that the correlation

[12]There are other two types of dark pools, one derives price from the lit venues, and the other acts like fast electronic market maker (Tabb Group, 2011; Zhu, 2014).

within each pair (i,j) is $\rho_{t,\mathcal{M}^*} = \rho_{i,j}$ and solve the bilateral equilibrium. With the equilibrium strategy solved in the second stage, we can write the ex-ante utility as a function of $\rho_{i,j}$ in the first stage, and the trader $j \neq i$ that gives the trader i the highest ex-ante utility will be the trader i's counterparty.

Given the market structure $\mathcal{M}_t^*$, at round t, traders submit a demand schedule $q_{i,t}$ to maximize the utility

$$\max_{q_{i,t}} \mathbb{E}[\theta_{i,t} q_{i,t} - \frac{\alpha}{2}(q_{i,t})^2 - p_t q_{i,t} | \mathcal{H}_t, s_{i,t}, p_t]$$

By taking first order condition with respect to $q_{i,t}$, we can solve the trader i's demand schedule,

$$q_{i,t} = \frac{\mathbb{E}[\theta_{i,t}|\mathcal{H}_t, s_{i,t}, p_t] - p_t}{\alpha + \lambda_{i,t}} \tag{1}$$

where $\lambda_{i,t} \equiv \frac{dp_t}{dq_{i,t}}$ is the price impact. By symmetry, the price impacts are the same for all traders in the same round $\lambda_{i,t} = \lambda_t, \forall i \in I_{t,\mathcal{M}^*}$. We can parameterize $\mathbb{E}[\theta_{i,t}|\mathcal{H}_t, s_{i,t}, p_t] = c_{\mathcal{H},i,t}\mathcal{H}_t + c_{s,i,t}s_{i,t} + c_{p,i,t}p_t$. By symmetry, the inference coefficients are the same for all traders in the same round, $c_{\mathcal{H},i,t} = c_{\mathcal{H},t}$, $c_{s,i,t} = c_{s,t}$ and $c_{p,i,t} = c_{p,t}$.

In equilibrium, by market clearing condition, λ_t is equal to the inverse of the slope of the residual demand,

$$\lambda_t = (-\sum_{j\neq i} \frac{dq_{j,t}}{dp_t})^{-1} = \frac{\alpha + \lambda_t}{(I_{t,\mathcal{M}^*} - 1)(1 - c_{p,t})}$$

Given the parameterization, the equilibrium price is,

$$p_t = (1 - c_{p,t})^{-1}(c_{\mathcal{H},t}\mathcal{H}_t + c_{s,t}\bar{s}_t) \tag{2}$$

where $\bar{s}_t = \frac{1}{I_{t,\mathcal{M}^*}} \sum_i s_{i,t}$ is the average signal in the exchange (for DM, it is the average signal in each pair).

The trader i's value $\theta_{i,t}$, the equilibrium price p_t given equation (2), the history $\mathcal{H}_t$ and the private signal $s_{i,t}$ are joint normally distributed. By the projection theorem, the inference coefficients $c_{\mathcal{H},t}$, $c_{s,t}$, and $c_{p,t}$ can be determined given the joint distribution of $(\theta_{i,t}, s_{i,t}, \mathcal{H}_t, p_t)$.

Theorem 1 (Trading Equilibrium)**.** *The equilibrium price impact is*

$$\lambda_t = \frac{\alpha}{(I_{t,\mathcal{M}^*} - 1)(1 - c_{p,t}) - 1}, \quad \forall i$$

where $c_{p,t} = \frac{I_{t,\mathcal{M}^*}(\rho_{t,\mathcal{M}^*}-\eta_t)\sigma^2}{\left(1-\rho_{t,\mathcal{M}^*}+\sigma^2\right)\left(1+(I_{t,\mathcal{M}^*}-1)\rho_{t,\mathcal{M}^*}-I_{t,\mathcal{M}^*}\eta_t\right)}$*.* $\eta_t = \frac{var(\theta_{i,t})-var(\theta_{i,t}|\mathcal{H}_t)}{var(\theta_{i,t})} = \frac{\boldsymbol{\tau}_t'(\boldsymbol{\Upsilon}_t)^{-1}\boldsymbol{\tau}_t}{\sigma_\theta^2} \in \mathbb{R}$*,* $\boldsymbol{\tau}_t \equiv cov(\mathcal{H}_t, \theta_{i,t}) \in \mathbb{R}^{|\mathcal{H}_t|}$*, and* $\boldsymbol{\Upsilon}_t \equiv cov(\mathcal{H}_t, \mathcal{H}_t') \in \mathbb{R}^{|\mathcal{H}_t|\times|\mathcal{H}_t|}$*.*

The utility conditional on $\mathcal{H}_t$ *for trader* i *is*

$$\mathbb{E}[U_{i,t}^{\mathcal{M}^*}|\mathcal{H}_t] = \frac{\alpha+2\lambda_t}{2(\alpha+\lambda_t)^2}\mathbb{E}[(\mathbb{E}[\theta_{i,t}|\mathcal{H}_t, s_{i,t}, p_t]-p_t)^2|\mathcal{H}_t] = \frac{\alpha+2\lambda_t}{2(\alpha+\lambda_t)^2}\frac{I_{t,\mathcal{M}^*}-1}{I_{t,\mathcal{M}^*}}\frac{(1-\rho_{t,\mathcal{M}^*})^2}{1-\rho_{t,\mathcal{M}^*}+\sigma^2}, \quad \forall i$$

3.2 First Stage Market Choice

Given the trading equilibrium in Theorem 1, we can obtain the ex-ante utility of the traders. By comparing the ex-ante utility of traders in DM and CM, we can determine the optimal market choice.

Ex-ante Utility in CM: If the market structure is CM, the ex-ante utility for trader i is

$$\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t] = \frac{\alpha+2\lambda_t^{CM}}{2(\alpha+\lambda_t^{CM})^2}\frac{I-1}{I}\frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2} \quad \forall i \in I \tag{3}$$

where $\lambda_t^{CM} = \frac{\alpha}{(I-1)(1-c_{p,t})-1}$, $c_{p,t} = \frac{I(\bar{\rho}-\eta_t)\sigma^2}{(1-\bar{\rho}+\sigma^2)(1+(I-1)\bar{\rho}-I\eta_t)}$.

Ex-ante Utility in DM: For traders in the DM, we will need to first determine the trader i's counterparty a la Irving (1985). The trader j that gives trader i the highest utility is matched with trader i. Given that the traders $j \neq i$ are ex-ante identical except for their correlation with trader i, equivalently, this optimal choice of counterparty can be framed as the optimal choice of $\rho_{i,j}$ among the pairwise correlations $\{\rho_{i,j}\}_{j\neq i}$,

$$\max_{\rho_{i,j}|j\in I, j\neq i} \mathbb{E}[U_{i,t}^{DM}(\rho_{i,j})|\mathcal{H}_t]$$

Lemma 1 (Ex-ante Utility With Respect to Correlation Across Traders)**.** *Keeping everything else constant, the ex-ante utility* $\mathbb{E}[U_{i,t}^{\mathcal{M}}(\rho)|\mathcal{H}_t]$ *is decreasing in the correlation* ρ*.*

By Lemma 1, the trader j with lowest correlation with i is matched as i's counterparty. By assumption, only one trader j has the lowest correlation ρ_ℓ with trader i, so the algorithm a la Irving (1985) generates a unique matching result. Given the matching result, trader i's ex-ante utility in DM is

$$\mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t] = \frac{\alpha+2\lambda_t^{DM}}{4(\alpha+\lambda_t^{DM})^2}\frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2} \quad \forall i \in I \tag{4}$$

where $\lambda_t^{DM} = \frac{\alpha}{-c_{p,t}}$, $c_{p,t} = \frac{2(\rho_\ell-\eta_t)\sigma^2}{(1-\rho_\ell+\sigma^2)(1+\rho_\ell-2\eta_t)}$.

3.3 Price History Informativeness

One observation from Theorem 1 is that the impact of the price history $\mathcal{H}_t$ on the market choice can be summarized by a sufficient statistic, the informativeness of the price history to the traders $\eta_t = \frac{var(\theta_{i,t}) - var(\theta_{i,t}|\mathcal{H}_t)}{var(\theta_{i,t})}$.[13] It measures how much traders can learn about their values from the price history.

Given the above observation that the informativeness of the price history η_t governs the impact of the past market choices on the current market choice, we will first discuss the impact of η_t on the market choice before we analyze the dynamics. We have the following comparative statics results for η_t.

Lemma 2 (Comparative Statics With Price History Informativeness η (Rostek and Wu, 2024))**.** *Keeping everything else constant, when η increases, the price impact λ_t decreases; and the ex-ante expected utility for any trader i increases.*

Lemma 2 implies that higher price history informativeness improves liquidity and utility for both centralized and decentralized market. To understand the intuition of Lemma 2, we can decompose the trader i's utility into two parts, the liquidity effect and the gain from heterogeneous values,

$$\mathbb{E}[U_{i,t}^{\mathcal{M}^*}|\mathcal{H}_t] = \underbrace{\frac{\alpha + 2\lambda_t}{2(\alpha+\lambda_t)^2}}_{\text{liquidity effect}} \underbrace{\mathbb{E}[(\mathbb{E}[\theta_{i,t}|\mathcal{H}_t, s_{i,t}, p_t] - p_t)^2|\mathcal{H}_t]}_{\text{gain from heterogeneous values}}$$

First, price impact decreases as price history informativeness increases. A more informative price history reduces adverse selection by providing additional information to calibrate the asset value. Specifically, given a more informative price history, the residual market (the counterparty) requires a smaller price increase to sell an additional unit to the trader, thus lowering the price impact.

Second, the gain from heterogeneous values remains constant regardless of price history informativeness. This is because price history only informs about the value that remains constant across rounds, not the idiosyncratic shocks that determine the gain from heterogeneous values. Since the liquidity effect increases with price history informativeness while the gain from heterogeneous values remains constant, traders' expected utilities rise as price history becomes more informative.

[13]When $\mathcal{H}_t$ is a scalar, η_t is the square of the correlation between the price history $\mathcal{H}_t$ and any trader i's value $\theta_{i,t}$.

Given heterogeneous trader values, when the price history is sufficiently informative, the traders' expected utility can be higher in DM than in CM.

Lemma 3 (Optimal Market Choice at Round t)**.** *Given $\mathcal{H}_t$, at round t,*

1. $\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t] - \mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$ *is decreasing in η_t;*
2. *if $\bar{\rho} \leq \bar{\rho}^* (I, \rho_\ell, \sigma^2)$, then any trader i will choose CM;*
3. *if $\bar{\rho} > \bar{\rho}^* (I, \rho_\ell, \sigma^2)$, there exists $\tilde{\eta} (I, \bar{\rho}, \rho_\ell, \sigma^2)$ the any trader i will choose CM if $\eta_t \leq \tilde{\eta}$, and otherwise if $\eta_t > \tilde{\eta}$.*

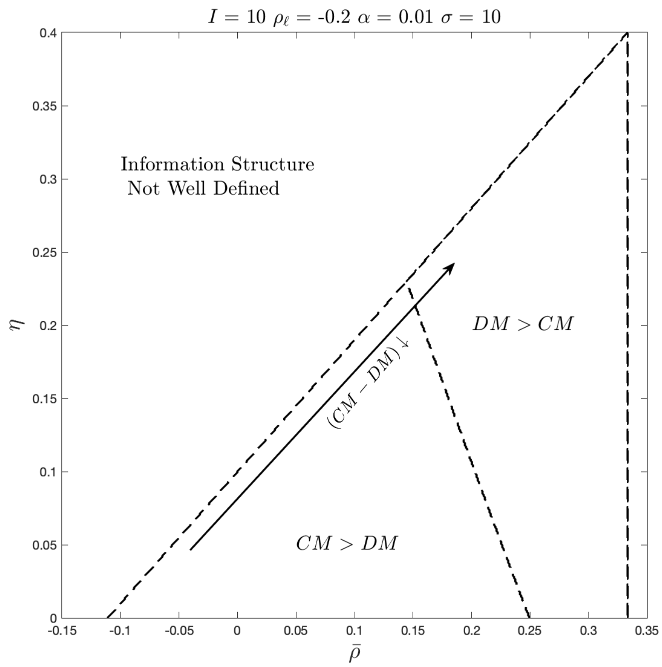
Figure 2 serves as an example for Lemma 3. It shows the comparison of current round utility in CM versus DM with respect to trader value correlation and past price informativeness. We can see that when $\bar{\rho}$ is sufficiently low, the utility in CM is always higher than DM. When $\bar{\rho}$ is high, there exists $\tilde{\eta}$ such that DM gives traders higher utility than CM. As η increases, the utility difference between CM and DM decreases.

To understand Lemma 3, we can start with the intuition in static models without price history (Rostek and Weretka, 2012; Yoon, 2017; Rostek and Wu, 2021; Babus and Parlatore, 2022). The horizontal axis of Figure 2 represents the static case $T = 1$. The trade-off between CM and DM is between market size and the value correlation with counterparty. CM always has a higher liquidity effect than the bilateral DM given a larger market size.[14] DM is better than CM in terms of counterparty. The gain from heterogeneous values reflects the highest potential gain from trade without price impacts with the residual market. In DM, the residual market is the best counterparty in DM - the one with the lowest correlation ρ_ℓ. In CM, traders not only interact with their best counterparty but also compete with $I - 2$ other counterparties with correlation ρ_h. The residual market in CM can be viewed as $I - 1$ average counterparties with correlation $\bar{\rho}$. When the best counterparty is sufficiently different from the average counterparty, i.e. the correlation ρ_ℓ is sufficiently lower than $\bar{\rho}$, the gain from heterogeneous value is lower in CM compared to DM.

In a dynamic model, having access to price history changes the static trade-off. Figure 2 shows that price history informativeness η expands another dimension that is independent of the value correlation $\bar{\rho}$. Price history enhances utility in DM more than that in CM.

[14] While empirically transaction volume is widely used to measure liquidity, we want to clarify that the liquidity effect in this paper refers merely to the effect due to price impact but not the transaction volume. Equation (1) implies that the transaction volume is determined by both liquidity and gain from heterogeneous values. Ex-ante, the variance of the transaction volume can be higher in DM than in CM when the gain from heterogeneous values is sufficiently large. Therefore, we may see higher transaction volume ex-post with lower liquidity (high price impact) in the DM than in the CM.

Figure 2: Utility in CM vs. DM With Traders' Value Correlation and Price History Informativeness



Note: This figure shows the comparison of utility (welfare) in CM versus DM with traders' value correlation and price history informativeness η. When the price history informativeness is higher or when the value correlation across all traders is higher, the difference between utilities in DM and CM becomes larger.

Higher price history informativeness improves liquidity in both markets, as demonstrated by Lemma 2, with the improvement being more pronounced in the DM. This is because the CM already boasts high liquidity, leaving less room for enhancement. With sufficiently heterogeneous correlation, i.e., $\bar{\rho} > \bar{\rho}^* (I, \rho_\ell, \sigma^2)$, the benefit from trading exclusively with the best counterparty is substantial. With high enough price history informativeness η, the loss of liquidity in DM is marginal. Consequently, traders tend to favor DM, prioritizing the optimal counterparty even if it means a slight sacrifice in liquidity.

Lemma 3 implies that, given sufficiently heterogeneous value correlation, when price history is sufficiently informative, the traders have incentives to switch to DM. This intuition applies to the analysis of dynamic market choices. In each round, we only need to compare the correlation $\bar{\rho}$ with the threshold $\bar{\rho}^* (I, \rho_\ell, \sigma^2)$ and the price history informativeness with its threshold $\tilde{\eta} (I, \bar{\rho}, \rho_\ell, \sigma^2)$ to determine the market choice.

3.4 Dynamic Equilibrium

Given the expected utility in DM and CM characterized by equations (3) and (4), if $\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t] \geq \mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$, then the optimal market choice at round t is CM and the price history $\mathcal{H}_{t+1} = \mathcal{H}_t \cup \{p_t\}$, otherwise, the optimal market choice at round t is DM and $\mathcal{H}_{t+1} = \mathcal{H}_t$. We have the following recursive algorithm to generate the equilibrium of dynamic market choice through updates of $\mathcal{H}_t$,

Theorem 2 (Algorithm for Dynamic Market Choice Equilibrium)**.** *The Bayesian Nash equilibrium is a set of price history $\{\mathcal{H}_t\}_t$, a sequence of market choice $\{\mathcal{M}_t^*\}_t$, and a set of inference coefficients $\{c_{s,t}, c_{p,t}, \boldsymbol{c}_{\mathcal{H},t}\}$ that is characterized forwardly recursively.*

1. *Initialize with $t = 1$, $\mathcal{H}_1 = \emptyset$.*

2. *Given $\mathcal{H}_t$, the equilibrium inference coefficients $\{c_{s,t}, c_{p,t}, \boldsymbol{c}_{\mathcal{H},t}\}$ is characterized in Theorem 1 with $\rho_{t,\mathcal{M}^*} = \rho_\ell$ $I_{t,\mathcal{M}^*} = 2$ if $\mathcal{M}^* = DM$, and $\rho_{t,\mathcal{M}^*} = \bar{\rho}$ $I_{t,\mathcal{M}^*} = I$ if $\mathcal{M}^* = CM$.*

3. *Given inference coefficients $\{c_{s,t}, c_{p,t}, \boldsymbol{c}_{\mathcal{H},t}\}$, If (i) $\bar{\rho} \leq \bar{\rho}^*(I, \rho_\ell, \sigma^2)$, or (ii) $\bar{\rho} > \bar{\rho}^*(I, \rho_\ell, \sigma^2)$ and $\eta_t \leq \tilde{\eta}(I, \bar{\rho}, \rho_\ell, \sigma^2)$ (equivalently $\frac{\alpha+2\lambda_t^{CM}}{2(\alpha+\lambda_t^{CM})^2} \frac{I_t-1}{I_t} \frac{(1-\bar{\rho}_t)^2}{1-\bar{\rho}_t+\sigma^2} \geq \frac{\alpha+2\lambda_t^{DM}}{4(\alpha+\lambda_t^{DM})^2} \frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2}$), then $\mathcal{M}_t^* = CM$, $\mathcal{H}_{t+1} = \mathcal{H}_t \cup \{p_t\}$; otherwise, $\mathcal{M}_t^* = DM$, $\mathcal{H}_{t+1} = \mathcal{H}_t$. Repeat Steps 2-3 with the next t, until t=T.*

The proof of Theorem 2 is immediate from the above analysis.

4 Dynamic Market Choice

In this section, we will explore how market choices evolve dynamically. In particular, we are interested in how trader value correlations and asset properties affect the dynamic market choice.

4.1 Constant Market Choice

In this part, we discuss sufficient conditions for traders to choose only one market structure in all rounds.

Homogeneous Correlation: First, let us consider a simple case where the traders' value correlations are homogeneous. In this case, traders will always choose CM.

Proposition 1 (Homogeneous Correlation)**.** *When the traders value correlations are sufficiently homogeneous $\bar{\rho} \leq \bar{\rho}^*(I, \rho_\ell, \sigma^2)$, traders will always stay in the CM.*

The proof of Proposition 1 directly follows from Lemma 3, as no price history informativeness η will allow traders to choose DM. When the lowest correlation and the correlation with all other traders are similar, the benefit of trading with one counterparty with lower adverse selection in the DM is dominated by the loss of lower liquidity, regardless of the price history. Thus the traders have no incentive to choose DM in any rounds.

Sufficiently Heterogeneous Correlation: On the opposite side, when the traders' value correlation is sufficiently heterogeneous, traders will always choose DM. If traders choose DM in the first round, they will choose DM for all rounds.

Lemma 4 (DM persisitency)**.** *Keep everything else constant, if $\mathcal{M}_1^* = DM$, then $\mathcal{M}_t^* = DM$ for all t.*

Proof. Suppose traders choose DM over CM in round 1. It is easy to see that $\eta_1 = 0$. Given that in each round primitives $(I, \bar{\rho}, \rho_\ell, \sigma^2)$ are the same, this means they prefer DM over CM if $\eta_t = 0$. As DM is opaque, if $\eta_t = 0$ and traders choose DM at round t, then $\eta_{t+1} = 0$. This implies traders will always choose DM by forward induction. ■

Given Lemma 4, if we find sufficiently heterogeneous correlation makes traders choose DM in the first round, then they will stay in DM for all rounds.

Proposition 2 (Sufficiently Heterogeneous Correlation)**.** *There exists $\underline{\rho}_\ell < 0$ and $\bar{\bar{\rho}} > \bar{\rho}^*$ such that for any $\rho_\ell < \underline{\rho}_\ell$ and $\bar{\rho} > \bar{\bar{\rho}}$, traders will stay in the DM for all rounds.*

Propositions 1 and 2 are consistent with Yoon (2017), where the traders' heterogeneous value correlation is crucial for traders to choose DM. Note that the condition for Propositions 2 is the same in both static ($T = 1$, Yoon (2017)) and dynamic models ($T \geq 2$) given Lemma 4. However, the threshold $\bar{\rho}^*$ for Proposition 1 in the dynamic model ($T \geq 2$) is lower than that in the static model ($T = 1$). This is because correlation should be more homogeneous to keep traders in a centralized market with a longer price history. Appendix Section B.3 provides simulations of the market choices as the number of rounds increases.

4.2 Alternating Market Choices

The equilibrium becomes more interesting when we the traders' value correlations are neither too homogenous nor too heterogeneous. Alternating between CM and DM can emerge endogenously as the optimal market choice. It is also worth mentioning that the optimal market choice generates the overall highest welfare, and Pareto dominates other market choices. We find that the asset properties, including asset sensitivity to shocks to fundamentals ξ, and the fundamental value volatility inversely related to autocorrelation κ, are crucial for the market choices.

Proposition 3 (Heterogeneous Correlation and Asset Sensitivity)**.** *With heterogeneous correlation $\bar{\rho} > \bar{\rho}^* (I, \rho_\ell, \sigma^2)$, $\rho_\ell \geq 0$, $\epsilon < \bar{\epsilon}(\sigma^2, I)$, $3 \leq T < \bar{T}(\sigma^2, I, \epsilon)$, and $\kappa < \bar{\kappa}(\sigma^2, I, \epsilon)$, there exists $\underline{\xi}$ and $\bar{\xi}$ such that such that traders will choose CM in the first round, and*

1. *When the asset sensitivity to shocks to fundamentals is sufficiently low $\xi \in [0, \underline{\xi})$, the traders shift to DM in the second round and stay there.*
2. *When the asset sensitivity to shocks to fundamentals is intermediate $\xi \in [\underline{\xi}, \bar{\xi})$, the traders will alternate between CM and DM.*
3. *When the asset sensitivity to shocks to fundamentals is sufficiently high $\xi \in [\bar{\xi}, \infty)$, the traders will always stay in the CM.*

Intuitively, when traders' value correlations are heterogeneous, i.e., when $\bar{\rho}$ and $\bar{\rho}_\ell$ are sufficiently different, the traders have incentives to shift to DM by our previous analysis. To further understand this result with respect to asset sensitivity, let us consider the following three-round example.

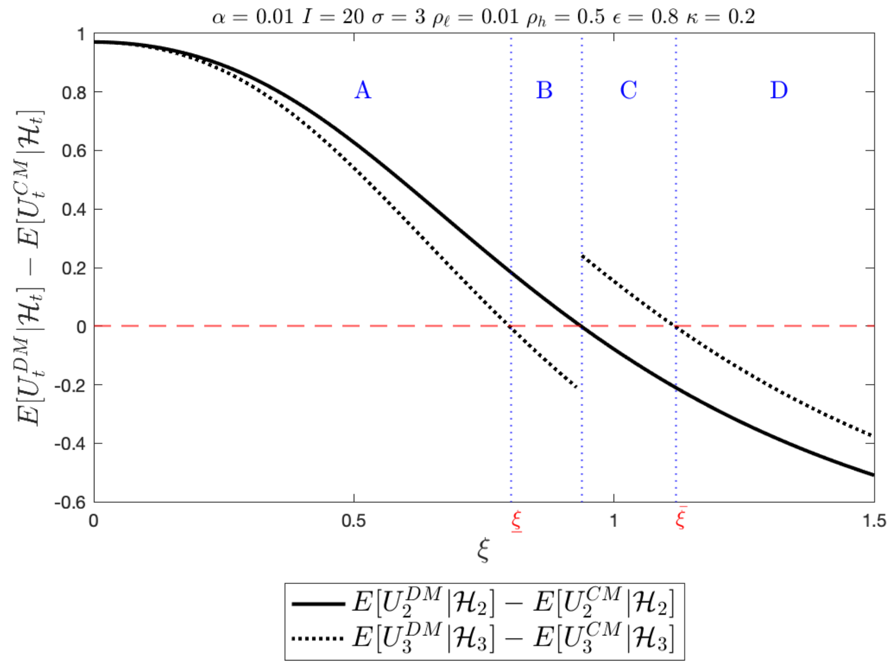
Example 1 (Three-round Market)**.** *We consider a market with $T = 3$. Assume that $\rho_\ell > 0$, such that in the 1st round the DM does not exist and the traders will always choose CM. Assume also that any pair of traders that do not have correlation ρ_ℓ have value correlation $\rho_h > \rho_\ell$.*

Figure 3 shows the market choice of the traders in the 2nd and 3rd round. In the Appendix, we also provided the price history informativeness in the 2nd and 3rd rounds with respect to ξ.

***Region A**: When the asset sensitivity is low, this implies the asset value is less susceptible to shocks to fundamentals and more correlated across rounds. This also implies that the price history is more informative to the traders. A more informative price history can lower the price impact and increase liquidity effect and utility. Such an increase is higher for DM than CM, as CM is already very liquid thus leaving less room for liquidity improvement. The higher liquidity improvement in DM can decrease the loss of liquidity effect for choosing DM and be dominated by the gain in heterogeneous values. This gives rise to a shift to DM in the 2nd round.*

When the asset sensitivity is sufficiently low, $\xi \in [0, \underline{\xi})$, traders will continue to stay in DM in the 3rd round, as the 1st round price is still informative enough for them to trade with better counterparty at just a bit higher price impact in DM.

Figure 3: Dynamic Market Choice With Respect to Asset Sensitivity ξ in $T = 3$ Market



Note: The black solid line plots the difference between the ex-ante expected utility of DM and that of CM in the 2nd round, and the black dotted line plots that difference in the 3rd round. The red dashed line is a reference line of 0. When the black solid(dotted) line is above the reference line, then the traders choose DM in the 2nd round(3rd round), and if it is below the reference line, the traders choose CM in the 2nd round(3rd round). The jump in the difference of utility in CM vs. DM in the third round comes from the difference in the second-round choice.
In region A, i.e. $\xi \in [0, \underline{\xi})$, traders choose DM in both 2nd and 3rd round. In region B, i.e. $\xi \in (\underline{\xi}, \bar{\xi}]$ and in the lower partition, traders choose DM in the 2nd round and CM in the 3rd round. In region C, $\xi \in (\underline{\xi}, \bar{\xi}]$ and in the higher partition, traders choose CM in the 2nd round and DM in the 3rd round. In region D, $\xi \in (\bar{\xi}, 1]$, traders choose CM in both the 2nd round and the 3rd round.

Region B*: However, when the asset sensitivity is not sufficiently low, i.e.,* $\xi \in [\underline{\xi}, \bar{\xi})$*, traders will alternate between DM and CM. traders will choose DM in the second round and choose CM in the 3rd round. As the asset value is not that stable across time and traders do not know the DM price, the price in the 1st round becomes stale and not informative enough for the 3rd round values. The liquidity difference in CM and DM again becomes large, making traders shift back to CM for liquidity improvement.*

Region C*: When the asset sensitivity is high but not high enough, i.e.,* $\xi \in [\underline{\xi}, \bar{\xi})$ *and higher than that in Case 2, traders will still alternate between DM and CM. Traders will choose CM in the first two rounds and shift to DM in the last round. This is because the asset sensitivity is not low enough such that traders will choose CM in the second round for higher liquidity. However,* *the asset sensitivity is low enough such that the price history is sufficiently informative in the 3rd round when traders see both the 1st round and 2nd round prices in CM. In the 3rd round, the liquidity improvement in DM is sufficiently higher than that in CM. The traders shift to DM in the 3rd round for a better counterparty.*

***Region D**: When the asset sensitivity is sufficiently high, i.e., $\xi \in [\bar{\xi}, \infty)$, traders will stay in the CM for both the 2nd and 3rd round. This is because the value of assets changes frequently across time, making price history not informative enough to largely boost the liquidity in the DM. Therefore, the liquidity difference between the DM and CM remains large, preventing the traders from choosing DM for the benefit of the best counterparty.*

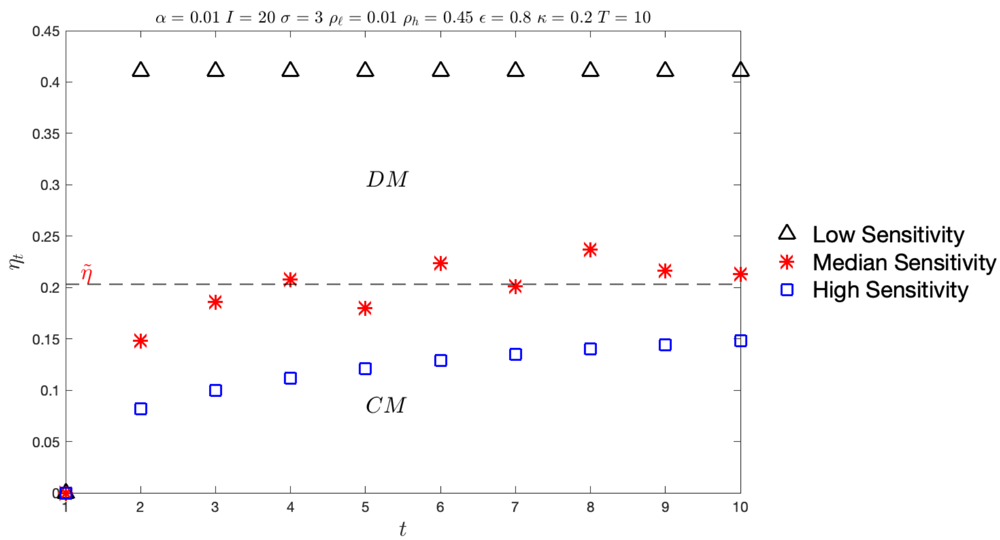
To summarize, Example 1 shows how the asset sensitivity affects the informativeness of the price history η, and then affects the liquidity effect and therefore current market choice. Past prices in the CM increase η and lower price impact. Better counterparties attracts traders to the DM. However, DM opacity can lower future price history informativeness η and push traders back to the CM. These intuitions from Example 1 can be extended to more than 3 rounds. Figure 4 shows the evolution of price history informativeness and market choice with $T = 10$ with respect to different levels of asset sensitivities. When the marker is above (below) the reference line $\tilde{\eta}$ which is defined by Lemma 3 and calculated according to trader value correlations, then traders choose DM (CM).

We would like to clarify that the mechanism for alternating market choice does not come from the tie-breaking rule which we do not impose any indeterminacy. It also differs from the mechanism as in Yoon (2017) where (i) traders in the DM do not access CM price; and (ii) marginal trader's (weak) indifference between DM and CM gives rise to coexistence. In this paper, traders choose DM when DM gives them a strictly higher utility than CM. DM emerges endogenously as a result of learning from price history, and fades endogenously when the price history becomes uninformative.

Proposition 3 is consistent with our real life observations. Securities that are designed to be insensitive to issuers' fundamentals, like bonds, are firstly traded in the centralized primary market and then mostly traded in the secondary over-the-counter market. Securities that are relatively more sensitive to issuers' fundamentals, like equities, are mostly traded in the centralized market, sometimes traded in dark pools. Securities that by design are most sensitive to issuers' fundamentals, like options, are only traded in the centralized market.

Proposition 4 (Heterogeneous Correlation and Autocorrelation)**.** *With heterogeneous correlation $\bar{\rho} > \bar{\rho}^*(I, \rho_\ell, \sigma^2)$, $\epsilon < \bar{\epsilon}(\sigma^2, I)$, $3 \leq T < \bar{T}(\sigma^2, I, \epsilon)$, and $\xi > \underline{\xi}(\sigma^2, I, \epsilon)$, there exists $\underline{\kappa}$ and*

Figure 4: Evolution of Price History Informativeness For Different Asset Sensitivities



Note: This figure shows the evolution of price history informativeness for different levels of asset sensitivity ξ for $T = 10$. The black dashed line is a reference line of threshold $\tilde{\eta}$. When the marker is above the reference line, then the history informativeness in that round is higher than $\tilde{\eta}$ and traders choose DM. If the marker is below the reference line, then the history informativeness in that round is lower than $\tilde{\eta}$ and traders choose CM.

$\bar{\kappa}$ such that traders will choose CM in the first round, and

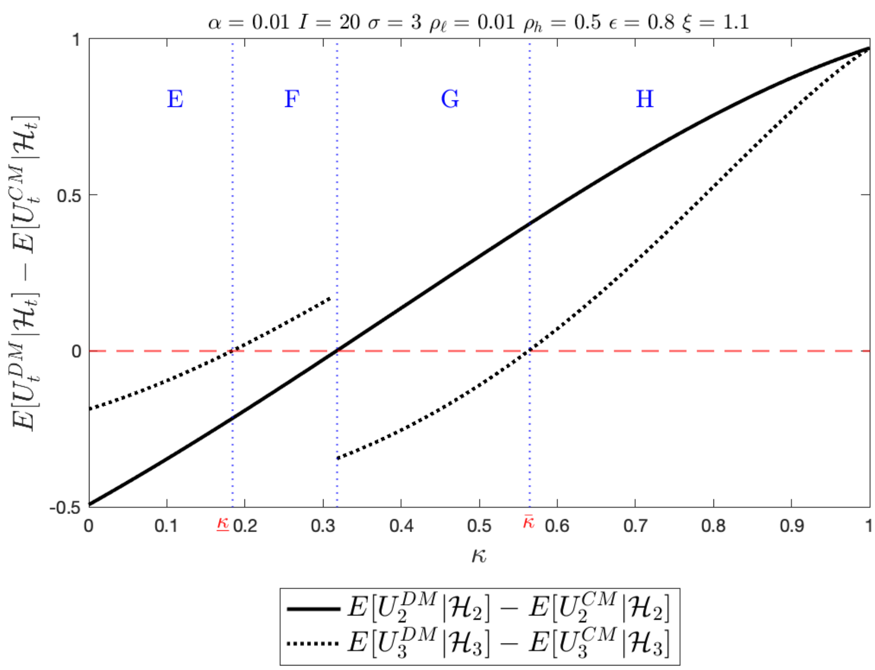
1. *When the autocorrelation is sufficiently low $\kappa \in [0, \underline{\kappa}]$, the traders will always stay in the CM.*

2. *When the autocorrelation is intermediate $\kappa \in (\underline{\kappa}, \bar{\kappa}]$, the traders will alternate between CM and DM.*

3. *When the autocorrelation is sufficiently high $\kappa \in (\bar{\kappa}, 1]$, the traders will choose DM over CM in the second round and never choose CM again.*

Figure 5 shows the market choice of the traders in the 2nd and 3rd round in Example 1 with respect to autocorrelation κ. In the Appendix, we also provided the price history informativeness in the 2nd and 3rd rounds with respect to κ. Similar to the analysis of Proposition 3, the intuition for Proposition 4 also works through the dynamics of the price history informativeness η. The price history informativeness η is increasing in autocorrelation κ. When autocorrelation is higher, this means the values are less volatile across rounds, the price history is more informative, and the traders are more likely to shift to DM. The intuition of Example 1 also applies to a market with more rounds. Figure 6 shows the evolution of price history informativeness and market choice for a $T = 10$ round market with respect to different levels of autocorrelation. When the marker is above (below) the reference line $\tilde{\eta}$ which is defined by Lemma 3 and calculated according to trader value correlations, then traders choose DM (CM).

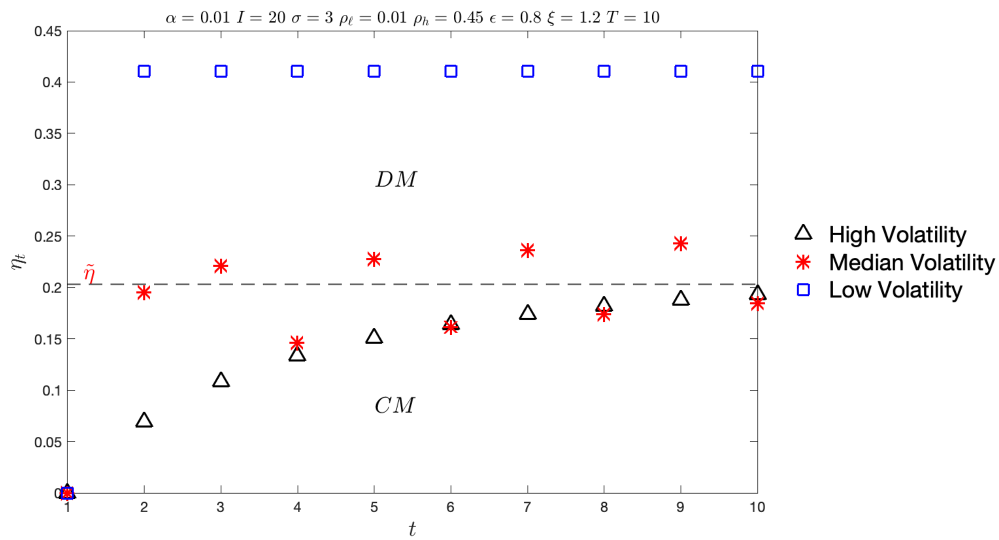
Figure 5: Dynamic Market Choices With Respect to Autocorrelation κ in $T = 3$ Market



Note: The black solid line plots the difference between the ex-ante expected utility of DM and that of CM in the 2nd round, and the black dotted line plots that difference in the 3rd round. The red dashed line is a reference line of 0. When the black solid (dotted) line is above the reference line, then the traders choose DM in the 2nd round (3rd round), and if it is below the reference line, the traders choose CM in the 2nd round (3rd round). The jump in the difference of utility in CM vs. DM in the third round comes from the difference in the second-round choice.
In region E, i.e. $\kappa \in [0, \underline{\kappa}]$, traders choose CM in both 2nd and 3rd round. In region F, i.e. $\kappa \in (\underline{\kappa}, \bar{\kappa}]$ and in the lower partition, traders choose CM in the 2nd round and DM in the 3rd round. In region G, $\kappa \in (\underline{\kappa}, \bar{\kappa}]$ and in the higher partition, traders choose DM in the 2nd round and CM in the 3rd round. In region H, $\kappa \in (\bar{\kappa}, 1]$, traders choose DM in both the 2nd round and the 3rd round.

The autocorrelation κ captures the volatility of the fundamentals. Proposition 4 implies that when the shocks are less volatile, i.e. high κ, then the traders are more likely to trade in DM. The implication of Proposition 4 is consistent with some existing empirical literature. Both Menkveld et al. (2017) and Buti et al. (2022) find that the market share of the dark pools (corresponding to DM in our model) relative to the lit venues (corresponding to CM in our model) decreases when the market is more volatile.

Figure 6: Evolution of Price History Informativeness For Different Autocorrelations



Note: This figure shows the evolution of price history informativeness for different levels of autocorrelation κ for $T = 10$. The black dashed line is a reference line of threshold $\tilde{\eta}$. When the marker is above the reference line, then the history informativeness in that round is higher than $\tilde{\eta}$ and traders choose DM. If the marker is below the reference line, then the history informativeness in that round is lower than $\tilde{\eta}$ and traders choose CM.

We also want to clarify the difference between asset sensitivity and volatility. Even if the issuer's fundamental has high volatility, it is possible for the issuer to design securities that have low asset sensitivity to be traded in the DM.

4.3 Discussions and Extensions

General Private Information Precision: In the baseline model, traders receive private signals with constant precision. Appendix Section B.1 extends the model to include varying precision of private information. We find that traders are more likely to choose CM after rounds with less precise private signals. Additionally, we demonstrate that traders can still alternate between CM and DM, even with post-trade transparent DM, when precise signals are infrequent.

Non-movers: In the baseline model, we focus on the behavior of traders who move across venues without frictions. This generates a pattern where all traders either choose CM or DM. In practice, some traders only have access to one type of the markets. For example, the retail traders in the U.S. equity market usually do not trade in the dark pools, and the bank dealers are prohibited from trading in the centralized bond market in China.

Appendix Section B.2 provides an extension to accommodate the non-movers in the market without changing the mechanisms and qualitative results in the baseline model.

Proportion of Time in CM: Appendix Section B.3 provides the analysis on the proportion of time traders choose CM over DM with sensitivity and volatility. Consistent with our intuition for Propositions 3 and 4, we find the proportion of time traders choosing CM is positively correlated with asset sensitivity and volatility.

Alternative Tie-breaking Rule: In this paper, we do not allow traders to choose DM and CM in the same round. Following Yoon (2017), we can allow the traders to choose DM or CM until no trader would like to deviate to the other market. Note that our results still hold qualitatively with this new tie-breaking rule, as price history informativeness η can still evolve endogenously with traders' past market choices and in turn determines traders future choices.

5 Market Designs with Endogenous Market Choices

In previous sections, we endogenize the traders' market choices given their value correlations and asset properties. Most of the literature focuses on comparing market designs with a fixed number of traders in each market. We may wonder how to improve market efficiency taking into account the flow of traders across venues. In this section, we will revisit some popular market designs given the endogenous market choice.

5.1 Transparency

So far, we have assumed that DM is opaque, i.e. future traders cannot see prices in DM and traders in DM cannot see prices in other pairs. In this section, we will consider introducing transparency designs in DM.

It is of policy interest to discuss the impact of transparency on market structures and welfare. In reality, traders have post-trade transparency in some decentralized markets, e.g. TRACE in the bond market, and blockchain technology in the crypto market. Some decentralized trading mechanism allows pre-trade transparency, e.g. request-for-quote. However, some decentralized markets are relatively opaque, e.g. dark pools for equities. The lack of transparency in dark pools has received critique and policy attention. However, the impact of introducing transparency to dark pools remains unclear. Our dynamic model allows us to explore the impact of transparency designs on traders' market choices and welfare.

5.1.1 Post-trade Transparency

In this section, we will consider introducing post-trade transparency to DM, i.e., prices in DM will enter the price history and affect future market choices. This definition of post-trade transparency follows Vairo and Dworczak (2023) and Rostek et al. (2024).

It is easy to see that Theorem 1 still applies to equilibrium with post-trade transparency. Denote the number of trading pairs in the DM as $N = \frac{I}{2}$, and each trading pair as n. We can slightly modify the price updating rule in Theorem 2 to characterize the new equilibrium.

Theorem 3 (Algorithm for Dynamic Market Choice Equilibrium with Post-trade Transparency)**.** *The Bayesian Nash equilibrium is a set of price history* $\{\mathcal{H}_t\}_t$*, a sequence of market choice* $\{\mathcal{M}_t^*\}_t$*, and a set of inference coefficients* $\{c_{s,t}, c_{p,t}, \boldsymbol{c}_{\mathcal{H},t}\}$ *that is characterized forwardly recursively.*

1. *Initialize with* $t = 1$, $\mathcal{H}_1 = \emptyset$.
2. *Given* $\mathcal{H}_t$*, the equilibrium inference coefficients* $\{c_{s,t}, c_{p,t}, \boldsymbol{c}_{\mathcal{H},t}\}$ *is characterized in Theorem 1 with* $\rho_{t,\mathcal{M}^*} = \rho_\ell\ I_{t,\mathcal{M}^*} = 2$ *if* $\mathcal{M}^* = DM$*, and* $\rho_{t,\mathcal{M}^*} = \bar{\rho}\ I_{t,\mathcal{M}^*} = I$ *if* $\mathcal{M}^* = CM$.
3. *Given inference coefficients* $\{c_{s,t}, c_{p,t}, \boldsymbol{c}_{\mathcal{H},t}\}$*, If* $\frac{\alpha+2\lambda_t^{CM}}{2(\alpha+\lambda_t^{CM})^2}\frac{I_t-1}{I_t}\frac{(1-\bar{\rho}_t)^2}{1-\bar{\rho}_t+\sigma^2} \geq \frac{\alpha+2\lambda_t^{DM}}{4(\alpha+\lambda_t^{DM})^2}\frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2}$*, then* $\mathcal{M}_t^* = CM$, $\mathcal{H}_{t+1} = \mathcal{H}_t \cup \{p_t\}$*; otherwise,* $\mathcal{M}_t^* = DM$, $\mathcal{H}_{t+1} = \mathcal{H}_t \cup \{p_{n,t}\}_n$*, where* $p_{n,t}$ *is the equilibrium price of bilateral trading pair* n*. Repeat Steps 2-3 with the next* t*, until t=T.*

The proof of Theorem 3 follows the analysis in Section 3.

First, we explore the impact of post-trade transparency on traders' optimal market choice. Perhaps surprisingly, we find that with post-trade transparency, traders will stay in DM once they have chosen it. This is because the price history informativeness η never decays, attracting traders to stay in DM.

Proposition 5 (Post-trade Transparency: Once DM, Always DM)**.** *With post-trade transparency, if* $\mathcal{M}_t^* = DM$*, then* $\mathcal{M}_\tau^* = DM$, $\forall \tau \geq t$.

By Proposition 5, the potential dynamic market choices will be (i) choosing DM for all rounds; (ii) choosing CM at first and DM thereafter; and (iii) choosing CM for all rounds. Alternating back and forth between DM and CM is no longer an optimal dynamic market choice. Note that this result is different from Lemma 4 which only describes one possible market choice, i.e., DM persists when traders choose DM in the first round. Proposition 5 implies that if we introduce post-trade transparency in dark pools, the traders will not return to the centralized market. Note that Proposition 5 holds only when the private signals and trading rounds arrive regularly, which may not hold in reality. If we slightly modify the model by allowing precise private signals to arrive less frequently, then the price history informativeness naturally decays during these long no-trade or uninformed-trade periods, and traders can return to CM from DM even with post-trade transparency (see Appendix Section B.1).

Still, regardless of its impact on the market choice, post-trade transparency in DM weakly increases overall welfare.

Proposition 6 (Post-trade Transparency Improves Welfare)**.** *Post-trade transparency weakly improves welfare regardless of market choices.*

Post-trade transparency in DM does not affect the utility of traders when they choose CM, but can weakly increase welfare when they choose DM. The intuition is as follows. η_t^{post} with post-trade transparency will always be weakly higher than η_t without post-trade transparency, as the DM prices are informationally equivalent to the average signal of each bilateral pair, which is at least as informative as the centralized market price in the same round if traders choose CM without post-trade transparency. Given $\eta_t^{post} \geq \eta_t$, any market choice without post-trade transparency will not give traders higher utility than DM with post-trade transparency.

5.1.2 Pre-trade Transparency

In this section, we will consider introducing pre-trade transparency in DM. The definition of post-trade transparency follows Rostek et al. (2024). We allow traders in each pair to not only submit demand schedules contingent on their price but also the prices in other pairs. Their demand schedule in DM at round t will be $q_{i,t}(\boldsymbol{p}_t) : \mathbb{R}^N \to \mathbb{R}$, where $\boldsymbol{p}_t \in \mathbb{R}^N$ is the vector all prices in all pairs whose n^{th} element is the price in pair n at round t, $p_{n,t}$. For tractability, besides that each trader will have a correlation ρ_ℓ with only one trader, we further assume that each trader has a correlation ρ_h with all other traders in the same round.

Equilibrium Characterization: It is easy to see that given history $\mathcal{H}_{i,t}$, the trading equilibrium in CM will not be affected by the pre-trade transparency in DM. We can still apply

Theorem 1 to characterize CM equilibrium. We need to solve for the new trading equilibrium for DM.

With pre-trade transparency, traders in the DM will have access to prices from other pairs and submit demand schedules contingent on them. Trader $i \in I(n)$ submit demand schedule $q_{i,t}(\boldsymbol{p}_t) : \mathbb{R}^N \to \mathbb{R}$ to maximize the expected utility conditional on the history $\mathcal{H}_t$, private signal $s_{i,t}$, and

$$\max_{q_{i,t}(\boldsymbol{p}_t)} \mathbb{E}[\theta_{i,t} q_{i,t} - \frac{1}{2}\alpha q_{i,t}^2 - p_{n,t} q_t | \boldsymbol{p}_t, \mathcal{H}_t, s_{i,t}]$$

trader i 's first-order condition as

$$q^i(\boldsymbol{p}_t) = \frac{\mathbb{E}[\theta_{i,t}|\boldsymbol{p}_t, \mathcal{H}_t, s_{i,t}] - p_{n,t}}{\alpha + \lambda_{i,t}}$$

where $\lambda_{i,t}$ is the trader i's price impact within pair n. Trader i also has cross-pair price impact as traders from other pairs will change their bids when price p_n changes with i's bid. Trader i's price impact over all pairs can be described with a price impact matrix $\boldsymbol{\Lambda}_{i,t} = (\frac{d\boldsymbol{p}}{d\boldsymbol{q}_{i,t}}) \in \mathbb{R}^{N\times N}$, where the n^{th} diagonal elements is $\lambda_{i,t}$. Each trader i 's price impact matrix is equal to the transpose of the Jacobian of trader i's inverse residual supply:

$$(\boldsymbol{\Lambda}_{i,t})' = \left(-\sum_{j\neq i} \frac{d\boldsymbol{q}_{j,t}}{d\boldsymbol{p}_t}\right)^{-1}$$

We can parameterize $\mathbb{E}[\theta_{i,t}|\boldsymbol{p}_t, \mathcal{H}_t, s_{i,t}] = \boldsymbol{c}_{\mathcal{H},i,t}\mathcal{H}_t + c_{s,i,t}s_{i,t} + \boldsymbol{c}_{p,i,t}\boldsymbol{p}_t$. $\boldsymbol{c}_{\mathcal{H},i,t} \in \mathbb{R}^{1\times|\mathcal{H}_t|}$, $c_{s,i,t} \in \mathbb{R}$, and $\boldsymbol{c}_{p,i,t} \in \mathbb{R}^{1\times N}$. Given symmetry within each pair, $\boldsymbol{c}_{\mathcal{H},i,t} = \boldsymbol{c}_{\mathcal{H},n,t}$, $c_{s,i,t} = c_{c,n,t}$, $\boldsymbol{c}_{p,i,t} = \boldsymbol{c}_{p,n,t}$ and $\lambda_{i,t} = \lambda_{n,t}$.

Given the market clearing condition, $\sum_{i\in I(n)} q_{i,t}(\boldsymbol{p}_t) = 0$, and trader symmetry within exchanges, we have the equilibrium price in all pairs in vector form,

$$\boldsymbol{p}_t = (\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1}(\boldsymbol{C}_{\mathcal{H},t}\mathcal{H}_t + \boldsymbol{C}_{s,t}\bar{\boldsymbol{s}}_t),$$

where $\boldsymbol{C}_{s,t} = \operatorname{diag}(c_{s,n,t})_n \in \mathbb{R}^{N\times N}, \boldsymbol{C}_{\mathcal{H},t} = (\boldsymbol{c}_{\mathcal{H},n,t})_n \in \mathbb{R}^{N\times|\mathcal{H}_t|}, \boldsymbol{C}_{p,t} = (\boldsymbol{c}_{p,n,t})_n \in \mathbb{R}^{N\times N}$. $\bar{\boldsymbol{s}}_t \in \mathbb{R}^N$ is the average signals for all pairs, where the n^{th} element is the average signal in pair n.

Given that value $\theta_{i,t}$, private signal $s_{i,t}$, prices $\boldsymbol{p}_t$, and price history $\mathcal{H}_t$ are jointly normally distributed, we can solve the inference coefficients $\boldsymbol{C}_{s,t}$, $\boldsymbol{C}_{\mathcal{H},t}$ and $\boldsymbol{C}_{p,t}$ through the projection theorem.

Theorem 4 (DM Trading Equilibrium with Pre-trade Transparency). *The price impact for trader i in pair n is*

$$\lambda_{n,t} = \left(\left(\left((\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} \right)_{nn} \right)^{-1} - 1 \right)^{-1} \alpha.$$

where $(A)_{nn}$ is an operator that gives the n^{th} diagonal element of matrix A.
$\boldsymbol{C}_{p,t} = \text{diag}\left(\frac{\sigma^2}{1-\rho_{n,t}+\sigma^2}\right)_n \left(\boldsymbol{Id} - \text{diag}\left(\frac{1-\rho_{n,t}}{2}\right)(\bar{\mathcal{C}} - \mathbf{1}\mathbf{1}'\eta_t)^{-1}\right)$. $\eta_t = \frac{\boldsymbol{\tau}_t' \boldsymbol{\Upsilon}_t^{-1} \boldsymbol{\tau}_t}{\sigma_\theta^2}$ *is price history informativeness.* $\bar{\mathcal{C}} = \frac{cov(\bar{\boldsymbol{\theta}}_t, \bar{\boldsymbol{\theta}}_t')}{\sigma_\theta^2} \in \mathbb{R}^{N \times N}$ *is the correlation of pairwise average values across all pairs, where $\bar{\boldsymbol{\theta}}_t \in \mathbb{R}^N$ is the vector of average value per trading pair where the n^{th} value is* $\bar{\theta}_{n,t} = \sum_{i \in I(n)} \theta_{i,t}$

The expect utility for trader i in pair n conditional on the price history is

$$\mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t] = \frac{\alpha + 2\lambda_t}{2(\alpha+\lambda_t)^2}\mathbb{E}[(\mathbb{E}[\theta_{i,t}|\mathcal{H}_t, s_{i,t}, \boldsymbol{p}_t] - p_{t,n})^2|\mathcal{H}_t] = \frac{\alpha + 2\lambda_{n,t}}{2(\alpha+\lambda_{n,t})^2}\frac{1}{2}\frac{(1-\rho_{n,t})^2}{1-\rho_{n,t}+\sigma^2}$$

Thoerem 4 shows that the price history's impact on the current round utility is still through the price impact, and can be summarized by the sufficient statistic, price history informativeness η_t.

Our next question is, will pre-trade transparency change the matching results in DM? We find that the expected utility is still monotonic in the correlation $\rho_{n,t}$ (see Lemma 5). Therefore, each trader will be matched with the counterparty that has the lowest correlation ρ_ℓ, same as the matching results in Section 3.

Lemma 5 (Monotonicity of Utility with Pre-trade Transparency). *With pre-trade transparency, $\mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$ is monotonically decreasing in $\rho_{n,t}$.*

Given that introducing pre-trade transparency does not change the matching results in DM, and the price history update rule remains the same, we can still apply Theorem 2 to characterize the equilibrium.

Pre-trade Transparency and Welfare: With the equilibrium characterization, we would be able to discuss the impact of pre-trade transparency on market choice and welfare.

First, we find that given price history $\mathcal{H}_t$, introducing pre-trade transparency always weakly increases the utility for all traders in DM.

Lemma 6 (Pre-trade Transparency Increases DM Utility). *Given price history $\mathcal{H}_t$, introducing pre-trade transparency weakly increases the utility for all traders in DM.*

Given Lemma 6, it is intuitive that, holding everything else constant, it is more likely for traders to choose DM over CM as the threshold of history informativeness $\tilde{\eta}$ for traders to opt for DM is weakly lower.

Proposition 7 (Pre-trade Transparency Precipitates DM)**.** *With pre-trade transparency, (i) the first time for traders to choose DM is no later than without transparency; (ii) if the round when traders first choose DM is the same as the round when traders first choose DM without pre-trade transparency, then they stay in DM for weakly longer.*

The fact that pre-trade transparency can make traders choose DM earlier creates nuances in terms of welfare. By Lemma 6 we know that transparency increases utility for traders in DM given the price history. However, choosing DM earlier and staying longer can potentially decrease the price history informativeness and welfare in later rounds. Pre-trade transparency can bring down welfare when the loss of history informativeness dominates the benefit in DM.

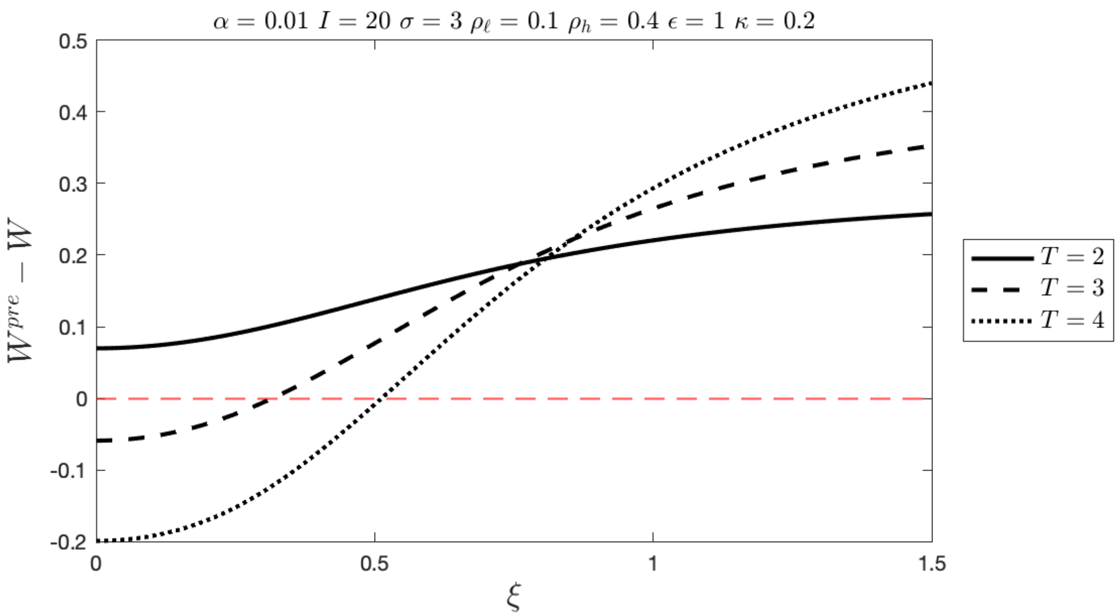
Proposition 8 (Pre-trade Transparency and Welfare)**.**

1. *For sufficiently heterogeneous trader value $\rho_\ell < \underline{\rho}_\ell < 0$ and $\bar{\rho} > \bar{\bar{\rho}}$, pre-trade transparency weakly improves welfare.*
2. *For sufficiently homogenous trader value, $\bar{\rho} < \bar{\rho}^{*,pre}(I, \rho_\ell, \sigma^2)$, pre-trade transparency does not change welfare.*
3. *When traders' value correlations are neither sufficiently heterogeneous nor sufficiently homogenous, pre-trade transparency can decrease welfare when the number of rounds T is sufficiently large, asset sensitivity ξ is low, or autocorrelation κ is high.*

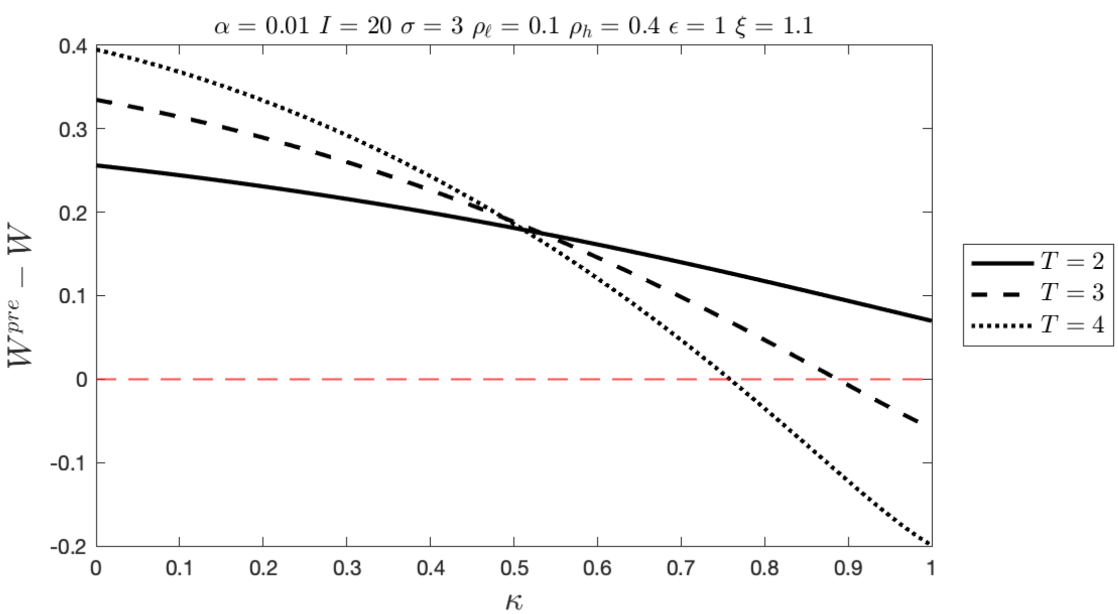
Intuitively, Proposition 8.1 corresponds to the constant DM choices both with and without pre-trade transparency, and given Lemma 6, pre-trade transparency should always weakly increase welfare. Proposition 8.2 corresponds to the constant CM choices both with and without pre-trade transparency. As traders do not choose DM, pre-trade transparency does not change welfare.

The impact of pre-trade transparency on welfare is ambiguous when traders' value correlation heterogeneity is of intermediate level (Proposition 8.3). This is when traders can have alternating market choices in the benchmark model. Despite that DM utility is higher with pre-trade transparency, price history informativeness may be lower as traders are more likely to choose DM earlier with pre-trade transparency. Figure 7 shows the difference between welfare with pre-trade transparency and welfare with opaque DM with respect to asset sensitivity ξ, volatility $(1 - \kappa^2)$, and the number of rounds T. When the

Figure 7: The Difference Between Welfare With Pre-trade Transparency and Welfare With Opaque DM



(a) Asset sensitivity ξ



(b) Autocorrelation κ

Note: Each black line plots the welfare with pre-trade transparency minus the welfare with opaque DM. The red dash line is a reference line of 0. If the black line is higher than (or at) the reference line, then pre-trade transparency (weakly) improves welfare, otherwise, it decreases welfare.

number of rounds T is large, and the asset value is stable either due to low sensitivity ξ or low volatility (high κ), low price history informativeness has a persistent and long-run impact. With these conditions, the loss of price history informativeness dominates the utility gain in DM, making pre-trade transparency welfare-decreasing.

This welfare result contrasts Vairo and Dworczak (2023) where they find pre-trade transparency always improves welfare. The key difference is that they focus on the impact of transparency given the decentralized market structure, but we endogenize the impact of pre-trade transparency on dynamic market choice and highlight the loss in price history informativeness.

5.2 Coexisting DM&CM vs. CM only

Besides lack of transparency, another concern on DM is market fragmentation. For example, the decentralized corporate bonds market in the U.S. has raised policy concerns about its lack of efficiency. There is an ongoing debate on whether to introduce a centralized market to the decentralized corporate bond market (e.g. Plante, 2017; Kutai et al., 2022; Allen and Wittwer, 2023).

We first consider the welfare impact of providing traders in DM with the option to trade in CM. In our model, we will compare the welfare under the baseline model with both opaque DM and CM and the welfare of the opaque DM market only. We find that introducing CM alongside DM weakly increases welfare (see Proposition 9). [15]

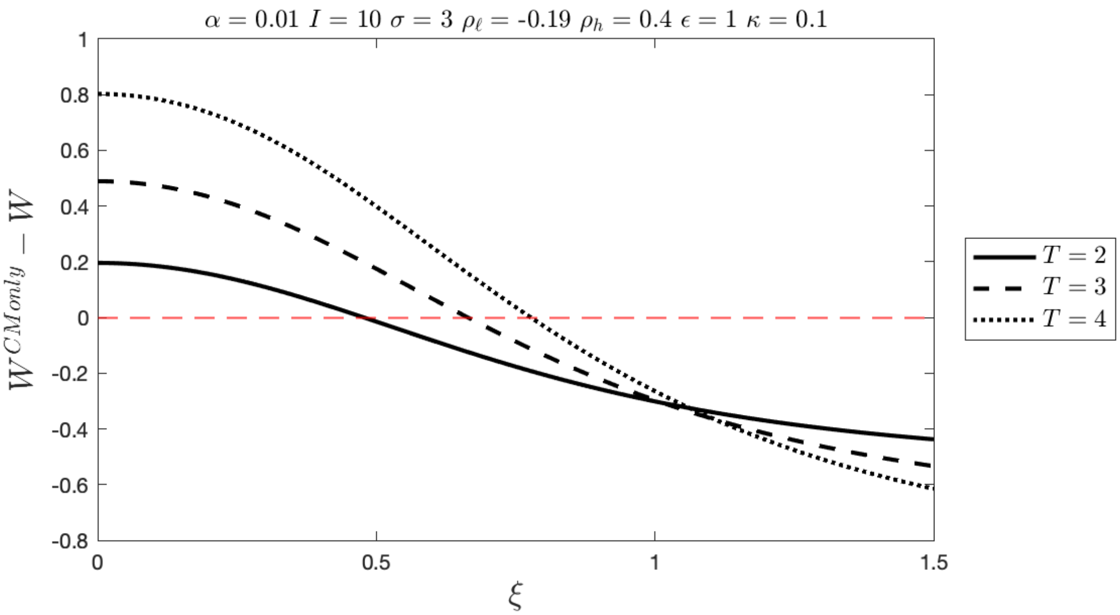
Proposition 9 (Welfare Comparison: DM vs. DM&CM)**.** *Compared with DM only, introducing CM weakly improves welfare.*

A more radical market design is to move all traders to the centralized market. We model this design as a design where traders no longer have the option to trade in DM.

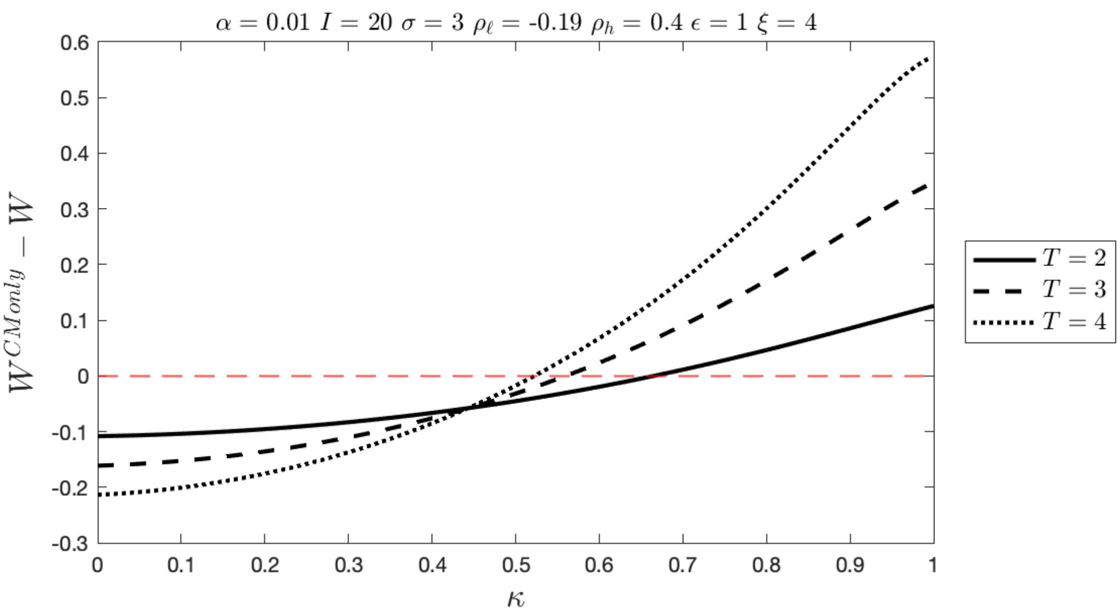
We find that compared with having access to both CM and DM, centralizing DM can decrease welfare when the value correlations are sufficiently heterogeneous, or when the asset sensitivity and volatility are sufficiently high with trading rounds $T < \bar{T}$. Intuitively, with sufficiently heterogeneous value correlation, traders' gain from trading with the lowest correlation trader is higher than the loss from the less deep market in the DM with no price history, so they prefer DM over CM in the first round. Over time, the transparent CM increases the price history informativeness and the CM welfare. If the asset sensitivity

[15] To allow a more interesting comparison, the DM-market-only case can be relaxed to allow traders to access both CM and DM in the first round but only DM later. Proposition 9 still holds. See proof of Proposition 9 for details.

Figure 8: The Difference Between Welfare With Centralized Market Only and Welfare With Parallel Markets with Sufficiently Heterogeneous Correlation



(a) Asset sensitivity ξ



(b) Autocorrelation κ

Note: Each black line plots the welfare with CM only minus the welfare with both DM and CM with sufficiently heterogeneous correlation $\rho_\ell < \underline{\rho}_\ell < 0$ and $\bar{\rho} > \bar{\bar{\rho}}$. The red dash line is a reference line of 0. If the black line is higher than (or at) the reference line, then centralizing the DM improves welfare, otherwise, it decreases welfare.

and volatility are sufficiently high with short trading horizons, the improvement in price history informativeness in CM is not large enough to offset the welfare loss in the beginning. Conversely, with sufficiently heterogeneous correlation, when the asset sensitivity and volatility are sufficiently low, with long enough trading horizons, the CM can improve welfare upon the coexisting markets (see Figure 8).

Proposition 10 (Welfare Comparison: CM vs. DM&CM)**.** *Compared with the parallel markets of CM and DM,*

1. *Centralizing DM does not change welfare, (i) if values are sufficiently homogenous, $\bar{\rho} < \bar{\rho}^{*,pre}(I, \rho_\ell, \sigma^2)$ or (ii) if traders' value correlations are neither sufficiently heterogeneous nor sufficiently homogenous, and the number of rounds $T < \bar{T}$, centralizing DM does not change welfare if the asset sensitivity is sufficiently high $\xi \in [\bar{\xi}, \infty)$, or volatility is sufficiently high $\kappa \in [0, \underline{\kappa}]$.*

2. *Centralizing DM decreases welfare if trader values are sufficiently heterogeneous $\rho_\ell < \underline{\rho}_\ell < 0$ and $\bar{\rho} > \bar{\bar{\rho}}$, the asset sensitivity is sufficiently high $\xi \in [\bar{\xi}^{CM\,only}, \infty)$ and when the volatility is sufficiently high $\kappa \in [0, \underline{\kappa}^{CM\,only})$ with finite rounds $T < \bar{T}$.*

3. *Centralizing DM improves welfare if trader value correlations satisfy $\bar{\bar{\rho}} < \bar{\rho} < \tilde{\bar{\rho}}$, $\underline{\underline{\rho}}_\ell < \rho_\ell < \underline{\rho}_\ell < 0$, and the number of rounds $T > \tilde{T}$ is sufficiently large.*

6 Empirical Evidence

The model provides us with some testable predictions. Proposition 3 implies that the assets with higher sensitivity (higher ξ) to shocks to fundamentals are more likely to be traded in CM. Proposition 4 implies that assets with higher volatility (lower κ) are more likely to be traded in CM. The model also predicts a dynamic market choice as price history informativeness grows. Lemma 3 implies that traders are more likely to switch to DM from CM with a more informative price history.

We collect and analyze the Chinese corporate bond market data to test the predictions.

6.1 Institutional Background

Before delving into the data, it is beneficial to offer an overview of the distinctive institutional framework of Chinese bond markets.

Parallel OTC and CM: In China, there exist two concurrent bond markets: the interbank market and the exchange market. The interbank bond market, established in 1997, operates similarly to the U.S. interbank bond market and is an over-the-counter (OTC) market. On the other hand, the exchange bond market, inaugurated in 1990, functions as part of the Shanghai and Shenzhen Stock Exchanges and operates as a centralized market. Both markets comprise a cash bond market for primary issuance and secondary trading, as well as a repo market.

Participants. Participants in the two bond markets exhibit some variation, but largely share most non-bank institutional investors. The interbank bond market caters to qualified institutional investors, including commercial banks, mutual funds, insurance companies, and security firms, functioning as a wholesale market. Conversely, the exchange-based bond market operates as a retail market, permitting non-bank institutions, corporate investors, and retail investors to engage in bond investments. Commercial banks' involvement in the exchange market is minimal due to restrictions on repo transactions. However, many non-bank financial institutions, such as mutual funds, insurance companies, and security firms actively participate in both markets. According to Chen et al. (2023), the non-bank financial institutions take up 76 percent and 57 percent of aggregate enterprise bond holdings, over 80 percent and nearly 50 percent of enterprise bonds' spot transactions on the exchange and interbank markets respectively by the end of 2014.

Bond Products. Bonds traded in the exchange market typically exhibit smaller sizes compared to those in the interbank market. Nonetheless, certain bond products, particularly some enterprise bonds and government bonds, are dual-listed, being traded in both markets. Enterprise bonds are corporate bonds issued by state-owned enterprises or entities with substantial state ownership. Access to enterprise bonds in the exchange market was limited until 2005 when the National Development and Reform Commission (NDRC) granted non-public-listed state-owned enterprises entry to the exchange market. Since then, dual-listed enterprise bonds have experienced significant growth. In 2018, over 28 percent of outstanding enterprise bonds were dual-listed. We will focus on these dual-listed corporate bonds in the following analysis.

Regulators and Clearing Houses. The two markets are overseen by different regulatory bodies. The People's Bank of China (PBOC) serves as the primary regulator of the interbank bond market, while the China Securities Regulatory Commission (CSRC) regulates the exchange market. In the interbank market, trading occurs via the China Foreign Exchange Trade System (CFETS), with clearing services provided by the Shanghai Clearing House (SHCH) and China Central Depository & Clearing Co. Ltd (CCDC), which exclusively offers custodial services. Conversely, in the exchange market, investor bids are consolidated in electronic order books, with the exchange acting as the central clearing house, and all matched trades are settled through the China Securities Depository & Clearing Co. Ltd (CSDC).

Limited Same-day Arbitrage: Several obstacles hinder a trader's ability to trade in both markets on the same day. Firstly, as outlined by Chen et al. (2023), the transfer of bonds between the interbank market (CCDC) and the exchange market (CSDC) took approximately 3-4 working days in 2014, with even longer durations (about 4-6 working days) required to move bonds from the exchange market to the interbank market. Although the transfer process has become faster in recent years, it still entails a significant waiting period. Secondly, transferring funds from the exchange market to the interbank market encounters settlement delays. While the interbank market operates on a "T+0" settlement basis, the exchange market follows a "T+1" settlement model, necessitating a day's wait for fund transfers. Transferring funds from the interbank market to the exchange market also involves time constraints. Typically, if a transaction concludes in the interbank market in the morning, settlement occurs in the afternoon. Given that the exchange market closes at 3:00 pm, executing same-day arbitrage between the two markets becomes nearly infeasible. Thirdly, the settlement fee is relatively high compared to the potential gains from cross-market arbitrage opportunities. Finally, shorting bonds is prohibited in bond markets in China. Therefore traders cannot apply a long-short strategy across the two markets to arbitrage. As a result of these barriers, it is unusual to see traders trading the same bonds in both markets on the same day.

6.2 Asset Properties and Market Choices

We obtain daily prices and transaction volume of corporate bonds in China from WIND. We focus on enterprise bonds dual-listed in both the interbank market and the exchange market. The sample period is January 1st 2018 to May 31st, 2018.[16] The observations are

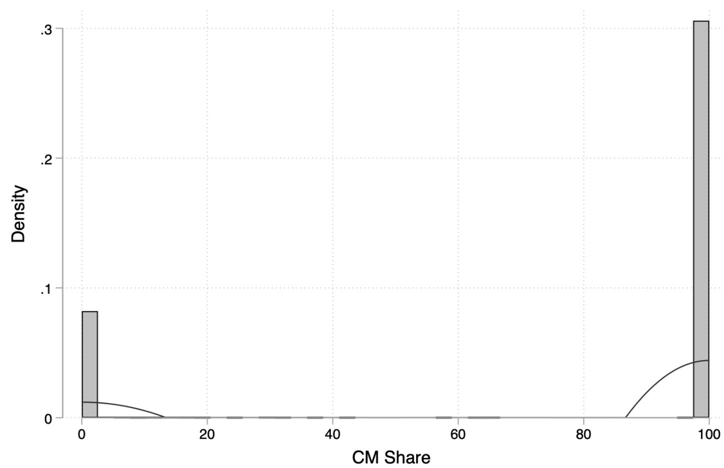
[16]We chose this sample period as there were no collateral-based monetary policies or any changes in bond pledgeability during this period. These policies can create a large divergence in the prices of the same bond in the two markets (see Fang et al. (2020) and Chen et al. (2023)), thus may introduce noises in the measurement of asset value volatility.

at the bond $\times$ day level. Hereafter, we will refer to the interbank market as DM and the exchange market as CM.

In this section, we tested the relationship between asset properties and market choices (Propositions 3 and 4).

We define CM share as the daily transaction volume in CM as a percentage of the total daily transaction volume at bond $\times$ day level. Figure 9 shows the distribution of CM share. We can see that most of the mass is distributed on either 0% or 100%. Table 1 shows the count of observations by their markets. We find that only around 3% of observations are bonds traded in both markets on the same day. This is largely consistent with our model assumption where traders choose the market before they bid instead of submitting orders to both markets. We define two bond choice dummies, the indicator CM is 1 if the bond is traded in CM on that day and 0 otherwise, and the indicator DM is 1 if the bond is traded in DM on that day and 0 otherwise.

Figure 9: Distribution of Centralized Market Share



Note: This figure shows the density of centralized market share for dual-listed corporate bonds traded between Jan. 1st, 2018 and May 31st, 2018.

We use default risk to proxy for the sensitivity of assets to the issuing firms' fundamental value. Given the hockey-stick-like bond payoff structure, we would expect when the default risk is higher, the bond payoff is more sensitive to the issuer's fundamentals. Therefore, we collect the 2017 year-end current ratio for each bond issuer as the proxy for their asset sensitivity.[17] The current ratio (CR) is defined as the ratio of the issuer's current

[17] We also use the 2017 year-end debt-to-asset ratio, cash interest ratio of the issuers, and the bond credit rating to proxy the default risk in robustness check.

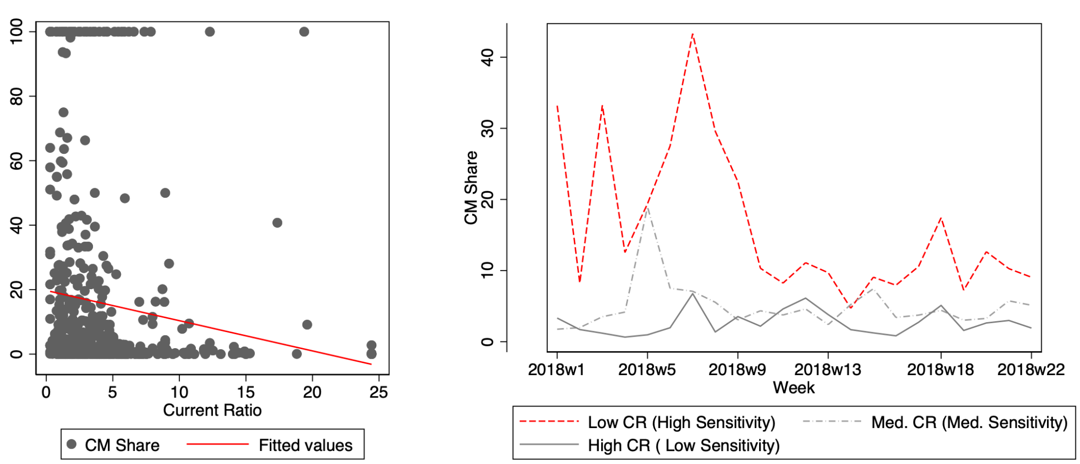
Table 1: Count of Observations by Their Markets

	Not traded in DM ($DM = 0$)	Traded in DM ($DM = 1$)	Total
Not traded in CM ($CM = 0$)	-	1,653	1,653
Traded in CM ($CM = 1$)	7,098	299	7,397
Total	7,098	1,952	9,050

Note: This table shows the number of observations by traders' market choice of the bond on that day.

assets to its current liabilities. When CR is lower, the default risk is higher and the bond sensitivity is higher. We winsorize these ratios at 1%.[18] Figure 10 (a) shows the scatter plots of average centralized market share across the sample period for each bond with respect to their current ratios. Figure 10 (b) shows the average centralized market share for each bond with high (above 75 percentile), median (25-75 percentile), and low (below 25 percentile) current ratios across time. We find that when the current ratio is lower, the bond has a larger overall centralized market share. This is consistent with the prediction of Proposition 3.

Figure 10: Current Ratio and the Centralized Market Share



(a) Current Ratio and CM Share

(b) Current Ratio and CM Share Over Time

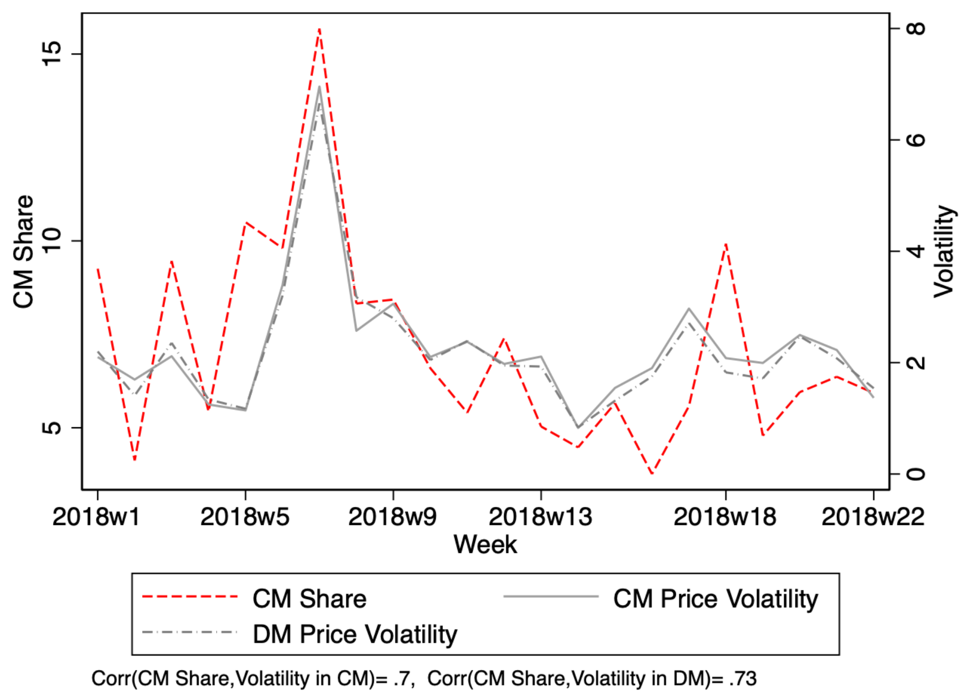
Note: This figure shows the correlation between the current ratio and the centralized market share of the dual-listed corporate bonds traded during Jan. 1st, 2018 - May 31st, 2018. Panel (a) shows the scatter plot of CM share against the current ratio of the bond issuer with a fitted linear curve. Panel (b) shows the time series of current ratios with respect to low, median, and high levels of current ratios.

We use the standard deviation of prices between the $t - 30$ to $t - 1$ trading days as

[18]The regression results are robust without winsorization.

a proxy for the volatility at day t for each bond. Given that we have two markets, we construct the price volatility for both CM and DM. Figure 11 shows the correlation of the average centralized market share and weekly average price volatility across bonds weighted by their transaction volume. We can see that the centralized market share moves in tandem with the volatility measures. The correlation between the centralized market share and the volatility in the CM (DM) market is 0.7 (0.73).[19] Figure 11 is consistent with the prediction of Proposition 4, that traders are more likely to choose CM with higher asset volatility.

Figure 11: Asset Volatility and the Centralized Market Share



Note: This figure shows the time series of asset price volatility and the centralized market share of the dual-listed corporate bonds traded during Jan. 1st, 2018 - May 31st, 2018. The correlation between the CM price volatility and the CM share is 0.70, and the correlation between the DM price volatility and d the CM share is 0.73.

Panel A of Table 2 provides the summary of statistics for the variables used in this section.

[19]There may be concerns that the positive correlation is driven by the high volatility in week 7 of 2018. As a robustness check, we drop the observations in week 7 and calculate the correlations between the weekly average CM share and price volatility measures weighted by bond transaction volume. These correlations remain positive without observations in week 7. The correlation between the centralized market share and the volatility measures in the CM (DM) market is 0.3 (0.39).

Table 2: Summary of Statistics

VARIABLES	(1) N	(2) Mean	(3) Std. Dev.	(4) Min	(5) Max
Panel A: Full Sample					
CM Share	9,050	78.54	40.97	0	100
CM	9,050	0.817	0.386	0	1
DM	9,050	0.216	0.411	0	1
Current Ratio	9,050	3.575	3.442	0.291	24.44
CM Price Volatility	9,050	2.992	3.997	0	17.53
DM Price Volatility	9,050	2.820	4.118	0	18.00
Panel B: Switching Sample					
Pr(From CM to DM)	848	0.508	0.500	0	1
Trade Frequency	848	0.586	0.155	0.0769	0.971

Note: This table shows the summary of statistics for the dual-listed corporate bonds traded between Jan. 1st, 2018 and May 31st, 2018. Panel A is statistics of the full sample for regression equations (5) and (6). Panel B is statistics of the sample where traders switch markets for the bond in the next round for regression equation (7).

We then formally tested Propositions 3 and 4 in a regression framework. We first test the model predictions using a probit regression, with the indicator dummy CM (or DM) as the dependent variable and the current ratio and the volatility measures as independent variables. Note that 3% of the observations are the same bonds traded in both CM and DM within a day, the coefficients in the following two probit regressions are not exactly opposite.

$$\begin{aligned} Pr(CM_{it} = 1) &= \Phi(\beta_0 + \beta_1 \text{Volatility}_{it} + \beta_2 \text{Current Ratio}_i) \\ Pr(DM_{it} = 1) &= \Phi(\beta_0 + \beta_1 \text{Volatility}_{it} + \beta_2 \text{Current Ratio}_i) \end{aligned} \tag{5}$$

where i is the index for bonds and t is the index for dates. Table 3 shows the regression results of equation (5). We find that higher price volatility and asset sensitivity increase the probability of traders choosing the centralized market, consistent with the patterns in Figures 10 and 11.

Instead of running regressions, we can also use CM share as the dependent variable to test Propositions 3 and 4, with the asset properties as the independent variable, controlling

Table 3: Market Choice and Asset Properties

VARIABLES	(1) Pr(CM=1)	(2) Pr(CM=1)	(3) Pr(DM=1)	(4) Pr(DM=1)
Current Ratio	-0.0443***	-0.0442***	0.0399***	0.0398***
	(0.00411)	(0.00410)	(0.00404)	(0.00403)
CM Price Volatility	0.0403***		-0.0371***	
	(0.00428)		(0.00402)	
DM Price Volatility		0.0353***		-0.0309***
		(0.00401)		(0.00378)
Constant	0.964***	0.982***	-0.833***	-0.854***
	(0.0244)	(0.0239)	(0.0236)	(0.0231)
Observations	9,050	9,050	9,050	9,050
Pseudo R-squared	0.0245	0.0223	0.0197	0.0171

Note: This table shows the impact of asset properties on trader's market choice from probit regression equation (5). We focus on dual-listed corporate bonds in China. The sample period is Jan. 1st, 2018, and May 31st, 2018. We include robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

for the date fixed effects,

$$\text{CM Share}_{it} = \beta_1 \text{Current Ratio}_{it} + \beta_2 \text{Volatility}_{it} + \text{Date FE} + \varepsilon_{it} \tag{6}$$

where i is the index for bonds and t is the index for dates, and ε are robust standard errors.

Table 4 shows the regression results of equation (6). We find that higher price volatility and asset sensitivity increase centralized market share, consistent with our previous findings and model predictions.

6.3 Price History and Market Choices

In this section, we directly test the mechanism of the model that when the price history is more informative, traders are more likely to choose DM (Lemma 3). This implies that bonds with more price history are more likely to be traded in DM.

The difficulty in testing this prediction is that we do not know the threshold $\tilde{\eta}$ and therefore the exact time for traders to switch from CM to DM. The price history informativeness η may be above the threshold, such that past CM trades lead to DM trades. It is also possible for η to be below the threshold, such that past CM trades still lead to more CM trades. Therefore, simply running a probit regression with the number of past trades as the independent variable and the current round market choice as the dependent

Table 4: CM Share and Asset Properties

VARIABLES	(1) CM Share	(2) CM Share
Current Ratio	-1.339***	-1.338***
	(0.142)	(0.142)
CM Price Volatility	0.847***	
	(0.118)	
DM Price Volatility		0.689***
		(0.111)
Date FE	Yes	Yes
Observations	9,050	9,050
R-squared	0.068	0.066

Note: This table shows the impact of asset properties on the CM share of the bond from regression equation (6). We focus on dual-listed corporate bonds in China. The sample period is Jan. 1st, 2018, and May 31st, 2018. We include robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

variable is not feasible.

To test Lemma 3, we can focus on the sample when traders change their market choice. Lemma 3 implies that conditional on traders changing their market, more (less) past trades incentivize traders to switch from CM to DM (from DM to CM).

We drop the observations when the same bond is traded in the same market for two consecutive trades, and focus on the observations that the next trading market differs from the current market. We define the dependent variable $FromCMtoDM_{it}$ as a dummy which takes 1 if the bond i is traded in CM on trading day t but in DM the next time when it is traded, and 0 otherwise. We use $TradeFrequency \equiv \frac{TradeCount}{TotalTradeCount}$ to proxy for price history informativeness. $TradeCount$ is the number of days traded in DM and CM for each bond during the last 60 trading days, i.e. $[t-60, t-1]$ for bonds traded on trading day t. The $TotalTradeCount$ is the total number of trades during the sample period from Jan. 1, 2018, to May 31, 2018, for each bond. Panel B of Table 2 provides the statistics of the variables used in this section.

We then tested the relationship between price history and traders' market choices with the following probit regression,

$$Pr(FromCMtoDM_{it}) = \Phi(\beta_0 + \beta_1 TradeFrequency) \tag{7}$$

Table 5 column (1) shows the regression results of equation (7). We find that consistent with the model prediction, traders are more likely to shift from CM to DM when they observe more past trades.

Table 5: The Impact of Past Trading Frequency on Market Choices

VARIABLES	(1) Pr(From CM to DM)
Trade Frequency	1.046***
	(0.284)
Constant	-0.593***
	(0.172)
Observations	848
Pseudo R-squared	0.0118

Note: This table shows the impact of past trade frequency on the probability for traders to switch from CM to DM from regression equation (7). We focus on dual-listed corporate bonds in China. The sample period is Jan. 1st, 2018, and May 31st, 2018. We include robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

6.4 Other Empirical Results

Amplification Effects We also examine whether the volatility could amplify the impact of asset sensitivity on traders' market choices. Table 6 shows the regression results of equation (5) with an additional interaction term of current ratio and volatility. We find that higher volatility together with higher sensitivity (i.e. lower current ratio) further increases the probability for traders to choose CM, and decreases the probability for traders to choose CM. Table 7 shows that the amplification effect, despite being consistent in the signs, is not statistically significant in regression equation (6) with the interaction term.

Robustness Check 1. Other Default Risk Measures One may wonder whether our results hold robustly to other default risk measures. To verify this, we include the 2017 year-end debt-to-asset ratio and cash interest ratio of the bond issuers, and the bond credit rating on each day in regressions (5) and (6) as a robustness check.[20] Table 8 Panel A shows the statistics of these additional default risk measures. Tables 9 and 10 show that the coefficients on the current ratio and the volatility measure do not differ much from the baseline

[20]The the 2017 year-end debt-to-asset ratio, and cash interest ratio of the bond issuers are winsorized at 1% to avoid extreme values. There are three credit ratings for bonds in the sample, AA, AA+, and AAA. The AAA corporate bonds are taken as the reference group in the regression.

regressions. The coefficients on the 2017 year-end debt-to-asset ratio and cash-interest ratio of the bond issuers are not significant. The coefficients on the bond credit rating AA and AA+ are negative, mostly driven by the fact that retail traders in CM trade more AAA bonds than those bonds with lower ratings.

Robustness Check 2. Alternative Trade Frequency Measure We include alternative trade frequency measure as a robustness check. We measure $TradeCount$ with the number of days traded in DM and CM for each bond in the last 30 instead of 60 trading days. Table 8 Panel B shows the statistics of this alternative trade count measure and other default risk measures for equation (7). Table 11 shows the regression results. We find with the new measure, the results are still consistent with the model prediction that more trade history makes traders more likely to switch from CM to DM.

Additional Evidence from the U.S. Markets: We provided additional empirical evidence for Proposition 4 with the U.S. equity market data (see Appendix **??**). Also, by comparing the average centralized market share of bonds in China and the equities in the U.S. we obtain an additional piece of evidence in support of Proposition 3: The assets more sensitive to shocks to fundamentals such as equities generally have higher centralized market share than those with lower sensitivity such as bonds.

7 Conclusion and Discussions

This paper presents a model examining the dynamic market choice between centralized and decentralized markets, where arriving traders must decide between a centralized market and a bilaterally matched decentralized market in each period. The emergence of dynamic market choice is observed as a consequence of learning from the centralized market price history. Optimal market choices, influenced by asset properties, include switching between centralized and decentralized markets when traders' value correlation is moderately heterogeneous. In cases where asset values are insensitive to shocks or shocks are predictable, traders switch between centralized and decentralized markets or stay in the decentralized market after one round in the centralized market. Conversely, when asset values are sensitive to unpredictable fundamentals, traders choose to stay in the centralized market.

It is interesting to see that learning from price history alone generates rich dynamic market choices. It is also important to recognize that we abstract away from the inventory held by traders by assuming short-lived traders. Adding dynamic inventory significantly reduces tractability in the linear-quadratic double auction setting like this paper. Inventory management across rounds is also an important aspect of trading strategies. We believe dynamic market choice with both dynamic inventory and learning warrants future research.

The empirical results can also be explained by alternative theories. For instance, the positive correlation between centralized market share and volatility measures might be attributed to retail traders, who typically trade in the centralized market, increasing their trading activity during periods of higher volatility. However, we cannot empirically rule out such alternative explanations since our dataset does not identify individual traders. These possibilities remain open for investigation in future studies.

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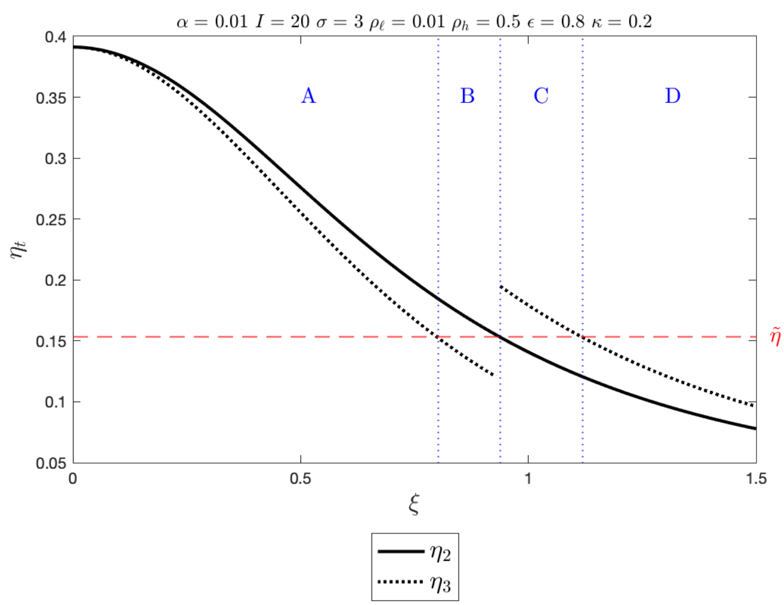
Appendices

A Additional Tables and Figures

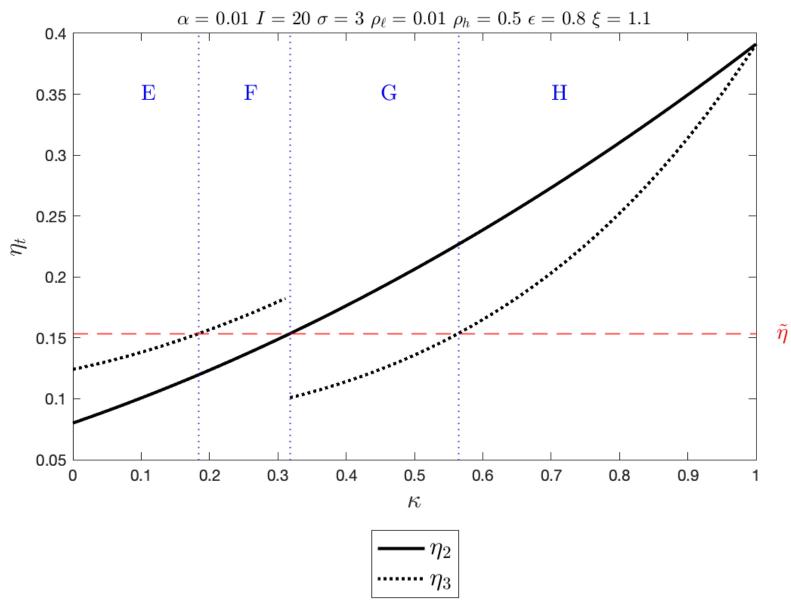
Figure 12: Price History Informativeness With Respect to Asset Sensitivity ξ in $T = 3$ Market



Note: $\eta_1 = 0$. The black solid line plots η_2, and the black dotted line plots η_3. The red dashed line is a reference line of $\tilde{\eta}$. When the black solid (dotted) line is above the reference line, then the traders choose DM in the 2nd round (3rd round), and if it is below the reference line, the traders choose CM in the 2nd round (3rd round). The jump in the difference of utility in CM vs. DM in the third round comes from the difference in the second-round choice.

In region A, i.e. $\xi \in [0, \underline{\xi})$, traders choose DM in both 2nd and 3rd round. In region B, i.e. $\xi \in (\underline{\xi}, \bar{\xi}]$ and in the lower partition, traders choose DM in the 2nd round and CM in the 3rd round. In region C, $\xi \in (\underline{\xi}, \bar{\xi}]$ and in the higher partition, traders choose CM in the 2nd round and DM in the 3rd round. In region D, $\xi \in (\bar{\xi}, 1]$, traders choose CM in both the 2nd round and the 3rd round.

Figure 13: Price History Informativeness With Respect to Autocorrelation κ in $T = 3$ Market



Note: $\eta_1 = 0$. The black solid line plots η_2, and the black dotted line plots η_3. The red dashed line is a reference line of $\tilde{\eta}$. When the black solid (dotted) line is above the reference line, then the traders choose DM in the 2nd round (3rd round), and if it is below the reference line, the traders choose CM in the 2nd round (3rd round). The jump in the difference of utility in CM vs. DM in the third round comes from the difference in the second-round choice.
In region E, i.e. $\kappa \in [0, \underline{\kappa}]$, traders choose CM in both 2nd and 3rd round. In region F, i.e. $\kappa \in (\underline{\kappa}, \bar{\kappa}]$ and in the lower partition, traders choose CM in the 2nd round and DM in the 3rd round. In region G, $\kappa \in (\underline{\kappa}, \bar{\kappa}]$ and in the higher partition, traders choose DM in the 2nd round and CM in the 3rd round. In region H, $\kappa \in (\bar{\kappa}, 1]$, traders choose DM in both the 2nd round and the 3rd round.

Table 6: Amplification Effect: Market Choice and Asset Properties with Interaction Term of Volatility Measures and Current Ratio

VARIABLES	(1) Pr(CM=1)	(2) Pr(CM=1)	(3) Pr(DM=1)	(4) Pr(DM=1)
Current Ratio	-0.0389***	-0.0401***	0.0353***	0.0366***
	(0.00473)	(0.00452)	(0.00467)	(0.00446)
CM Price Volatility	0.0535***		-0.0476***	
	(0.00718)		(0.00673)	
Current Ratio × Exchange Volatility	-0.00321**		0.00264**	
	(0.00139)		(0.00134)	
DM Price Volatility		0.0474***		-0.0397***
		(0.00682)		(0.00640)
Current Ratio × Interbank Volatility		-0.00293**		0.00217*
		(0.00132)		(0.00128)
Constant	0.943***	0.966***	-0.816***	-0.842***
	(0.0260)	(0.0250)	(0.0251)	(0.0242)
Observations	9,050	9,050	9,050	9,050
Pseudo R-squared	0.0251	0.0229	0.0201	0.0174

Note: This table shows the impact of asset properties on the CM share of the bond from regression equation (5) with an additional interaction term between the current ratio and the volatility measure. We focus on dual-listed corporate bonds in China. The sample period is Jan. 1st, 2018, and May 31st, 2018. We include robust standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

Table 7: Amplification Effect: CM Share and Asset Properties with Interaction Term of Volatility Measures and Current Ratio

VARIABLES	(1) CM Share	(2) CM Share
Current Ratio	-1.314***	-1.321***
	(0.156)	(0.154)
CM Price Volatility	0.895***	
	(0.170)	
Exchange Volatility × Current Ratio	-0.0129	
	(0.0377)	
DM Price Volatility		0.725***
		(0.167)
Interbank Volatility × Current Ratio		-0.00970
		(0.0371)
Observations	9,050	9,050
R-squared	0.068	0.066
Date FE	Yes	Yes

Note: This table shows the impact of asset properties on the CM share of the bond from regression equation (6) with an additional interaction term between the current ratio and the volatility measure. We focus on dual-listed corporate bonds in China. The sample period is Jan. 1st, 2018, and May 31st, 2018. We include robust standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

Table 8: Robustness Check: Summary of Statistics for Other Default Risk Measures

VARIABLES	(1) N	(2) Mean	(3) Std. Dev.	(4) Min	(5) Max
Panel A: Full Sample					
Debt-to-Asset Ratio	9,050	54.92	11.97	26.03	77.36
Cash Interest Ratio	9,050	-21.91	211.8	-1,935	385.9
AA	9,050	0.429	0.495	0	1
AA+	9,050	0.344	0.475	0	1
Panel B: Switching Sample					
Trade Frequency (in 30 days)	1,193	0.336	0.159	0.0294	0.969
Pr(From CM to DM)	1,193	0.490	0.500	0	1

Note: This table shows the summary of statistics for risk measures other than the current ratio. We focus on dual-listed corporate bonds in China. The sample period is Jan. 1st, 2018, and May 31st, 2018. Panel A is statistics of the full sample for regression equations (5) and (6). Panel B is statistics of the sample where traders switch markets for the bond in the next round for regression equation (7), where the trade frequency is measured in the last 30 days instead of the last 60 days.

Table 9: Robustness Check: Market Choices with Other Default Risk Measures

VARIABLES	(1) Pr(CM=1)	(2) Pr(CM=1)	(3) Pr(DM=1)	(4) Pr(DM=1)
Current Ratio	-0.0395***	-0.0410***	0.0357***	0.0373***
	(0.00508)	(0.00487)	(0.00500)	(0.00480)
CM Price Volatility	0.0533***		-0.0477***	
	(0.00729)		(0.00684)	
Current Ratio × Exchange Volatility	-0.00323**		0.00267**	
	(0.00140)		(0.00135)	
DM Price Volatility		0.0475***		-0.0400***
		(0.00699)		(0.00655)
Current Ratio × Interbank Volatility		-0.00294**		0.00219*
		(0.00133)		(0.00129)
Debt-to-Asset Ratio	-0.000610	-0.000823	0.000559	0.000739
	(0.00147)	(0.00147)	(0.00140)	(0.00140)
Cash Interest Ratio	8.44e-05	8.21e-05	-9.55e-05	-9.18e-05
	(6.74e-05)	(6.73e-05)	(6.74e-05)	(6.72e-05)
AA	-0.0338	-0.0385	0.0520	0.0529
	(0.0420)	(0.0425)	(0.0405)	(0.0409)
AA+	-0.0989**	-0.108**	0.130***	0.137***
	(0.0418)	(0.0419)	(0.0403)	(0.0404)
Constant	1.030***	1.070***	-0.918***	-0.957***
	(0.0980)	(0.0982)	(0.0940)	(0.0940)
Observations	9,050	9,050	9,050	9,050
Pseudo R-squared	0.0260	0.0239	0.0215	0.0190

Note: This table shows the impact of asset properties on the CM share of the bond from regression equation (5) with (1) an additional interaction term between the current ratio and the volatility measure; and (2) default risk measures other than the current ratio, Debt-to-Asset Ratio, Cash Interest Ratio and bond rating dummies (AA and AA+). We focus on dual-listed corporate bonds in China. The sample period is Jan. 1st, 2018, and May 31st, 2018. We include robust standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

Table 10: Robustness Check: CM Share with Other Default Risk Measures

VARIABLES	(1) CM Share	(2) CM Share
Current Ratio	-1.332***	-1.344***
	(0.164)	(0.162)
CM Price Volatility	0.922***	
	(0.173)	
Current Ratio $\times$ Exchange Volatility	-0.0147	
	(0.0384)	
DM Price Volatility		0.766***
		(0.172)
Current Ratio $\times$ Interbank Volatility		-0.0122
		(0.0379)
Debt-to-Asset Ratio	-0.0368	-0.0389
	(0.0414)	(0.0415)
Cash Interest Ratio	0.00245	0.00236
	(0.00187)	(0.00187)
AA	-3.268**	-3.262**
	(1.349)	(1.356)
AA+	-5.584***	-5.751***
	(1.235)	(1.240)
Date FE	Yes	Yes
Observations	9,050	9,050
R-squared	0.070	0.069

Note: This table shows the impact of asset properties on the CM share of the bond from regression equation (6) with (1) an additional interaction term between the current ratio and the volatility measure; and (2) default risk measures other than the current ratio, Debt-to-Asset Ratio, Cash Interest Ratio and bond rating dummies (AA and AA+). We focus on dual-listed corporate bonds in China. The sample period is Jan. 1st, 2018, and May 31st, 2018. We include robust standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

Table 11: Robustness Check: Market Choice and Alternative Measure of Price History

VARIABLES	(1) Pr(From CM to DM)
Trade Frequency	0.460**
	(0.228)
Constant	-0.179**
	(0.0849)
Observations	1,193
Pseudo R-squared	0.00246

Note: This table shows the impact of past trading frequency on the probability for traders to switch from CM to DM from regression equation (7) with past trading frequency measured by the bond's trade count in the last 30 trading days over the total trade counts. We focus on dual-listed corporate bonds in China. The sample period is Jan. 1st, 2018, and May 31st, 2018. We include robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

B Extensions

B.1 General Private Information Precision

In the baseline model, traders receive private signals with constant precision. In this section, we provide an extension with general private information precision. We find that traders are more likely to choose CM after rounds with less precise private signals. We show that the traders can still switch back and forth between CM and DM, even with post-trade transparent DM (as opposed to Proposition 5), when the precise signals do not arrive frequently.

We generalize the private signals' precision to accommodate different information arrival frequencies. Assume trader will receive a private signal $s_{i,t} = \theta_{i,t} + \varepsilon_{i,t}$, where $\varepsilon_{i,t} \sim \mathcal{N}(0, \sigma_{i,t}^2 \sigma_\theta^2)$, $\sigma_{i,t} \geq ((\frac{2(I-1)}{I})^{1/3} - 1)^{-1/2}$. Note that $\sigma_{i,t} \to \infty$ is equivalent to traders not receiving private signals.

With the above generalization, we can solve the trading equilibrium as follows:

Proposition 11 (Trading Equilibrium with General Private Information)**.** *Given the price history $\mathcal{H}_t$ and the market structure $\mathcal{M}_t^*$, the equilibrium at round t can be characterized by a fixed point of inference coefficients,*

$$c_{s,t} = \frac{1-\rho_{t,\mathcal{M}^*}}{1-\rho_{t,\mathcal{M}^*}+\sigma_{i,t}^2}$$

$$\boldsymbol{c}_{\mathcal{H},t} = \frac{(1-\rho_{t,\mathcal{M}^*})\sigma_{i,t}^2}{\left(1-\rho_{t,\mathcal{M}^*}+\sigma_{i,t}^2\right)(1+(I_{t,\mathcal{M}^*}-1)\rho_{t,\mathcal{M}^*}-I_{t,\mathcal{M}^*}\eta_t)}\boldsymbol{\tau}_t'\boldsymbol{\Upsilon}_t^{-1}$$

$$c_{p,t} = \frac{I_{t,\mathcal{M}^*}(\rho_{t,\mathcal{M}^*}-\eta_t)\sigma_{i,t}^2}{\left(1-\rho_{t,\mathcal{M}^*}+\sigma_{i,t}^2\right)(1+(I_{t,\mathcal{M}^*}-1)\rho_{t,\mathcal{M}^*}-I_{t,\mathcal{M}^*}\eta_t)}$$

where $\eta_t = \frac{var(\theta_{i,t})-var(\theta_{i,t}|\mathcal{H}_t)}{var(\theta_{i,t})} = \frac{\boldsymbol{\tau}_t'(\boldsymbol{\Upsilon}_t)^{-1}\boldsymbol{\tau}_t}{\sigma_\theta^2}$*,* $\boldsymbol{\tau}_t \equiv cov(\mathcal{H}_t, \theta_{i,t}) \in \mathbb{R}^{|\mathcal{H}|}$*, and* $\boldsymbol{\Upsilon}_t \equiv cov(\mathcal{H}_t, \mathcal{H}_t') \in \mathbb{R}^{|\mathcal{H}|\times|\mathcal{H}|}$.

The equilibrium price impact is

$$\lambda_t = \frac{\alpha}{(I_{t,\mathcal{M}^*}-1)(1-c_{p,t})-1}, \quad \forall i$$

The utility conditional on $\mathcal{H}_t$ for trader i is

$$\mathbb{E}[U_{i,t}^{\mathcal{M}^*}|\mathcal{H}_t] = \frac{\alpha+2\lambda_t}{2(\alpha+\lambda_t)^2}\mathbb{E}[(\mathbb{E}[\theta_{i,t}|\mathcal{H}_t, s_{i,t}, p_t]-p_t)^2|\mathcal{H}_t] = \frac{\alpha+2\lambda_t}{2(\alpha+\lambda_t)^2}\frac{I_{t,\mathcal{M}^*}-1}{I_{t,\mathcal{M}^*}}\frac{(1-\rho_{t,\mathcal{M}^*})^2}{1-\rho_{t,\mathcal{M}^*}+\sigma_{i,t}^2}, \quad \forall i$$

The dynamic market equilibrium is characterized as follows:

Theorem 5 (Algorithm for Dynamic Market Choice Equilibrium with General Private Information). *The Bayesian Nash equilibrium is a set of price history $\{\mathcal{H}_t\}_t$, a sequence of market choice $\{\mathcal{M}_t^*\}_t$, and a set of inference coefficients $\{c_{s,t}, c_{p,t}, \boldsymbol{c}_{\mathcal{H},t}\}$ that characterized forwardly recursively.*

1. *Initialize with $t = 1$, $\mathcal{H}_1 = \emptyset$.*

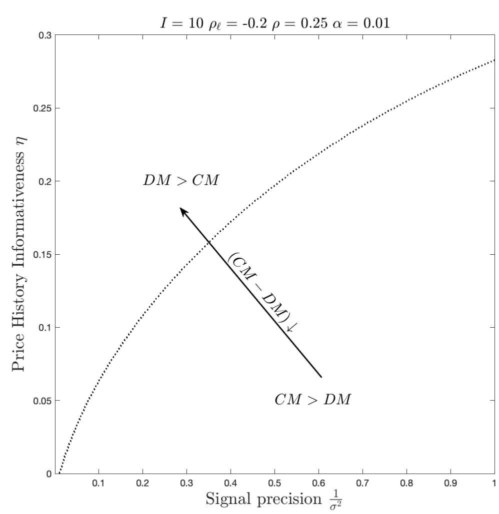
2. *Given $\mathcal{H}_t$, the equilibrium inference coefficients $\{c_{s,t}, c_{p,t}, \boldsymbol{c}_{\mathcal{H},t}\}$ is characterized in Proposition 11 with $\rho_{t,\mathcal{M}^*} = \rho_\ell$ $I_{t,\mathcal{M}^*} = 2$ if $\mathcal{M}^* = DM$, and $\rho_{t,\mathcal{M}^*} = \bar{\rho}$ $I_{t,\mathcal{M}^*} = I$ if $\mathcal{M}^* = CM$.*

3. *Given inference coefficients $\{c_{s,t}, c_{p,t}, \boldsymbol{c}_{\mathcal{H},t}\}$, If $\frac{\alpha+2\lambda_t^{CM}}{2(\alpha+\lambda_t^{CM})^2} \frac{I_t-1}{I_t} \frac{(1-\bar{\rho}_t)^2}{1-\bar{\rho}_t+\sigma_{i,t}^2} \geq \frac{\alpha+2\lambda_t^{DM}}{4(\alpha+\lambda_t^{DM})^2} \frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma_{i,t}^2}$, then $\mathcal{M}_t^* = CM$, $\mathcal{H}_{t+1} = \mathcal{H}_t \cup \{p_t\}$; otherwise, $\mathcal{M}_t^* = DM$, $\mathcal{H}_{t+1} = \mathcal{H}_t$. Repeat Steps 2-3 with the next t, until $t = T$.*

Differences Compared to the Baseline Model: There are two major differences in the model with general private signals compared with the baseline model.

First, the traders are more likely to go to DM in rounds with less precise private information. The threshold of price history informativeness to choose DM over CM in rounds with noisy private signals is lower than the threshold with precise private signals. Figure 14 compares the utility of each trader in CM versus DM. We can see that when the private signals become less precise, the threshold of price history informativeness for traders to choose DM over CM becomes lower.

Figure 14: Utility in CM vs. DM With Private Signal Precision and Price History Informativeness



Note: This figure shows the comparison of utility (welfare) in CM versus DM with private signal precision $\frac{1}{\sigma^2}$ and price history informativeness η. When the price history informativeness is higher given precision, or when the signal precision is lower (i.e. σ is higher) given price history informativeness, the difference between utilities in DM and CM becomes larger.

Second, the traders are more likely to go to CM after rounds without private signals, as price history informativeness decreases. Intuitively, the price aggregates private signals, and it is less informative when the private signals are less precise. The price history informativeness η_t will decrease after a round with extremely noisy signals. Traders will choose CM if the price history informativeness falls below the threshold by Lemma 2. In rounds traders do not receive private signals ($\sigma_{i,t} \to \infty$), there will be no trade in those rounds, and the price history informativeness decays with time.[21]

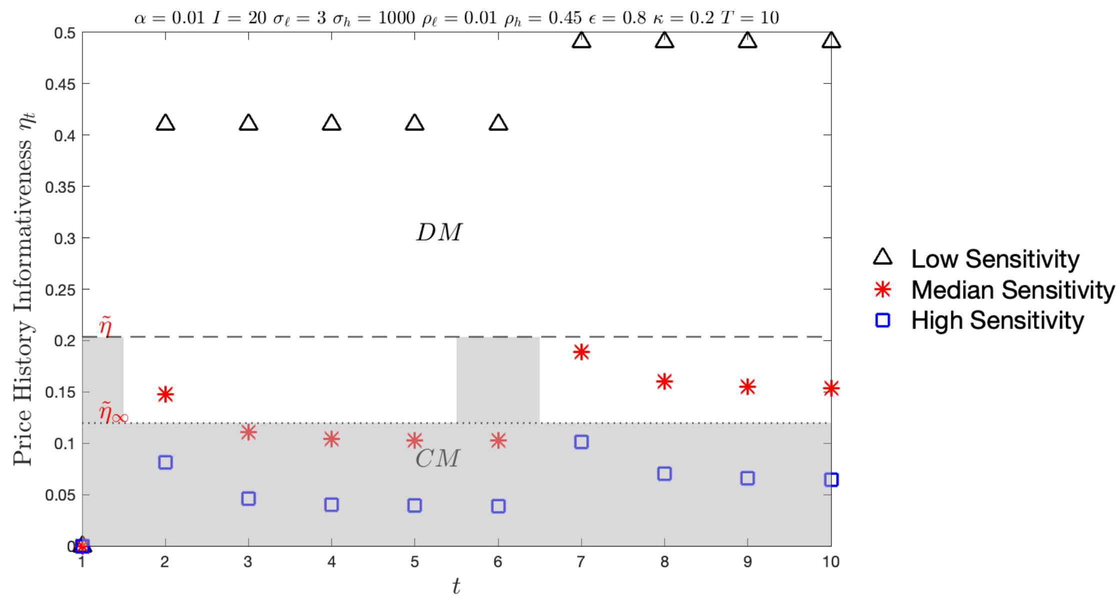
With Post-trade Transparency: When DM is post-trade transparent, different from Proposition 5 where traders stay in DM once they choose DM, traders can switch back and forth with the general information structure. By Lemma 2, we would expect the traders to choose CM if the price history informativeness is below the threshold $\tilde{\eta}$ after sufficient rounds of extremely noisy signals, regardless of post-trade transparency.

Example 2 (Infrequent Arrival of Private Signals)**.** *We consider a $T = 10$ market where DM is post-trade transparent. Each trader will receive a private signal $s_{i,t} = \theta_{i,t} + \varepsilon_{i,t}$, where $\varepsilon_{i,t} \sim \mathcal{N}(0, \sigma_{i,t}^2 \sigma_\theta^2)$, $\sigma_{i,t} = \sigma_\ell$ in rounds $t = 5k + 1$, $k \in \mathbb{N}$, and $\sigma_{i,t} = \sigma_h \gg \sigma_\ell$ otherwise. This setup captures a case where traders receive precise private signals in rounds 1 and 6, and extremely noisy signals in other rounds.*

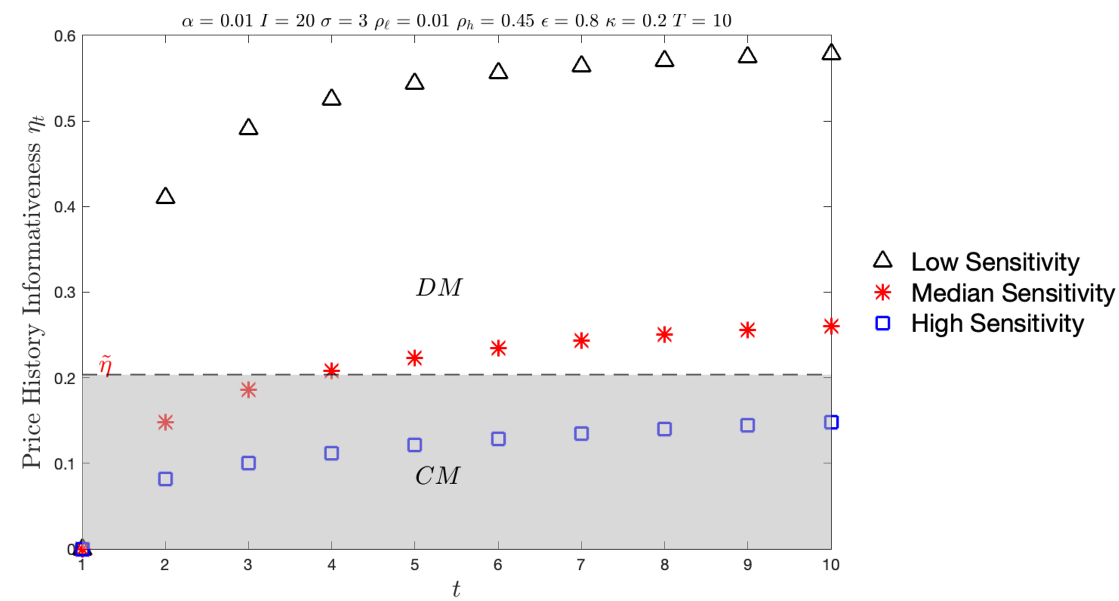
Figure 15 shows the simulated evolution of price history informativeness for different asset sensitivities. Panel (a) shows the price history informativeness and corresponding traders' market choices with the infrequent arrival of precise private signals. In panel (b) we include an example as the baseline model with post-trade transparent DM for comparison. Panel (a) indicates that when the sensitivity is neither too high nor too low and precise private signals arrive infrequently, the traders switch back and forth between CM and DM. In this example, traders choose DM in rounds 2, 7, 8, 9, and 10 and CM in other rounds. In contrast, with private signals being precise in every round (Panel (b)), traders choose to stay in DM once they enter DM (see Proposition 5). Figure 16 shows a similar pattern for the evolution of price history informativeness with different levels of asset volatilities (autocorrelations).

[21]The price is a linear combination of past prices in rounds without private signals. $\mathcal{H}_t$ is a sufficient statistic for p_t when $\sigma \to \infty$. Even if traders can observe the price, they do not learn from it conditional on $\mathcal{H}_t$.

Figure 15: Evolution of Price History Informativeness For Different Asset Sensitivities with Post-trade Transparent DM



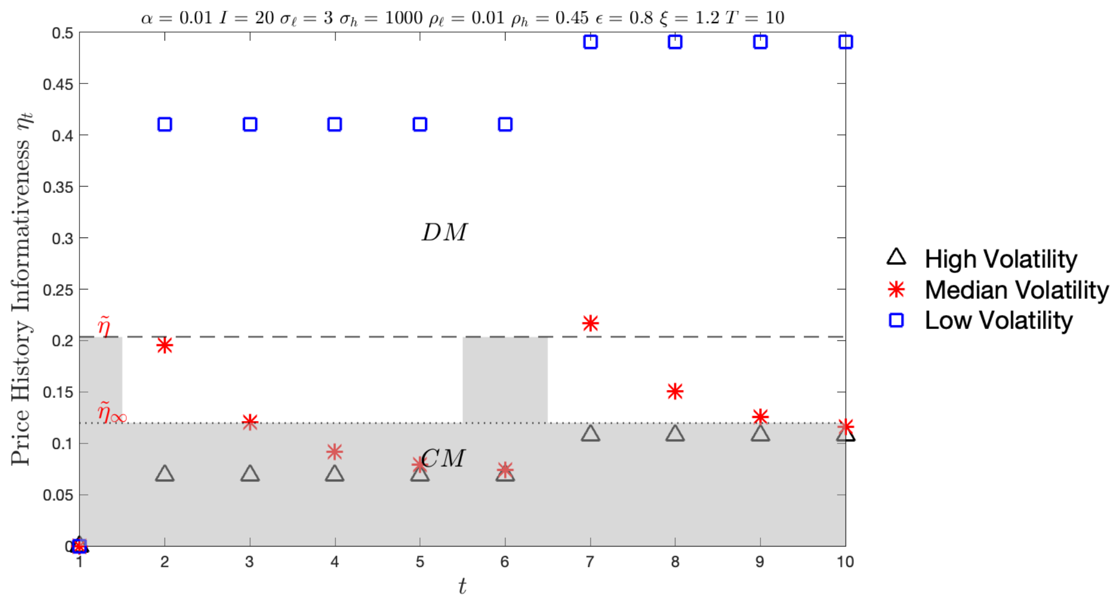
(a) Infrequent Precise Private Signals (Example 2)



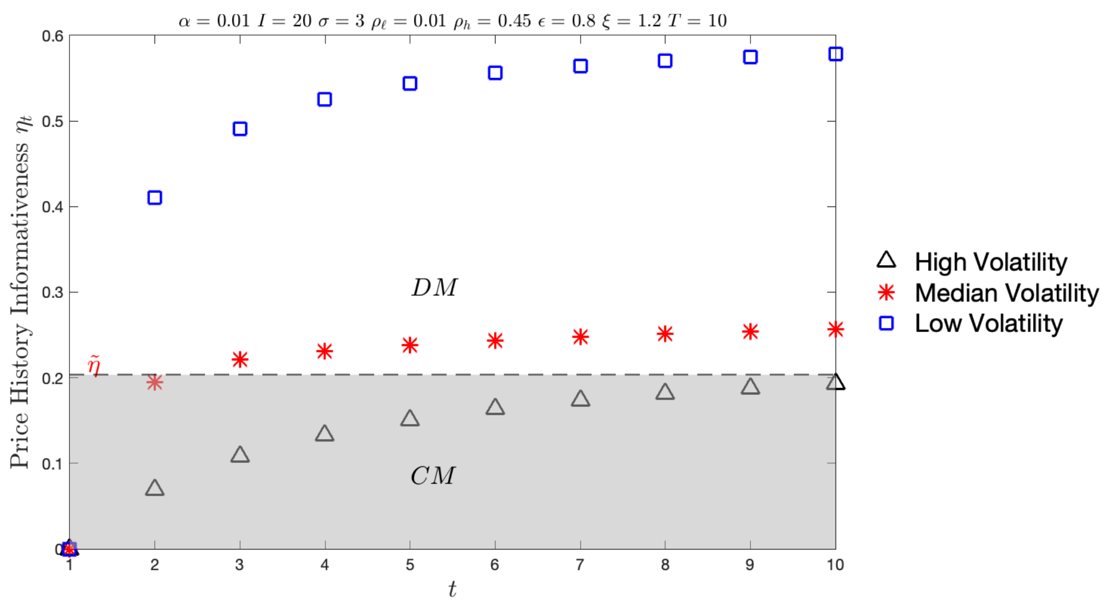
(b) Frequent Precise Private Signals

Note: This figure shows the evolution of price history informativeness for different levels of asset sensitivity ξ for $T = 10$ when DM is post-trade transparent. The black dashed line is a reference line of threshold $\tilde{\eta}$ with private signals. The black dotted line is a reference line of threshold $\tilde{\eta}_\infty$ with noisier private signals. When the marker is in the shaded area, the history informativeness in that round is lower than $\tilde{\eta}$ (or $\tilde{\eta}_\infty$ in rounds with noisier private signals) and traders choose CM. When the marker is in the unshaded area, the history informativeness in that round is higher than $\tilde{\eta}$ (or $\tilde{\eta}_\infty$ in rounds with noisier private signals) and traders choose DM.

Figure 16: Evolution of Price History Informativeness For Different Autocorrelations with Post-Trade Transparent DM



(a) Infrequent Precise Private Signals (Example 2)



(b) Frequent Precise Private Signals

Note: This figure shows the evolution of price history informativeness for different levels of autocorrelation κ for $T = 10$ when DM is post-trade transparent. The black dashed line is a reference line of threshold $\tilde{\eta}$ with private signals. The black dotted line is a reference line of threshold $\tilde{\eta}_\infty$ with noisier private signals. When the marker is in the shaded area, the history informativeness in that round is lower than $\tilde{\eta}$ (or $\tilde{\eta}_\infty$ in rounds with noisier private signals), and traders choose CM. When the marker is in the unshaded area, the history informativeness in that round is higher than $\tilde{\eta}$ (or $\tilde{\eta}_\infty$ in rounds with noisier private signals) and traders choose DM.

B.2 Non-movers and Trading Volumes

The baseline model in the paper models traders who move without frictions across decentralized and centralized markets. However, in practice, there are non-movers who trade only in one market due to regulations, lack of information, or entry cost. For example, banks are not allowed to trade in the centralized Exchange Bond Market in China, and retail traders hardly trade in dark pools in the United States. We can extend the model by adding non-movers in both the centralized and decentralized markets.

Let us add to the baseline model $I_d = 2N_d$ traders that always trade in the decentralized market. These traders are ex-ante identical to the traders in the baseline model except that they are non-movers. We also have a continuum of non-strategic retail traders who always trade in the centralized market with exogenous demand, such that the market clearing price is $p_t = A_t + \frac{B_t}{I} \sum_i q_{i,t}$, where A_t and B_t are constant known to all traders. Intuitively, if DM is opaque, adding these non-movers does not affect the price informativeness and movers' choice of venues, but the price levels and total trading volumes in CM and DM. If DM is post-trade transparent, more non-movers in DM increases the price history informativeness, and thus future traders' incentive to choose DM.

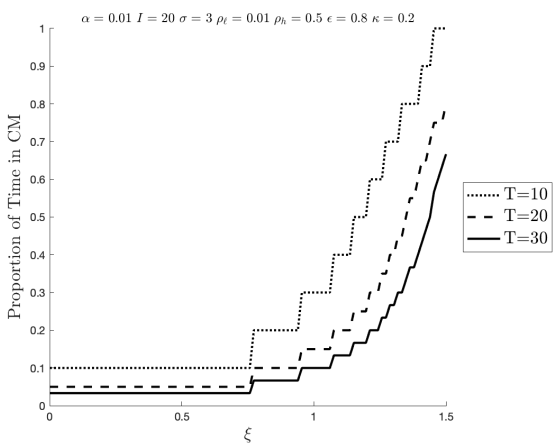
B.3 Proportion of Time in CM

Figure 17 shows the proportion of time when traders choose CM with respect to asset sensitivity ξ and volatility $(1 - \kappa^2)$. Consistent with our intuition for Propositions 3 and 4, we find that the proportion of time in CM increases with ξ, and decreases with κ.

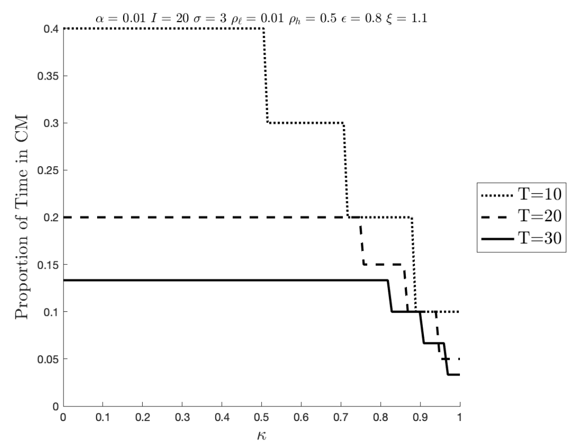
Figure 17 also shows the proportion of time when traders in CM with respect to rounds T. Numerically, we find that the choice of alternating markets between DM and CM is generally more prevalent as the trading round T increases. Note that with a small probability the proportion of time in CM with a smaller T can be lower than that with larger T, this is because the last round can end at different stages of an alternating cycle.

Intuitively, with longer T the price history informativeness η increases as its length accumulates, and it is more likely for traders to choose DM over CM. This implies assets with shorter terms are more likely to be traded in the centralized market, e.g. most options are less than 90 days. Assets with the longer term are more likely to be traded in the decentralized market or alternating market structure, e.g. bonds have maturities as long as 30 years, and equities usually do not have maturity.

Figure 17: Proportion of Time in CM



(a) Asset sensitivity ξ



(b) Autocorrelation κ

Note: This figure shows the proportion of time in CM for (a) asset sensitivities and (b) autocorrelation. The solid, dashed, and dotted lines plot the proportions of rounds in CM with total rounds $T = 30$, $T = 20$, and $T = 10$ respectively.

C Proofs

Proof of Theorem 1. Given the market structure $\mathcal{M}^*$, at round t, traders submit a demand schedule $q_{i,t}$ to maximize the utility

$$\max_{q_{i,t}} \mathbb{E}[\theta_{i,t} q_{i,t} - \frac{\alpha}{2}(q_{i,t})^2 - p_t q_{i,t} | \mathcal{H}_t, s_{i,t}, p_t]$$

By taking first order condition with respect to $q_{i,t}$, we can solve the trader i's demand schedule,

$$q_{i,t} = \frac{\mathbb{E}[\theta_{i,t}|\mathcal{H}_t, s_{i,t}, p_t] - p_t}{\alpha + \lambda_{i,t}}$$

where $\lambda_{i,t} \equiv \frac{dp_t}{dq_{i,t}}$ is the price impact. By symmetry, the price impacts are the same for all traders in the same round $\lambda_{i,t} = \lambda_t, \forall i \in I_{t,\mathcal{M}^*}$. We can parameterize $\mathbb{E}[\theta_{i,t}|\mathcal{H}_t, s_{i,t}, p_t] = \boldsymbol{c}_{\mathcal{H},i,t}\mathcal{H}_t + c_{s,i,t}s_{i,t} + c_{p,i,t}p_t$, where $\boldsymbol{c}_{\mathcal{H},i,t} \in \mathbb{R}^{1\times|\mathcal{H}_t|}$, $c_{s,i,t} \in \mathbb{R}$, and $c_{p,i,t} \in \mathbb{R}$. By symmetry, the inference coefficients are the same for all traders in the same round, $\boldsymbol{c}_{\mathcal{H},i,t} = \boldsymbol{c}_{\mathcal{H},t}$, $c_{s,i,t} = c_{s,t}$ and $c_{p,i,t} = c_{p,t}$.

In equilibrium, by market clearing condition, λ_t is equal to the inverse of the slope of the residual demand,

$$\lambda_t = (-\sum_{j\neq i} \frac{dq_{j,t}}{dp_t})^{-1} = \frac{\alpha}{(I_t - 1)(1 - c_{p,t}) - 1}$$

Given the parameterization, the equilibrium price is,

$$p_t = (1 - c_{p,t})^{-1}(\boldsymbol{c}_{\mathcal{H},t}\mathcal{H}_t + c_{s,t}\bar{s}_t) \tag{8}$$

where $\bar{s}_t = \frac{1}{I_t}\sum_i s_{i,t}$ is the average signal in the exchange (for DM, it is the average signal in each pair).

(Step 1: Inference Coefficients) The trader i's value $\theta_{i,t}$, the equilibrium price p_t given equation (8), the history $\mathcal{H}_t$ and the private signal $s_{i,t}$ are jointly normally distributed. By projection theorem, the inference coefficients $c_{\mathcal{H},t}$, $c_{s,t}$, and $c_{p,t}$ can be determined given the joint distribution of $(\theta_{i,t}, s_{i,t}, \mathcal{H}_t, p_t)$,

$$\begin{pmatrix} \theta_{i,t} \\ s_{i,t} \\ \mathcal{H}_t \\ p_t \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} \mathbb{E}[\theta] \\ \mathbb{E}[\theta] \\ \mathbb{E}[\boldsymbol{\theta}] \\ \mathbb{E}[\theta] \end{pmatrix}, \begin{pmatrix} var(\theta_{i,t}) & cov(\theta_{i,t}, s_{i,t}) & cov(\theta_{i,t}, \mathcal{H}'_t) & cov(\theta_{i,t}, p'_t) \\ cov(s_{i,t}, \theta_{i,t}) & var(s_{i,t}) & cov(s_{i,t}, \mathcal{H}'_t) & cov(s_{i,t}, p'_t) \\ cov(\mathcal{H}_t, \theta_{i,t}) & cov(\mathcal{H}_t, s_{i,t}) & cov(\mathcal{H}_t, \mathcal{H}'_t) & cov(\mathcal{H}_t, p'_t) \\ cov(p_t, \theta_{i,t}) & cov(p_t, s_{i,t}) & cov(p_t, \mathcal{H}'_t) & cov(p_t, p'_t) \end{pmatrix}\right]$$

where

$$
\begin{aligned}
cov\left(p_t, \theta_{i,t}\right) &= \left(1-c_{p,t}\right)^{-1}\left(c_{s,t} cov\left(\bar{s}_t, \theta_{i,t}\right) + \boldsymbol{c}_{\mathcal{H},t} cov(\mathcal{H}_t, \theta_{i,t})\right) \\
cov\left(p_t, s_{i,t}\right) &= \left(1-c_{p,t}\right)^{-1}\left(c_{s,t} cov\left(\bar{s}_t, s_{i,t}\right) + \boldsymbol{c}_{\mathcal{H},t} cov(\mathcal{H}_t, s_{i,t})\right) \\
cov(p_t, \mathcal{H}_t') &= \left(1-c_{p,t}\right)^{-1}\left(c_{s,t} cov\left(\bar{s}_t, \mathcal{H}_t'\right) + \boldsymbol{c}_{\mathcal{H},t} cov(\mathcal{H}_t, \mathcal{H}_t')\right) \\
cov\left(p_t, p_t'\right) &= \left(1-c_{p,t}\right)^{-1}\left(c_{s,t} var(\bar{s}_t) + c_{s,t} cov\left(\bar{s}_t, \mathcal{H}_t'\right)\boldsymbol{c}_{\mathcal{H},t}' + \boldsymbol{c}_{\mathcal{H}} cov(\mathcal{H}_t, \mathcal{H}_t')\,\boldsymbol{c}_{\mathcal{H}}'\right)
\end{aligned}
$$

By projection theorem, we have

$$
[c_{s,t}, \boldsymbol{c}_{\mathcal{H},t}, c_{p,t}] \begin{pmatrix} cov(s_{i,t}) & cov(s_{i,t}, \mathcal{H}_t') & cov\left(s_{i,t}, p_t\right) \\ cov\left(\mathcal{H}_t, s_{i,t}\right) & cov\left(\mathcal{H}_t, \mathcal{H}_t'\right) & cov\left(\mathcal{H}_t, p_t\right) \\ cov\left(p_t, s_{i,t}\right) & cov\left(p_t, \mathcal{H}_t'\right) & cov\left(p_t, p_t'\right) \end{pmatrix} = [cov(\theta_{i,t}, s_{i,t}), cov(\theta_{i,t}, \mathcal{H}_t'), cov(\theta_{i,t}, p_t)] \tag{9}
$$

From equation (22), we have the following equations,

$$
cov(c_{s,t}s_{i,t} + \boldsymbol{c}_{\mathcal{H},t}\mathcal{H}_t + c_{p,t}p_t, s_{i,t}) = cov(\theta_{i,t}, s_{i,t}) \tag{10}
$$

$$
cov(c_{s,t}s_{i,t} + \boldsymbol{c}_{\mathcal{H}_t,t}\mathcal{H}_t + c_{p,t}p_t, \mathcal{H}_t') = cov(\theta_{i,t}, \mathcal{H}_t') \tag{11}
$$

$$
cov(c_{s,t}s_{i,t} + \boldsymbol{c}_{\mathcal{H}_t,t}\mathcal{H}_t + c_{p,t}p_t, p_t') = cov(\theta_{i,t}, p_t) \tag{12}
$$

Given that $p_t = (1-c_{p,t})^{-1}(\boldsymbol{c}_{\mathcal{H},t}\mathcal{H}_t + c_{s,t}\bar{s}_t)$, subtracting $\boldsymbol{c}_{\mathcal{H}}$ times equation (24) from equation (25) gives us

$$
cov(c_{s,t}s_{i,t} + \boldsymbol{c}_{\mathcal{H},t}\mathcal{H}_t + c_{p,t}p_t, \bar{s}_t) = cov(\theta_{i,t}, \bar{s}_t) \tag{13}
$$

Averaging equation (23) over i in the same exchange gives

$$
c_{s,t}(1+\sigma^2)\sigma_\theta^2 + cov(\boldsymbol{c}_{\mathcal{H},t}\mathcal{H}_t + c_{p,t}p_t, \bar{s}_t) = \sigma_\theta^2 \tag{14}
$$

Comparing equation (26) and (27), we have

$$
c_{s,t} = \frac{cov(\theta_{i,t}, \bar{s}_t) - \sigma_\theta^2}{cov(s_{i,t}, \bar{s}_t) - (1+\sigma^2)\sigma_\theta^2} = \frac{1-\rho_{t,\mathcal{M}^*}}{1-\rho_{t,\mathcal{M}^*} + \sigma^2} \tag{15}
$$

where $\rho_{t,\mathcal{M}^*}$ is the correlation of traders given market structure $\mathcal{M}^*$.

Given equation (28), we can rewrite equation (24) as

$$
\left(1-c_{p,t}\right)^{-1}\left(c_{s,t} cov(s_{i,t}, \mathcal{H}_t') + \boldsymbol{c}_{\mathcal{H},t} cov(\mathcal{H}_t, \mathcal{H}_t')\right) = cov(\theta_{i,t}, \mathcal{H}_t') \tag{16}
$$

and equation (26) as

$$\left(1-c_{p,t}\right)^{-1}\left(c_{s,t}var(\bar{s}_t)+\boldsymbol{c}_{\mathcal{H}}cov(\mathcal{H}_t,\bar{s}_t)\right)=cov(\theta_{i,t},\bar{s}_t) \tag{17}$$

Given that $cov(\mathcal{H},\theta_i)=cov(\mathcal{H},s_i)=cov(\mathcal{H},s_j)$, $\forall j\neq i$, and c_s in equation (28) , we can solve the term $\boldsymbol{c}_{\mathcal{H},t}$ and $c_{p,t}$ by equation (16) and equation (17),

$$\boldsymbol{c}_{\mathcal{H},t}=\frac{(1-\rho_{t,\mathcal{M}^*})\sigma^2}{(1-\rho_{t,\mathcal{M}^*}+\sigma^2)\left(1+(I_{t,\mathcal{M}^*}-1)\rho_{t,\mathcal{M}^*}-I_{t,\mathcal{M}^*}\eta\right)}\boldsymbol{\tau}_t'\boldsymbol{\Upsilon}_t^{-1}$$

$$c_{p,t}=\frac{I_{t,\mathcal{M}^*}(\rho_{t,\mathcal{M}^*}-\eta)\sigma^2}{(1-\rho_{t,\mathcal{M}^*}+\sigma^2)\left(1+(I_{t,\mathcal{M}^*}-1)\rho_{t,\mathcal{M}^*}-I_{t,\mathcal{M}^*}\eta\right)}$$

where $\eta=\boldsymbol{\tau}_t'(\boldsymbol{\Upsilon}_t)^{-1}\boldsymbol{\tau}_t$, $\boldsymbol{\tau}_t\equiv\frac{cov(\mathcal{H}_t,\theta_{i,t})}{\sigma_\theta^2}\in\mathbb{R}^{|\mathcal{H}|}$, and $\boldsymbol{\Upsilon}_t\equiv\frac{cov(\mathcal{H}_t,\mathcal{H}_t')}{\sigma_\theta^2}\in\mathbb{R}^{|\mathcal{H}|\times|\mathcal{H}|}$.

The equilibrium price impact is

$$\lambda_t=\frac{\alpha}{(I_{t,\mathcal{M}^*}-1)(1-c_{p,t})-1},\quad\forall i$$

The ex-ante utility for trader i is

$$\mathbb{E}[U_{i,t}|\mathcal{H}_t]=\frac{\alpha+2\lambda_t}{2(\alpha+\lambda_t)^2}\mathbb{E}[(\mathbb{E}[\theta_{i,t}|\mathcal{H}_t,s_{i,t},p_t]-p_t)^2|\mathcal{H}_t]=\frac{\alpha+2\lambda_t}{2(\alpha+\lambda_t)^2}\frac{I_{t,\mathcal{M}^*}-1}{I_{t,\mathcal{M}^*}}\frac{(1-\rho_{t,\mathcal{M}^*})^2}{1-\rho_{t,\mathcal{M}^*}+\sigma^2},\quad\forall i$$

■

Proof of Lemma 1. We leave out the subscripts t and $\mathcal{M}^*$ to ease the notation. Taking the derivative of welfare over ρ, we have

$$\frac{d\mathbb{E}[U_i|\mathcal{H}]}{d\rho}=-\frac{\alpha+2\lambda}{2(\alpha+\lambda)^2}\frac{I-1}{I}\frac{(1-\rho)\left(1-\rho+2\sigma^2\right)}{(1-\rho+\sigma^2)^2}-\frac{\lambda}{(\alpha+\lambda)^3}\frac{I-1}{I}\frac{(1-\rho)^2}{1-\rho+\sigma^2}\frac{d\lambda}{d\rho}<0.$$

Keep everything else constant,

$$\frac{d\lambda}{d\rho}=\lambda^2\frac{I\,\sigma^2\,(I-1)\,\left(I(\eta-\frac{(I-1)\rho+1}{I})^2+\frac{I-1}{I}(1-\rho)^2+(1-\eta)\sigma^2\right)}{\alpha\,\left(1-\rho+\sigma^2\right)^2\,\left(1+(I-1)\rho-I\eta\right)^2}>0.$$

Thus $\frac{d\mathbb{E}[U_i|\mathcal{H}]}{d\rho}<0$. The traders' welfare decreases with trader value correlation ρ. ■

Proof of Lemma 2. We leave out the subscripts t and $\mathcal{M}^*$ to ease the notation. Keep everything else constant,

$$\frac{d\lambda}{d\eta}=-\lambda^2\frac{I\,\sigma^2\,(I-1)\,(1-\rho)}{\alpha\,\left(1-\rho+\sigma^2\right)\,\left(1+(I-1)\rho-I\eta\right)^2}<0.$$

Therefore the price impact decreases with price history informativeness η.

The expected utility of any trader i is

$$\mathbb{E}[U_i|\mathcal{H}] = \frac{\alpha + 2\lambda}{2(\alpha+\lambda)^2}\frac{I-1}{I}\frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2}.$$

Taking the derivative of welfare over η, we have

$$\frac{d\mathbb{E}[U_i|\mathcal{H}]}{d\eta} = \frac{\sigma^2}{\alpha}\frac{\left(1-\rho+\sigma^2\right)\,\left(1+(I-1)\rho - I\,\eta\right)}{(I-1)\,\left(1+(I-1)\rho+\sigma^2-I\,\eta\right)^3} > 0.$$

Therefore the traders' welfare increases with price history informativeness η. ■

Proof of Lemma 3. We leave out the subscript t to ease the notation.

Monotonicity: The difference between trader i's utility in the CM and DM is

$$\mathbb{E}[U_i^{CM}|\mathcal{H}] - \mathbb{E}[U_i^{DM}|\mathcal{H}] = \frac{\alpha+2\lambda^{CM}}{2(\alpha+\lambda^{CM})^2}\frac{I-1}{I}\frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2} - \frac{\alpha+2\lambda^{DM}}{2(\alpha+\lambda^{DM})^2}\frac{1}{2}\frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2}.$$

Taking its derivative over the public informativeness η, we have

$$\frac{d(\mathbb{E}[U_i^{CM}|\mathcal{H}] - \mathbb{E}[U_i^{DM}|\mathcal{H}])}{d\eta} = \frac{\sigma^2}{\alpha}\left(\frac{\left(1-\bar{\rho}+\sigma^2\right)\,\left(1+(I-1)\bar{\rho}-I\,\eta\right)}{(I-1)\,\left(1+(I-1)\bar{\rho}+\sigma^2-I\,\eta\right)^3} - \frac{\left(1-\rho_\ell+\sigma^2\right)\,\left(1+\rho_\ell-2\,\eta\right)}{\left(1+\rho_\ell+\sigma^2-2\,\eta\right)^3}\right) < 0.$$

given that $\sigma \geq ((\frac{2(I-1)}{I})^{1/3}-1)^{-1/2}$, $\bar{\rho} > \rho_\ell$, and $\eta \leq \frac{1+(I-1)\bar{\rho}}{I} \leq \frac{1+\rho_\ell}{2}$, for the joint correlation matrix of values to be positive semidefinite.

CM vs. DM: The lowest possible η is ρ_ℓ for equilibrium existence in the DM. $\lim_{\eta\to\rho_\ell}\lambda^{DM} = \infty$ and $\lim_{\eta\to\rho_\ell}\mathbb{E}[U_i^{DM}|\mathcal{H}] = 0$, therefore

$$\lim_{\eta\to\rho_\ell}(\mathbb{E}[U_i^{CM}|\mathcal{H}] - \mathbb{E}[U_i^{DM}|\mathcal{H}]) = \lim_{\eta\to\rho_\ell}\mathbb{E}[U_i^{CM}|\mathcal{H}] > 0. \tag{18}$$

Given $\frac{(\rho_\ell+1)}{2} - \frac{1+(I-1)\bar{\rho}}{I} \geq 0$ for the joint correlation matrix of values to be positive semidefinite, the maximum η is $\frac{1+(I-1)\bar{\rho}}{I}$,

$$\lim_{\eta\to\frac{1+(I-1)\bar{\rho}}{I}}(\mathbb{E}[U_i^{CM}|\mathcal{H}] - \mathbb{E}[U_i^{DM}|\mathcal{H}]) = \frac{1}{2\alpha}\frac{I-1}{I}\frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2} - \frac{\alpha+2\lim_{\eta\to\frac{1+(I-1)\bar{\rho}}{I}}\lambda_n}{4(\alpha+\lim_{\eta\to\frac{1+(I-1)\bar{\rho}}{I}}\lambda_n)^2}\frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2}. \tag{19}$$

where $\lim_{\eta\to\frac{1+(I-1)\bar{\rho}}{I}}\lambda_n = \frac{\alpha\left(1-\rho_\ell+\sigma^2\right)\left(\frac{1+\rho_\ell}{2}-\frac{1+(I-1)\bar{\rho}}{I}\right)}{\left(\rho_\ell-\frac{1+(I-1)\bar{\rho}}{I}\right)\sigma^2}$. There exists unique $\bar{\rho}^*$ as a function of (I,ρ_ℓ,σ^2) such that $\lim_{\eta\to\frac{1+(I-1)\bar{\rho}}{I}}(\mathbb{E}[U_i^{CM}|\mathcal{H}] - \mathbb{E}[U_i^{DM}|\mathcal{H}]) = 0$ if $\bar{\rho} = \bar{\rho}^*(I,\rho_\ell,\sigma^2)$. If $\bar{\rho} > \bar{\rho}^*$,

$$\lim_{\eta\to\frac{1+(I-1)\bar{\rho}}{I}}(\mathbb{E}[U_i^{CM}|\mathcal{H}] - \mathbb{E}[U_i^{DM}|\mathcal{H}]) < 0. \tag{20}$$

Given that the difference between the ex-ante utility of the centralized market and that of the decentralized market is continuous and monotonically decreasing in η, by equations (18) and (20), if $\bar{\rho} > \bar{\rho}^*(I, \rho_\ell, \sigma^2)$, there exist $\tilde{\eta}(I, \bar{\rho}, \rho_\ell, \sigma^2)$ such that the centralized market has equal welfare as the decentralized market if $\eta = \tilde{\eta}$, the centralized market has higher welfare than the decentralized market if $\eta < \tilde{\eta}$, and otherwise if $\eta \geq \tilde{\eta}$.

If $\bar{\rho} \leq \bar{\rho}^*(I, \rho_\ell, \sigma^2)$,

$$\lim_{\eta \to \frac{1+(I-1)\bar{\rho}}{I}} (\mathbb{E}[U_i^{CM}|\mathcal{H}] - \mathbb{E}[U_i^{DM}|\mathcal{H}]) > 0. \tag{21}$$

Given that the difference between the utility of the centralized market and that of the decentralized market is continuous and monotonically decreasing in η, by equation (21) the utility in the centralized market is always higher than the utility in the decentralized market regardless of η. ■

Proof of Proposition 1. The proof of Proposition 1 directly follows from Lemma 3, as no price history informativeness η will allow traders to choose DM. ■

Proof of Proposition 2. By Lemma 1, the expected utility $\mathbb{E}[U_i^{CM}|\mathcal{H}]$ decreases with $\bar{\rho}$, $\mathbb{E}[U_i^{DM}|\mathcal{H}]$ decreases with ρ_ℓ, if $\mathbb{E}[U_i^{CM}(\bar{\bar{\rho}})|\mathcal{H}] - \mathbb{E}[U_i^{DM}(\underline{\rho_\ell})|\mathcal{H}] < 0$, then $\mathbb{E}[U_i^{CM}(\bar{\rho})|\mathcal{H}] - \mathbb{E}[U_i^{DM}(\rho_\ell)|\mathcal{H}]$ for any $\rho_\ell < \underline{\rho_\ell}$ and $\bar{\rho} > \bar{\bar{\rho}}$.

By Lemma 4 we are subject to find $\underline{\rho_\ell}$ and $\bar{\bar{\rho}}$ that makes $\mathbb{E}[U_i^{CM}(\bar{\bar{\rho}})|\mathcal{H}] - \mathbb{E}[U_i^{DM}(\underline{\rho_\ell})|\mathcal{H}] < 0$ when $\eta = 0$. It is easy to see $\underline{\rho_\ell} < 0$ for DM to exist. And by Lemma 3, $\bar{\bar{\rho}} > \bar{\rho}^*$ given there exists η for traders to choose DM over CM.

When $\eta = 0$, the trader's utility in the CM is

$$\mathbb{E}[U_i^{CM}] = \frac{\alpha + 2\lambda^{CM}}{2(\alpha + \lambda^{CM})^2} \frac{I-1}{I} \frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2} \quad \forall i \in I$$

where $\lambda^{CM} = \frac{\alpha}{(I-1)(1-c_p^{CM})-1}$, $c_p^{CM} = \frac{I\bar{\rho}\sigma^2}{(1-\bar{\rho}+\sigma^2)(1+(I-1)\bar{\rho})}$.

The trader's utility in the DM is

$$\mathbb{E}[U_i^{DM}] = \frac{\alpha + 2\lambda^{DM}}{4(\alpha + \lambda_1^{DM})^2} \frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2} \quad \forall i \in I$$

where $\lambda^{DM} = \frac{\alpha}{-c_p^{DM}}$, $c_p^{DM} = \frac{2\rho_\ell\sigma^2}{(1-\rho_\ell+\sigma^2)(1+\rho_\ell)}$.

For the correlation matrix to be well-defined (positive-semidefinite), the maximum $\bar{\rho}$ as a function of ρ_ℓ is $\frac{\frac{I(1+\rho_\ell)}{2}-1}{I-1}$.

$$\lim_{\rho_\ell \to -1} \lim_{\bar{\rho} \to \frac{\frac{I(1+\rho_\ell)}{2}-1}{I-1}} \mathbb{E}[U_i^{DM}|\mathcal{H}] - \mathbb{E}[U_i^{CM}|\mathcal{H}] = \frac{1}{\alpha(2+\sigma^2)} - \frac{1}{2\alpha} \frac{I}{(I+(I-1)\sigma^2)} > 0$$

Given $\mathbb{E}[U_i^{CM}]$ decreases with $\bar{\rho}$, $\mathbb{E}[U_i^{DM}]$ decreases with ρ_ℓ, and $\mathbb{E}[U_i^{DM}] - \mathbb{E}[U_{i,1}^{CM}]$ is continuous in $\bar{\rho}$ and ρ_ℓ, there exists $\underline{\rho_\ell} < 0$ and $\bar{\bar{\rho}} > \bar{\rho}^*$ such that $\mathbb{E}[U_i^{DM}] = \mathbb{E}[U_i^{CM}]$, $\mathbb{E}[U_i^{DM}] - \mathbb{E}[U_i^{CM}] > 0$ if $\rho_\ell < \underline{\rho_\ell}$ and $\bar{\rho} > \bar{\bar{\rho}}$. ■

Proof of Proposition 3. With $\rho_\ell \geq 0$, the DM equilibrium does not exist due to extreme adverse selection. Traders will choose CM in the first round.

Step 1. Less (More) history, lower (higher) η_t: $\eta_t = \frac{var(\theta_{i,t}) - var(\theta_{i,t}|\mathcal{H}_t)}{var(\theta_{i,t})}$. To see this point, consider $\tilde{\eta}_t$ derived from $\tilde{\mathcal{H}}_t$. $\tilde{\mathcal{H}}_t$ is a strict subset of the price history $\tilde{\mathcal{H}}_t \subset \mathcal{H}_t$. $\tilde{\eta}_t = \frac{var(\theta_{i,t}) - var(\theta_{i,t}|\tilde{\mathcal{H}}_t)}{var(\theta_{i,t})}$. As $\tilde{\mathcal{H}}_t$ is a sub-sigma-algebra of $\mathcal{H}_t$, $var(\theta_{i,t}|\mathcal{H}_t) \leq var(\theta_{i,t}|\tilde{\mathcal{H}}_t)$. Thus $\tilde{\eta}_t \leq \eta_t$. This result tells us to keep everything else including the market choices in other rounds constant, if the trader chooses DM (CM) instead at round t, the informativeness in any round $\tau > t$ decreases (increases).

Step 2. Higher ξ, lower η_t: With symmetric market assumption, the price history is a linear combination of the past average signals in the CM. Let $\mathcal{H}_t = \boldsymbol{L}\bar{\boldsymbol{s}}^{CM}_{\tau<t}$, where $\bar{\boldsymbol{s}}^{CM}_{\tau<t} \in \mathbb{R}^{|\mathcal{H}_t|}$ is the vector of the average signals in past rounds where CM is the optimal market choice, and $\boldsymbol{L} \in \mathbb{R}^{|\mathcal{H}_t|\times|\mathcal{H}_t|}$ is a linear operator. We have the following equivalence:

$$
\begin{aligned}
\eta_t &= \frac{cov(\theta_{i,t}, \mathcal{H}_t) cov(\mathcal{H}_t, \mathcal{H}_t')^{-1} cov(\mathcal{H}_t, \theta_{i,t})}{\sigma^2_\theta} \\
&= \frac{cov(\theta_{i,t}, \boldsymbol{L}'(\bar{\boldsymbol{s}}^{CM}_{\tau<t})') cov(\boldsymbol{L}\bar{\boldsymbol{s}}^{CM}_{\tau<t}, (\bar{\boldsymbol{s}}^{CM}_{\tau<t})'\boldsymbol{L}')^{-1} cov(\boldsymbol{L}\bar{\boldsymbol{s}}^{CM}_{\tau<t}, \theta_{i,t})}{\sigma^2_\theta} \\
&= \frac{cov(\theta_{i,t}, (\bar{\boldsymbol{s}}^{CM}_{\tau<t})') cov(\bar{\boldsymbol{s}}^{CM}_{\tau<t}, (\bar{\boldsymbol{s}}^{CM}_{\tau<t})')^{-1} cov(\bar{\boldsymbol{s}}^{CM}_{\tau<t}, \theta_{i,t})}{\sigma^2_\theta}
\end{aligned}
$$

We only need to compute the joint distribution of $\{\bar{s}_\tau\}_{\tau<t}$ and $\{\theta_{i,t}\}_i$ to obtain the η_t given the above equivalence.

Given the primitive, we have $cov(\bar{s}^{CM}_\tau, \bar{s}^{CM}_\tau) = \frac{1+(I-1)\bar{\rho}+\sigma^2}{I}\sigma^2_\theta$, $cov(\bar{s}^{CM}_\tau, \bar{s}^{CM}_{\tau'}) = \frac{1+\xi^2\kappa^{|\tau'-\tau|}}{(1+\xi^2)(1+\epsilon^2)}\sigma^2_\theta$, $cov(\bar{s}^{CM}_\tau, \theta_{i,t}) = \frac{1+\xi^2\kappa^{t-\tau}}{(1+\xi^2)(1+\epsilon^2)}\sigma^2_\theta$ for $\tau < t$. Fixing the past market choice, we have the following comparative static:

$$
\frac{d\eta_t}{d\xi} = \frac{1}{\sigma^2_\theta} \frac{dcov(\theta_{i,t}, (\bar{\boldsymbol{s}}^{CM}_{\tau<t})') cov(\bar{\boldsymbol{s}}^{CM}_{\tau<t}, (\bar{\boldsymbol{s}}^{CM}_{\tau<t})')^{-1} cov(\bar{\boldsymbol{s}}^{CM}_{\tau<t}, \theta_{i,t})}{d\xi} < 0
$$

which means, the price history informativeness is decreasing in asset sensitivity ξ given past market choice.

Step 3. Existence of $\underline{\xi}$: By Lemma 3, to show the existence of $\underline{\xi}$, we will need to check if there exists ξ that $\eta_t \geq \tilde{\eta}$, $\forall t$. By Step 1 the lowest possible η_t over t and all possible market choices is the $\underline{\eta}_T$ with price history set including only p^{CM}_1. Given $\frac{d\eta_t}{d\xi} < 0$, we are subject to check if the smallest ξ makes $\underline{\eta}_T \geq \tilde{\eta}$.

$$
\lim_{\xi\to 0} \underline{\eta}_T = \frac{1}{(1+\epsilon^2)^2} \frac{I}{1+(I-1)\bar{\rho}+\sigma^2}
$$

To show that $\underline{\eta}_T \geq \tilde{\eta}$, we are subject to show $\lim_{\xi\to 0} \mathbb{E}[U^{CM}_i(\underline{\eta}_T)|\mathcal{H}] - \mathbb{E}[U^{DM}_i(\underline{\eta}_T)|\mathcal{H}] < 0$.

$$
\lim_{\xi\to 0} \mathbb{E}[U^{CM}_i(\underline{\eta}_T)|\mathcal{H}] - \mathbb{E}[U^{DM}_i(\underline{\eta}_T)|\mathcal{H}] = \frac{\alpha+2\lambda^{CM}}{2(\alpha+\lambda^{CM})^2} \frac{I-1}{I} \frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2} - \frac{\alpha+2\lambda^{DM}}{2(\alpha+\lambda^{DM})^2} \frac{1}{2} \frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2}.
$$

$\frac{d\lim_{\xi\to 0}\mathbb{E}[U_i^{CM}(\underline{\eta}_T)|\mathcal{H}]-\mathbb{E}[U_i^{DM}(\underline{\eta}_T)|\mathcal{H}]}{d\epsilon} < 0$. There exist $\bar{\epsilon}(\sigma^2, I)$ such that for any $\epsilon < \bar{\epsilon}(\sigma^2, I)$, $\lim_{\xi\to 0}\mathbb{E}[U_i^{CM}(\underline{\eta}_T)|\mathcal{H}]-\mathbb{E}[U_i^{DM}(\underline{\eta}_T)|\mathcal{H}] < 0$. Given $\frac{d\eta_t}{d\xi} < 0$ and η_t is continuous in ξ, and $\epsilon < \bar{\epsilon}(\sigma^2, I)$, there exists $\underline{\xi}$, such that for any $\xi \in [0, \underline{\xi})$, traders will stay in the DM since the 2nd round.

Step 4. Existence of $\bar{\xi}$: By Lemma 3, to show the existence of $\bar{\xi}$, we will need to check if there exists ξ that $\eta_t \leq \tilde{\eta}$, $\forall t$. By Step 1 the highest possible η_t over t and all possible market choices is $\bar{\eta}_T$ when all past market choices are CMs and all past prices are available. Therefore, we are subject to check a hypothetical $\bar{\eta}_T$ that is generated with the history of all past CM prices. Given Step 2, $\frac{d\eta_t}{d\xi} < 0$ and η_t is continuous in ξ, we are subject to check if the highest ξ makes $\bar{\eta}_T \leq \tilde{\eta}$.

$$\lim_{\xi\to\infty}\bar{\eta}_T < (\frac{\kappa}{1+\epsilon^2})^2\frac{I(T-1)}{1+(I-1)\bar{\rho}+\sigma^2}$$

There exists $\bar{\kappa}$ such that for $\kappa < \bar{\kappa}(\sigma^2, I, \epsilon)$, $3 \leq T < \bar{T}(\sigma^2, I, \epsilon)$, $\frac{d\lim_{\xi\to 0}\mathbb{E}[U_i^{CM}(\bar{\eta}_T)|\mathcal{H}]-\mathbb{E}[U_i^{DM}(\bar{\eta}_T)|\mathcal{H}]}{dI} > 0$. Given $\frac{d\eta_t}{d\xi} < 0$, η_t is continuous in ξ, and $\kappa < \bar{\kappa}(\sigma^2, I, \epsilon)$, there exist $\bar{\xi}$, for any $\xi \in [\bar{\xi}, \infty)$, traders will stay in the CM.

Step 5. Summarize: Given $3 \leq T < \bar{T}(\sigma^2, I, \epsilon)$, $\epsilon < \bar{\epsilon}(\sigma^2, I)$ and $\kappa < \bar{\kappa}(\sigma^2, I, \epsilon)$, there exists $\underline{\xi}$ and $\bar{\xi}$ such that traders will choose CM in the first round, and

1. When the asset sensitivity to shocks to fundamentals is sufficiently low $\xi \in [0, \underline{\xi})$, the traders shift to DM in the second round and stay there.

2. When the asset sensitivity to shocks to fundamentals is intermediate $\xi \in [\underline{\xi}, \bar{\xi})$, the traders will alternate between CM and DM. This is because, for $\xi \in [\underline{\xi}, \bar{\xi})$, there exists t such that $\eta_t > \tilde{\eta}$, and there also exists t such that $\eta_t < \tilde{\eta}$.

3. When the asset sensitivity to shocks to fundamentals is sufficiently high $\xi \in [\bar{\xi}, \infty)$, the traders will always stay in the CM.

■

Proof of Proposition 4. With $\rho_\ell \geq 0$, the DM equilibrium does not exist due to extreme adverse selection. Traders will choose CM in the first round.

Step 1. Less (More) history, lower (higher) η_t: See proof of Proposition 3.

Step 2. Higher ξ, lower η_t: The derivation of η as a function of the joint distribution of signals and values follows from the proof of Proposition 3. Fixing the past market choice, we have the following comparative static:

$$\frac{d\eta_t}{d\kappa} > 0$$

which means, the price history informativeness is decreasing in autocorrelation κ given past market choice.

Step 3. Existence of $\bar{\kappa}$: By Lemma 3, to show the existence of $\bar{\kappa}$, we will need to check if there exists κ that $\eta_t \geq \tilde{\eta}, \forall t$. By Step 1 the lowest possible η_t over t and all possible market choices is the $\underline{\eta}_T$ with price history set including only p_1^{CM}. Given $\frac{d\eta_t}{d\kappa} > 0$, we are subject to check if the highest κ makes $\underline{\eta}_T \geq \tilde{\eta}$.

$$\lim_{\kappa \to 1} \underline{\eta}_T = \frac{1}{(1+\epsilon^2)^2} \frac{I}{1+(I-1)\bar{\rho}+\sigma^2}$$

There exist $\bar{\epsilon}(\sigma^2, I)$ such that for any $\epsilon < \bar{\epsilon}(\sigma^2, I)$, $\lim_{\kappa \to 0} \mathbb{E}[U_i^{CM}(\underline{\eta}_T)|\mathcal{H}] - \mathbb{E}[U_i^{DM}(\underline{\eta}_T)|\mathcal{H}] < 0$. Given $\frac{d\eta_t}{d\kappa} > 0$ and η_t is continuous in κ, and $\epsilon < \bar{\epsilon}(\sigma^2, I)$, there exists $\bar{\kappa}$, such that for any $\kappa \in (\bar{\kappa}, 1]$, traders will stay in the DM since the 2nd round.

Step 4. Existence of $\underline{\kappa}$: By Lemma 3, to show the existence of $\underline{\kappa}$, we will need to check if there exists κ that $\eta_t \leq \tilde{\eta}, \forall t$. By Step 1 the highest possible η_t over t and all possible market choices is the $\bar{\eta}_T$ when all past market choices are CMs and all past prices are available. Therefore, we are subject to check a hypothetical $\bar{\eta}_T$ that is generated with the history of all past CM prices. Given Step 2, $\frac{d\eta_t}{d\kappa} > 0$ and η_t is continuous in κ, we are subject to check if the highest κ makes $\bar{\eta}_T \leq \tilde{\eta}$.

$$\lim_{\kappa \to 0} \bar{\eta}_T < \left(\frac{1}{(1+\xi^2)(1+\epsilon^2)}\right)^2 \frac{I(T-1)}{1+(I-1)\bar{\rho}+\sigma^2}$$

There exists $\underline{\xi}$ such that for $\xi > \underline{\xi}(\sigma^2, I, \epsilon)$, $3 \leq T < \bar{T}(\sigma^2, I, \epsilon)$, $\frac{d \lim_{\xi \to 0} \mathbb{E}[U_i^{CM}(\bar{\eta}_T)|\mathcal{H}] - \mathbb{E}[U_i^{DM}(\bar{\eta}_T)|\mathcal{H}]}{dI} > 0$. Given $\frac{d\eta_t}{d\kappa} > 0$, η_t is continuous in κ, and $3 \leq T < \bar{T}(\sigma^2, I, \epsilon)$, $\xi > \underline{\xi}(\sigma^2, I, \epsilon)$, there exist $\underline{\kappa}$, for any $\kappa \in [0, \underline{\kappa}]$, traders will stay in the CM.

Step 5. Summarize: Given $\epsilon < \bar{\epsilon}(\sigma^2, I)$ and $\xi > \underline{\xi}(\sigma^2, I, \epsilon)$, $3 \leq T < \bar{T}(\sigma^2, I, \epsilon)$, there exists $\underline{\kappa}$ and $\bar{\kappa}$ such that traders will choose CM in the first round, and

1. When the autocorrelation is sufficiently low $\kappa \in [0, \underline{\kappa}]$, the traders will always stay in the CM.
2. When the autocorrelation is intermediate $\kappa \in (\underline{\kappa}, \bar{\kappa}]$, the traders will alternate between CM and DM, as there exists t such that $\eta_t > \tilde{\eta}$, and there also exists t such that $\eta_t < \tilde{\eta}$.
3. When the autocorrelation is sufficiently high $\kappa \in (\bar{\kappa}, 1]$, the traders will choose DM over CM in the second round and never choose CM again.

■

Proof of Proposition 5. The proof of Proposition 5 is simple and intuitive. By the first monotonicity result in Lemma 3, if $\mathcal{M}_t^* = DM$ for η_t, and price history informativeness increases $\eta_{t+1} \geq \eta_t$, then $\mathcal{M}_{t+1}^* = DM$. We are subject to show that $\eta_{t+1} \geq \eta_t$ if traders choose DM at round t. $\eta_t = \frac{var(\theta_i) - var(\theta_i|\mathcal{H}_t)}{var(\theta_i)}$. If traders choose DM at round t, then $\mathcal{H}_{t+1} = \mathcal{H}_t \cup \{p_{n,t}\}_n$, and $\eta_{t+1} = \frac{var(\theta_i) - var(\theta_i|\mathcal{H}_{t+1})}{var(\theta_i)}$. Given that $\mathcal{H}_t \subset \mathcal{H}_{t+1}$, $var(\theta_i|\mathcal{H}_{t+1}) \leq var(\theta_i|\mathcal{H}_t)$, and therefore $\eta_{t+1} \geq \eta_t$. ■

Proof of Proposition 6. If traders always stay in CM. Post-trade transparency has no impact on welfare.

If traders have ever chosen DM, denote the round that traders first choose DM as t^*. For $t \le t^*$, post-trade transparency has no impact on traders' utility. For $t > t^*$, denote the price history informativeness as $\eta_t^{post} = \frac{var(\theta_i) - var(\theta_i|\mathcal{H}_t^{post})}{var(\theta_i)}$. By symmetry, the price history is a linear combination of the average signal in the market and is informationally equivalent to the average signal per exchange. Thus $\eta_t^{post} = \frac{var(\theta_i) - var(\theta_i|\mathcal{S}_t^{post})}{var(\theta_i)}$, where $\mathcal{S}_t^{post} \equiv \{\bar{s}_\tau\}_{\tau<t^*}, \{\bar{s}_{n,\tau}\}_{n,t^* \le \tau < t})$. Without post-trade transparency in DM, $\eta_t = \frac{var(\theta_i) - var(\theta_i|\mathcal{S}_t)}{var(\theta_i)}$, where $\mathcal{S}_t \subset \{\bar{s}_\tau\}_{\tau<t}$. filtration generated by $\mathcal{S}_t$ is a sub σ-algebra of filtration generated by $\mathcal{S}_t^{post}$, therefore, $var(\theta_i|\mathcal{S}_t^{post}) \le var(\theta_i|\mathcal{S}_t)$, and $\eta_t^{post} \ge \eta_t$, $\forall t$.

If the traders choose DM at round t without post trade transparency, given that $\eta_t^{post} \ge \eta_t$, $\mathbb{E}[U_{i,t}^{DM,post}|\mathcal{H}_t^{post}] \ge \mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$.

If the traders choose CM at round t without post-trade transparency, $\mathbb{E}[U_{i,t}^{DM,post}] > \mathbb{E}[U_{i,t}^{CM,post}|\mathcal{H}_t^{post}] \ge \mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]$, the first inequality follows from the fact that traders prefer DM over CM at round t given proof of Proposition 5, the second equality follows from $\eta_t^{post} > \eta_t$. ■

Proof of Theorem 4. **(Step 1: Optimization)** Let the cross pair price information be $\boldsymbol{p}_t \in \mathbb{R}^N$, whose n^{th} element is the price in pair n at round t, $p_{n,t}$. Trader $i \in I(n)$ submit demand schedule $q_{i,t}(\boldsymbol{p}_t) : \mathbb{R}^N \to \mathbb{R}$ to maximize the expected utility conditional on the history $\mathcal{H}_t$, private signal $s_{i,t}$, and

$$\max_{q_{i,t}(\boldsymbol{p}_t)} \mathbb{E}[\theta_{i,t} q_{i,t} - \frac{1}{2}\alpha q_{i,t}^2 - p_{n,t} q_t | \boldsymbol{p}_t, \mathcal{H}_t, s_{i,t}]$$

trader i 's first-order condition as

$$q^i(\boldsymbol{p}_t) = \frac{\mathbb{E}[\theta_{i,t}|\boldsymbol{p}_t, \mathcal{H}_t, s_{i,t}] - p_{n,t}}{\alpha + \lambda_{i,t}}$$

where $\lambda_{i,t}$ is the trader i's price impact within pair n. Trader i also has cross-pair price impact as traders from other pairs will change their bids when price p_n changes with i's bid. Trader i's price impact over all pairs can be described with a price impact matrix $\boldsymbol{\Lambda}_{i,t} = (\frac{d\boldsymbol{p}}{d\boldsymbol{q}_{i,t}}) \in \mathbb{R}^{N \times N}$, where the n^{th} diagonal elements is $\lambda_{i,t}$. Each trader i 's price impact matrix is equal to the transpose of the Jacobian of trader i 's inverse residual supply:

$$(\boldsymbol{\Lambda}_{i,t})' = \left(-\sum_{j \ne i} \frac{d\boldsymbol{q}_{j,t}}{d\boldsymbol{p}_t}\right)^{-1}$$

We can parameterize $\mathbb{E}[\theta_{i,t}|\boldsymbol{p}_t, \mathcal{H}_t, s_{i,t}] = \boldsymbol{c}_{\mathcal{H},i,t}\mathcal{H}_t + c_{s,i,t}s_{i,t} + \boldsymbol{c}_{p,i,t}\boldsymbol{p}_t$. $\boldsymbol{c}_{\mathcal{H},i,t} \in \mathbb{R}^{1 \times |\mathcal{H}_t|}$, $c_{s,i,t} \in \mathbb{R}$, and $\boldsymbol{c}_{p,i,t} \in \mathbb{R}^{1 \times N}$. Given symmetry within each pair, $\boldsymbol{c}_{\mathcal{H},i,t} = \boldsymbol{c}_{\mathcal{H},n,t}$, $c_{s,i,t} = c_{c,n,t}$, $\boldsymbol{c}_{p,i,t} = \boldsymbol{c}_{p,n,t}$ and $\lambda_{i,t} = \lambda_{n,t}$.

Given the market clearing condition, $\sum_{i \in I(n)} q_{i,t}(\boldsymbol{p}_t) = 0$, and trader symmetry within exchanges, we have the equilibrium price in all pairs in vector form,

$$\boldsymbol{p}_t = (\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} (\boldsymbol{C}_{\mathcal{H},t}\mathcal{H}_t + \boldsymbol{C}_{s,t}\bar{\boldsymbol{s}}_t),$$

where $\boldsymbol{C}_{s,t} = \operatorname{diag}(c_{s,n,t})_n \in \mathbb{R}^{N\times N}$, $\boldsymbol{C}_{\mathcal{H},t} = (\boldsymbol{c}_{\mathcal{H},n,t})_n \in \mathbb{R}^{N\times|\mathcal{H}_t|}$, $\boldsymbol{C}_{p,t} = (\boldsymbol{c}_{p,n,t})_n \in \mathbb{R}^{N\times N}$. $\bar{\boldsymbol{s}}_t \in \mathbb{R}^N$ is the average signals for all pairs, where the n^{th} element is the average signal in pair n.

(Step 2: Inference Coefficients) We determine the inference coefficients as a function of the primitives (and in closed form). Random vector $(\theta_{i,t}, s_{i,t}, \mathcal{H}_t, \boldsymbol{p}_t)$ is jointly normally distributed:

$$\begin{pmatrix} \theta_{i,t} \\ s_{i,t} \\ \mathcal{H}_t \\ \boldsymbol{p}_t \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} \mathbb{E}[\theta] \\ \mathbb{E}[\theta] \\ \mathbb{E}[\boldsymbol{\theta}] \\ \mathbb{E}[\boldsymbol{\theta}] \end{pmatrix}, \begin{pmatrix} var(\theta_{i,t}) & cov(\theta_{i,t}, s_{i,t}) & cov(\theta_{i,t}, \mathcal{H}_t') & cov(\theta_{i,t}, \boldsymbol{p}_t') \\ cov(s_{i,t}, \theta_{i,t}) & var(s_{i,t}) & cov(s_{i,t}, \mathcal{H}_t') & cov(s_{i,t}, \boldsymbol{p}_t') \\ cov(\mathcal{H}_t, \theta_{i,t}) & cov(\mathcal{H}_t, s_{i,t}) & cov(\mathcal{H}_t, \mathcal{H}_t') & cov(\mathcal{H}_t, \boldsymbol{p}_t') \\ cov(\boldsymbol{p}_t, \theta_{i,t}) & cov(\boldsymbol{p}_t, s_{i,t}) & cov(\boldsymbol{p}_t, \mathcal{H}_t') & cov(\boldsymbol{p}_t, \boldsymbol{p}_t') \end{pmatrix}\right]$$

where

$$\begin{aligned}
&cov(\boldsymbol{p}_t, \theta_{i,t}) = (\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} (\boldsymbol{C}_{s,t} cov(\bar{\boldsymbol{s}}_t, \theta_{i,t}) + \boldsymbol{C}_{\mathcal{H},t} cov(\mathcal{H}_t, \theta_{i,t})) \\
&cov(\boldsymbol{p}_t, s_{i,t}) = (\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} (\boldsymbol{C}_{s,t} cov(\bar{\boldsymbol{s}}_t, s_{i,t}) + \boldsymbol{C}_{\mathcal{H},t} cov(\mathcal{H}_t, s_{i,t})) \\
&cov(\boldsymbol{p}_t, \mathcal{H}_t') = (\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} \left(\boldsymbol{C}_{s,t} cov\left(\bar{\boldsymbol{s}}_t, \mathcal{H}_t'\right) + \boldsymbol{C}_{\mathcal{H},t} cov(\mathcal{H}_t, \mathcal{H}_t')\right) \\
&cov\left(\boldsymbol{p}_t, \boldsymbol{p}_t'\right) = (\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} \left(\boldsymbol{C}_{s,t} cov\left(\bar{\boldsymbol{s}}_t, \bar{\boldsymbol{s}}_t'\right)(\boldsymbol{C}_{s,t})' + \boldsymbol{C}_{\mathcal{H},t} cov(\mathcal{H}_t, \mathcal{H}_t')\boldsymbol{C}_{\mathcal{H},t}' + \boldsymbol{C}_{\mathcal{H},t} cov(\mathcal{H}_t, \bar{\boldsymbol{s}}_t')\boldsymbol{C}_{s,t}' \right. \\
&\left. + \boldsymbol{C}_{s,t} cov(\bar{\boldsymbol{s}}_t, \mathcal{H}_t')\boldsymbol{C}_{\mathcal{H},t}'\right) \left((\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1}\right)'
\end{aligned}$$

By projection theorem, we have

$$[c_{s,n,t}, \boldsymbol{c}_{\mathcal{H},n,t}, \boldsymbol{c}_{p,n,t}] \begin{pmatrix} var(s_{i,t}) & cov(s_{i,t}, \mathcal{H}_t') & cov(s_{i,t}, \boldsymbol{p}_t') \\ cov(\mathcal{H}_t, s_{i,t}) & cov(\mathcal{H}_t, \mathcal{H}_t') & cov(\mathcal{H}_t, \boldsymbol{p}_t') \\ cov(\boldsymbol{p}_t, s_{i,t}) & cov(\boldsymbol{p}_t, \mathcal{H}_t') & cov(\boldsymbol{p}_t, \boldsymbol{p}_t') \end{pmatrix} = [cov(\theta_{i,t}, s_{i,t}), cov(\theta_{i,t}, \mathcal{H}_t'), cov(\theta_{i,t}, \boldsymbol{p}_t')] \tag{22}$$

From equation (22), we have the following equations,

$$cov(c_{s,n,t}s_{i,t} + \boldsymbol{c}_{\mathcal{H},n,t}\mathcal{H}_t + \boldsymbol{c}_{p,n,t}\boldsymbol{p}_t, s_{i,t}) = \sigma_\theta^2 \tag{23}$$

$$cov(c_{s,n,t}s_{i,t} + \boldsymbol{c}_{\mathcal{H},n,t}\mathcal{H}_t + \boldsymbol{c}_{p,n,t}\boldsymbol{p}_t, \mathcal{H}_t) = cov(\theta_{i,t}, \mathcal{H}_t') \tag{24}$$

$$cov(c_{s,n,t}s_{i,t} + \boldsymbol{c}_{\mathcal{H},n,t}\mathcal{H}_t + \boldsymbol{c}_{p,n,t}\boldsymbol{p}_t, \boldsymbol{p}_t') = cov(\theta_{i,t}, \boldsymbol{p}_t') \tag{25}$$

Given that $\boldsymbol{p}_t = (\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} (\boldsymbol{C}_{\mathcal{H},t}\mathcal{H} + \boldsymbol{C}_{s,t}\bar{\boldsymbol{s}}_t)$, subtracting $\boldsymbol{C}_{\mathcal{H},t}$ times equation (24) from equation (25) gives us

$$cov(c_{s,n,t}s_{i,t} + \boldsymbol{c}_{\mathcal{H},n,t}\mathcal{H}_t + \boldsymbol{c}_{p,n,t}\boldsymbol{p}_t, \bar{\boldsymbol{s}}_t') = cov(\theta_{i,t}, \bar{\boldsymbol{s}}_t'). \tag{26}$$

Averaging equation (23) over $i \in I(n)$ gives

$$c_{s,n,t}(1+\sigma^2)\sigma_\theta^2 + cov(\boldsymbol{c}_{\mathcal{H},n,t}\mathcal{H}_t + \boldsymbol{c}_{p,n,t}\boldsymbol{p}_t, \bar{s}_n) = \sigma_\theta^2, \quad \forall n. \tag{27}$$

Comparing equation (26) and (27), we have

$$c_{s,n,t} = \frac{cov(\theta_{i,t}, \bar{s}_n) - \sigma_\theta^2}{cov(s_{i,t}, \bar{s}_n) - (1+\sigma^2)\sigma_\theta^2} = \frac{1-\rho_{n,t}}{1-\rho_{n,t}+\sigma^2}. \tag{28}$$

where $\rho_{n,t}$ is the correlation for traders in pair n.

Given $\boldsymbol{C}_{s,t} = \text{diag}(c_{s,n,t})$ solved in equation (28), we can rewrite equation (24) in matrix form,

$$(\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1}\left(\boldsymbol{C}_{s,t}\mathbf{1}\boldsymbol{\tau}_t' + \boldsymbol{C}_{\mathcal{H},t}\boldsymbol{\Upsilon}_t\right) = \mathbf{1}\boldsymbol{\tau}_t'. \tag{29}$$

and equation (26) as

$$(\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1}\left(\boldsymbol{C}_{s,t}cov(\bar{\boldsymbol{s}}_t, \bar{\boldsymbol{s}}_t') + \boldsymbol{C}_{\mathcal{H},t}\boldsymbol{\tau}_t\mathbf{1}'\right) = cov(\bar{\boldsymbol{\theta}}_t, \bar{\boldsymbol{s}}_t') \tag{30}$$

where $\boldsymbol{\tau}_t = cov(\mathcal{H}_t, \theta_{i,t})$, $\boldsymbol{\Upsilon}_t = cov(\mathcal{H}_t, \mathcal{H}_t')$.

We can solve the term $\boldsymbol{C}_{\mathcal{H},t}$ and $\boldsymbol{C}_{p,t}$ by the above two equations,

$$\boldsymbol{C}_{p,t} = \boldsymbol{Id} - \boldsymbol{C}_{s,t} - \boldsymbol{C}_{s,t}\,\text{diag}\left(\tfrac{\sigma^2}{I_n}\right)_n (\bar{\mathcal{C}} - \mathbf{1}\mathbf{1}'\eta_t)^{-1} = \text{diag}\left(\tfrac{\sigma^2}{1-\rho_{n,t}+\sigma^2}\right)_n \left(\boldsymbol{Id} - \text{diag}\left(\tfrac{1-\rho_{n,t}}{2}\right)(\bar{\mathcal{C}} - \mathbf{1}\mathbf{1}'\eta_t)^{-1}\right)$$

$$\boldsymbol{C}_{\mathcal{H},t} = (\boldsymbol{Id} - \boldsymbol{C}_{p,t} - \boldsymbol{C}_{s,t})\mathbf{1}\boldsymbol{\tau}_t'\boldsymbol{\Upsilon}_t^{-1} = \text{diag}\left(\frac{(1-\rho_{n,t})\sigma^2}{2(1-\rho_{n,t}+\sigma^2)}\right)_n (\bar{\mathcal{C}} - \mathbf{1}\mathbf{1}'\eta_t)^{-1}\mathbf{1}\boldsymbol{\tau}_t'\boldsymbol{\Upsilon}_t^{-1}$$

$\eta_t = \frac{\boldsymbol{\tau}_t'\boldsymbol{\Upsilon}_t^{-1}\boldsymbol{\tau}_t}{\sigma_\theta^2}$ is price history informativeness. $\bar{\mathcal{C}} = \frac{cov(\bar{\boldsymbol{\theta}}_t, \bar{\boldsymbol{\theta}}_t')}{\sigma_\theta^2} \in \mathbb{R}^{N\times N}$ is the correlation of pairwise average values across all pairs, where $\bar{\boldsymbol{\theta}}_t \in \mathbb{R}^N$ is the vector of average value per trading pair where the n^{th} value is $\bar{\theta}_{n,t} = \sum_{i\in I(n)} \theta_{i,t}$.

(Step 3: Price impacts) In equilibrium, each trader i 's price impact is equal to the transpose of the Jacobian of trader i 's inverse residual supply:

$$(\boldsymbol{\Lambda}_{i,t})' = \left(-\sum_{j\neq i} \frac{d\boldsymbol{q}_{j,t}}{d\boldsymbol{p}_t}\right)^{-1} = (\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1}\,\text{diag}\left(\frac{\alpha + \lambda_{n,t}}{2 - \mathbf{1}_{i\in I(n)}}\right)_n.$$

From the last equation, we can solve for the within-exchange price impact for all $i \in I(n)$,

$$\lambda_{n,t} = \left(\left(\left((\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} \right)_{nn} \right)^{-1} - 1 \right)^{-1} \alpha.$$

where $(A)_{nn}$ is an operator that gives the n^{th} diagonal element of matrix A. Denote the matrix of within-exchange price impacts by $\hat{\boldsymbol{\Lambda}}_t \equiv \text{diag}\,(\lambda_{n,t})_n$. In equilibrium,

$$\hat{\boldsymbol{\Lambda}}_t = \left(\left(\left[(\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} \right]_{nn} \right)^{-1} - \boldsymbol{Id} \right)^{-1} \alpha,$$

where $[A]_{nn}$ is an operator that gives the diagonal elements of matrix A while setting all off-diagonal elements to zero.

In this paper, we focus on nonnegative price impacts such that the residual supply curve is downward-sloping, i.e., $\lambda_n \geq 0$, for all n. This is satisfied under the following conditions:

$$((\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1})_{nn} \leq 1$$

$\left((\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} \right)_{nn} = (1 + \frac{\sigma^2}{1-\rho_{n,t}}) \left(1 - \frac{\sigma^2}{2} (\frac{1+\rho_{n,t}+\sigma^2}{2} - \eta_t - A_t)^{-1} \right)$, where $A_t = (\frac{cov(\bar{\theta}_{n,t}, \bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2} - \boldsymbol{1}'\eta_t)(\frac{cov(\bar{\boldsymbol{\theta}}_{-n,t}, \bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2} - \boldsymbol{1}\boldsymbol{1}'\eta_t)^{-1}(\frac{cov(\bar{\boldsymbol{\theta}}_{-n,t}, \bar{\theta}_{n,t})}{\sigma_\theta^2} - \boldsymbol{1}\eta_t)$. Therefore, the following conditions are needed for equilibrium existence,

$$\eta_t + A_t \geq \rho_{n,t} \quad \forall n$$

The second-order condition for the trader i 's optimization problem is, $\lambda_n \geq -\frac{1}{2}\alpha$, and is trivially satisfied with nonnegative price impacts.

(Step 4: Utility) Given the inference coefficients and price impacts solved in the previous section, the expected utility conditional on price history is

$$\mathbb{E}[U_{i,t}^{DM,pre} | \mathcal{H}_t] = \frac{\alpha + 2\lambda_t}{2(\alpha + \lambda_t)^2} \mathbb{E}[(\mathbb{E}[\theta_{i,t} | \mathcal{H}_t, s_{i,t}, \boldsymbol{p}_t] - p_{t,n})^2 | \mathcal{H}_t] = \frac{\alpha + 2\lambda_{n,t}}{2(\alpha + \lambda_{n,t})^2} \frac{1}{2} \frac{(1 - \rho_{n,t})^2}{1 - \rho_{n,t} + \sigma^2}$$

■

Proof of Lemma 5. Taking derivative of $U_{i,t}^{DM}$ with respect to $\rho_{n,t}$, we have

$$\frac{d\mathbb{E}[U_{i,t}^{DM,pre} | \mathcal{H}_t]}{\rho_{n,t}} = - \underbrace{\frac{\alpha + 2\lambda_{n,t}}{4(\alpha + \lambda_{n,t})^2} \frac{(1 - \rho_{n,t})(1 - \rho_{n,t} + 2\sigma^2)}{(1 - \rho_{n,t} + \sigma^2)^2}}_{>0} - \underbrace{\frac{\lambda_{n,t}}{2(\alpha + \lambda_{n,t})^3} \frac{(1 - \rho_{n,t})^2}{1 - \rho_{n,t} + \sigma^2}}_{>0} \frac{d\lambda_{n,t}}{d\rho_{n,t}}$$

.

The derivative of price impact to correlation $\frac{d\lambda_{n,t}}{d\rho_{n,t}}$ is

$$\begin{aligned}
\frac{d\lambda_{n,t}}{d\rho_{n,t}} &= \frac{\lambda_{n,t}^2}{\alpha}\left(\left((\boldsymbol{Id}-\boldsymbol{C}_{p,t})^{-1}\right)_{nn}\right)^{-2}\frac{d\left((\boldsymbol{Id}-\boldsymbol{C}_{p,t})^{-1}\right)_{nn}}{d\rho_{n,t}} \\
&= \frac{\lambda_{n,t}^2}{\alpha\left(\left((\boldsymbol{Id}-\boldsymbol{C}_{p,t})^{-1}\right)_{nn}\right)^2}\left(\frac{\sigma^2\left((\boldsymbol{Id}-\boldsymbol{C}_{p,t})^{-1}\right)_{nn}}{(1-\rho_{n,t})(1-\rho_{n,t}+\sigma^2)}+(1+\frac{\sigma^2}{1-\rho_{n,t}})\frac{\sigma^2}{4}(\frac{1+\rho_{n,t}+\sigma^2}{2}-\eta_t-A_t)^{-2}\right) \\
&> 0
\end{aligned}$$

given that $\left((\boldsymbol{Id}-\boldsymbol{C}_{p,t})^{-1}\right)_{nn} = (1+\frac{\sigma^2}{1-\rho_{n,t}})\left(1-\frac{\sigma^2}{2}(\frac{1+\rho_{n,t}+\sigma^2}{2}-\eta_t-A_t)^{-1}\right)$, where $A_t = (\frac{cov(\bar{\theta}_{n,t},\bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2}-\boldsymbol{1}'\eta)(\frac{cov(\bar{\boldsymbol{\theta}}_{-n,t},\bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2}-\boldsymbol{1}\boldsymbol{1}'\eta)^{-1}(\frac{cov(\bar{\boldsymbol{\theta}}_{-n,t},\bar{\theta}_{n,t})}{\sigma_\theta^2}-\boldsymbol{1}\eta)$, and $\bar{\boldsymbol{\theta}}_{-n,t}\in\mathbb{R}^{N-1}$ is the vector of average values in pairs $m\neq n$. The last inequality follows from the fact that $\frac{1+\rho_{n,t}}{2}-\eta_t-A_t>0$ give positive-semidefinite joint correlation matrix of $\bar{\theta}_{n,t}$, $\bar{\boldsymbol{\theta}}_{-n,t}$ and history $\mathcal{H}_t$.

Therefore, $\frac{d\mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]}{\rho_{n,t}}<0$. The expected utility in DM is decreasing in $\rho_{n,t}$. ■

Proof of Lemma 6. To show that given $\mathcal{H}_t$ (and therefore given η_t) the utility for any trader i weakly increases, we are subject to show that the expected utility $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]\geq\mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$. Comparing $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]$ in Theorem 4 and $\mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$ in Theorem 1, we find if and only if $\lambda_{n,t}^{DM,pre}\leq\lambda_{n,t}^{DM}$, then $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]\geq\mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$.

$\lambda_{n,t}^{DM,pre}\leq\lambda_{n,t}^{DM}$ if and only if

$$\left((\boldsymbol{Id}-\boldsymbol{C}_{p,t})^{-1}\right)_{nn}\leq\frac{1}{1-c_p^{DM}} \tag{31}$$

Following proof of Lemma 5, $\left((\boldsymbol{Id}-\boldsymbol{C}_{p,t})^{-1}\right)_{nn} = (1+\frac{\sigma^2}{1-\rho_{n,t}})\left(1-\frac{\sigma^2}{2}(\frac{1+\rho_{n,t}+\sigma^2}{2}-\eta_t-A_t)^{-1}\right)\leq(1+\frac{\sigma^2}{1-\rho_{n,t}})\frac{\frac{1+\rho_{n,t}}{2}-\eta_t}{\frac{1+\rho_{n,t}+\sigma^2}{2}-\eta_t}$ as $A_t = (\frac{cov(\bar{\theta}_{n,t},\bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2}-\boldsymbol{1}'\eta_t)(\frac{cov(\bar{\boldsymbol{\theta}}_{-n,t},\bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2}-\boldsymbol{1}\boldsymbol{1}'\eta_t)^{-1}(\frac{cov(\bar{\boldsymbol{\theta}}_{-n,t},\bar{\theta}_{n,t})}{\sigma_\theta^2}-\boldsymbol{1}\eta_t)\geq0$ given it has a quadratic form and $\frac{cov(\bar{\boldsymbol{\theta}}'_{-n,t},\bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2}-\boldsymbol{1}\boldsymbol{1}'\eta_t=\frac{cov(\bar{\boldsymbol{\theta}}_{-n,t},\bar{\boldsymbol{\theta}}'_{-n,t}|\mathcal{H}_t)}{\sigma_\theta^2}$ is positive semidefinite. By Theorem 1, $\frac{1}{1-c_p^{DM}}=(1+\frac{\sigma^2}{1-\rho_{n,t}})\frac{\frac{1+\rho_{n,t}}{2}-\eta_t}{\frac{1+\rho_{n,t}+\sigma^2}{2}-\eta_t}$. Therefore, equation 31 holds, $\lambda_{n,t}^{DM,pre}\leq\lambda_{n,t}^{DM}$ and $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]\geq\mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$. ■

Proof of Proposition 7. **First Time Choosing DM:** Let the threshold to choose DM without pre-trade transparency by $\tilde{\eta}$ (see Lemma 3), and the threshold to choose DM with pre-trade transparency by $\tilde{\eta}^{pre}$. Suppose $\bar{\rho}>\bar{\rho}^*$, for any $\mathcal{H}_t$ generating $\eta_t\geq\tilde{\eta}$, given results of Lemma 6, $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]\geq\mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]\geq\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]$. This implies (i) the threshold to choose DM without pre-trade transparency is at least as low as $\tilde{\eta}$, $\tilde{\eta}^{pre}\leq\tilde{\eta}$; (ii) and the first round that traders choose DM with pre-trade transparency is no later than without pre-trade transparency, i.e. if $t_1^{DM}\equiv\min_t\{\mathcal{M}_t^*=DM\}$, then $t_1^{DM,pre}=\min_t\{\mathcal{M}_t^{*,pre}=DM\}\leq t_1^{DM}$.

First Time Stay in DM: If traders choose DM with pre-trade transparency in the same round as with opaque DM, i.e., $t_1^{DM} = t_1^{DM,pre}$, then the length of stay in DM when traders first choose DM is (weakly) longer with pre-trade transparency. This is because, given that they enter the DM at the same round, the evolution of η_t is the same before they first exit the DM after the first time they choose DM. And $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t] \geq \mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$ implies, the first time traders exit DM with pre-trade transparency is no earlier than the first time when they exit the opaque DM. Thus, the length of stay in DM when traders first choose DM is (weakly) longer with pre-trade transparency.

We are not sure about the following rounds of choosing DM, as the evolution of η_t will not be the same with and without pre-trade transparency, except for the $\tilde{\eta} = 0$ special case. If $\tilde{\eta} = 0$ then $\tilde{\eta}^{pre} = 0$, trader will always choose DM.

■

Lemma 7. *If A and $A + B$ are invertible, and B has rank 1, then let $g = \operatorname{trace}\left(BA^{-1}\right)$. Then $g \neq -1$ and*

$$(A + B)^{-1} = A^{-1} - \frac{1}{1+g} A^{-1} B A^{-1}.$$

Proof of Proposition 8. **Constant CM regardless of pre-trade transparency:** It is trivial that when traders choose CM for all rounds with or without pre-trade transparency, then pre-trade transparency should not have any impact on welfare. Given that $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t] > \mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$ for any η, choosing CM constantly implies $\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t] - \mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t] \geq 0$ for any η_t. We know that if $\bar{\rho} = \rho_h = \rho_\ell$,

$$\begin{aligned}
\left((\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1}\right)_{nn} &= (1 + \frac{\sigma^2}{1-\rho_\ell}) \left(1 - \frac{\sigma^2}{2} (\frac{1+\rho_\ell+\sigma^2}{2} - \eta_t - A_t)^{-1}\right) \\
&> \frac{1}{(I-1)(1-c_{p,t}^{CM})} = \frac{1}{I-1}(1 + \frac{\sigma^2}{1-\rho_\ell})(\frac{1+(I-1)\rho_\ell}{I} - \eta_t)(\frac{1+(I-1)\rho_\ell+\sigma^2}{I} - \eta_t)^{-1}
\end{aligned}$$

where $A_t = (\frac{cov(\bar{\theta}_{n,t}, \bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2} - \mathbf{1}'\eta_t)(\frac{cov(\bar{\boldsymbol{\theta}}'_{-n,t}, \bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2} - \mathbf{1}\mathbf{1}'\eta_t)^{-1}(\frac{cov(\bar{\boldsymbol{\theta}}_{-n,t}, \bar{\theta}_{n,t})}{\sigma_\theta^2} - \mathbf{1}\eta_t) = (\rho_h - \eta_t)\frac{g}{1+g}$, $g = N\frac{\rho_h - \eta_t}{\frac{1+\rho_\ell}{2} - \rho_h}$ by Lemma 7, and $\bar{\boldsymbol{\theta}}_{-n,t} \in \mathbb{R}^{N-1}$ is the vector of average values in pairs $m \neq n$. Therefore, $\lambda_{n,t}^{CM} < \lambda_{n,t}^{DM,pre}$ and

$$\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t] = \frac{\alpha + 2\lambda_{n,t}^{CM}}{2(\alpha + \lambda_{n,t}^{CM})^2} \frac{I-1}{I} \frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2} > \mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t] = \frac{\alpha + 2\lambda_{n,t}^{DM,pre}}{4(\alpha + \lambda_{n,t}^{DM,pre})^2} \frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2}$$

Given that $\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]$ is decreasing in $\bar{\rho}$, there exists $\bar{\rho}^{*,pre}(\rho_\ell, I, \sigma^2)$ such that if $\bar{\rho} < \bar{\rho}^{*,pre}(\rho_\ell, I, \sigma^2)$, $\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t] - \mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t] \geq 0$ for any η. So if $\bar{\rho} < \bar{\rho}^{*,pre}(\rho_\ell, I, \sigma^2)$, traders always choose CM with and without pre-trade transparency.

Transparency changes market choice: Without pre-trade transparency, when traders' value correlations are neither sufficiently heterogeneous nor sufficiently homogeneous, alternating market choice can be optimal according to Proposition 3 and Proposition 4. With pre-trade transparency, as the utility in DM is higher given the same parameters, traders are more likely to choose DM (weakly) earlier (see Proposition 7). And this can potentially decrease the price history informativeness and welfare. We can find a non-trivial set of parameters such that the pre-trade transparency can decrease welfare. A most intuitive case is a set of parameters such that (i) traders always choose CM or alternate between CM and DM with $\eta_t > 0$ for $t > 1$ without pre-trade transparency; (ii) traders always choose DM with pre-trade transparency, resulting $\eta_t = 0$ for all t; (iii) the total welfare over all rounds is higher without pre-trade transparency. We provide proof of the existence of such parameters below.

To satisfy condition (i), the traders' expected utility in CM is higher than the expected utility in opaque DM when $\eta_t = 0$, i.e. $\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]|_{\eta_t=0} \geq \mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]|_{\eta_t=0}$.

To satisfy condition (ii), we require the traders in DM with pre-trade transparency to have higher utility than in CM given $\eta_t = 0$. When $\eta_t = 0$, the expected utility in CM is

$$\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]|_{\eta_t=0} = \frac{\alpha + 2\lambda^{CM}}{2(\alpha + \lambda^{CM})^2} \frac{I-1}{I} \frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2} \quad \forall i \in I$$

where $\lambda^{CM} = \frac{\alpha}{(I-1)(1-c_p)-1}$, $c_p = \frac{I\bar{\rho}\sigma^2}{(1-\bar{\rho}+\sigma^2)(1+(I-1)\bar{\rho})}$. And when $\eta_t = 0$, the expected utility in DM with pre-trade transparency is

$$\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0} = \frac{\alpha + 2\lambda^{DM,pre}}{2(\alpha + \lambda^{DM,pre})^2} \frac{1}{2} \frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2}$$

where $\lambda^{DM,pre} = (1 + \frac{\sigma^2}{1-\rho_\ell})(\frac{1+\rho_\ell}{2} - A_0)\left(\frac{\sigma^2}{2} - (\frac{\sigma^2}{1-\rho_\ell})(\frac{1+\rho_\ell}{2} - A_0)\right)^{-1}\alpha$,
$A_0 = \frac{cov(\bar{\theta}_{n,t},\bar{\boldsymbol{\theta}}'_{-n,t})cov(\bar{\boldsymbol{\theta}}_{-n,t},\bar{\boldsymbol{\theta}}'_{-n,t})^{-1}cov(\bar{\boldsymbol{\theta}}_{-n,t},\bar{\theta}_{n,t})}{\sigma_\theta^2} \geq 0$.

Given that $\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]|_{\eta_t=0}$ is decreasing in $\bar{\rho}$ and $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0}$ is decreasing in ρ_ℓ given Lemma 1, and $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0} > \mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]|_{\eta_t=0}$ when $\rho_\ell = 0$ and $\bar{\rho} = \frac{\frac{I}{2}-1}{I-1}$, there exists $0 \leq \rho_\ell < \bar{\rho}_\ell$ and $\bar{\rho} > \bar{\bar{\rho}}^{pre}\left(I, \rho_\ell, \sigma^2\right)$ such that $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0} \geq \mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]|_{\eta_t=0}$.

To satisfy condition (iii), we require the total welfare over all rounds is higher without pre-trade transparency than with pre-trade transparency, i.e. $W^{pre} \equiv IT\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0} \leq W \equiv I\sum_{t=1}^{T}\mathbb{E}[U_{i,t}^{\mathcal{M}^*}|\mathcal{H}_t]$. Given the same market choice, the expected utility weakly increases with the length of price history, i.e. $\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_\tau] \geq \mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]$ for $\tau > t$. For sufficiently small ξ and sufficiently large κ, there exists $\underline{t}$, such that $\eta_{\underline{t}}$ is sufficiently large, $\mathbb{E}[U_{i,\underline{t}}^{CM}|\mathcal{H}_{\underline{t}}] > \mathbb{E}[U_{i,\underline{t}}^{DM,pre}|\mathcal{H}_{\underline{t}}]|_{\eta_t=0}$. Therefore, we can rewrite the difference between welfare without and with pre-trade transparency as

$$
\begin{aligned}
(W - W^{pre})/I &= \sum_{t=1}^{T} \mathbb{E}[U_{i,t}^{\mathcal{M}^*}|\mathcal{H}_t] - T\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0} \\
&\geq (T - \underline{t}) \underbrace{\left(\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t] - \mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0}\right)}_{>0} + \underbrace{\sum_{t=1}^{\underline{t}} \mathbb{E}[U_{i,t}^{\mathcal{M}^*}|\mathcal{H}_t] - \underline{t}\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0}}_{>-\underline{t}\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0}}
\end{aligned}
$$

Easy to see $W - W^{pre}$ is increasing in T. Given that the second part is bounded below, $\lim_{T\to\infty} W - W^{pre} > 0$. This implies we can find sufficiently small ξ and sufficiently large κ and sufficiently large T, such that condition (iii) is satisfied. ■

Proof of Proposition 9.

1. Introducing CM does not change welfare when traders choose DM in all rounds. This is the case when trader values are sufficiently heterogeneous $\rho_\ell < \underline{\rho}_\ell < 0$ and $\bar{\rho} > \bar{\bar{\rho}}$. If traders are allowed to have access to CM in the first round, the welfare also does not change when conditions of Proposition 3.1 and Proposition 4.3 are satisfied.

2. Introducing CM weakly improves welfare for any other cases (where there could be no trade with DM only). This is because, (1) when traders have access to both CM and DM, traders choose CM when the expected utility conditional on the history in CM is weakly higher than that in DM, and (2) choosing CM weakly increases price history informativeness and weakly increase utility for all future traders. In math,

$$\mathbb{E}[U_{i,t}^{\mathcal{M}^*}|\mathcal{H}_t] \geq \mathbb{E}[U_{i,t}^{\mathcal{M}^*}|\mathcal{H}_t^{\text{DM only}}] \geq \mathbb{E}[U_{i,t}^{\text{DM only}}|\mathcal{H}_t^{\text{DM only}}], \quad \forall t.$$

   The first inequality comes from the fact that $\mathcal{H}_t^{\text{DM only}} = \emptyset \subseteq \mathcal{H}_t$, $\eta_t \geq \eta_t^{\text{DM only}} = 0$ and that the expected utility is increasing in η_t (Lemma 2). The second inequality comes from the optimal market choice $\mathcal{M}^*$.

■

Proof of Proposition 10.

1. For sufficiently homogeneous value, $\bar{\rho} < \bar{\rho}^{*,pre}(I, \rho_\ell, \sigma^2)$, by Proposition 1 traders will always choose CM, and therefore centralizing DM does not change welfare.

   When traders' value correlations are neither sufficiently heterogeneous nor sufficiently homogeneous, and the number of rounds $T < \bar{T}$, by Proposition 3 and 4, if the asset sensitivity is sufficiently high $\xi \in [\bar{\xi}, \infty)$, or volatility is sufficiently high $\kappa \in [0, \underline{\kappa}]$, traders will always choose CM, and therefore centralizing DM does not change welfare.

2. For sufficiently heterogeneous value $\rho_\ell < \underline{\rho}_\ell < 0$ and $\bar{\rho} > \bar{\bar{\rho}}$, by the proof of Proposition 2, traders will always choose DM if they have the choice of choosing either CM or DM and $\mathbb{E}[U_{i,1}^{CM}] < \mathbb{E}[U_{i,1}^{DM}]$. If we centralize the market, the traders will be worse off

in the first round, as they will prefer DM with zero price history informativeness. Given that $var(\theta_i|\mathcal{H}_t^{\text{CM only}}) \geq var(\theta_i|\mathcal{H}_{t+1}^{\text{CM only}})\ \forall t$, $\eta_t^{\text{CM only}} = \frac{var(\theta_i)-var(\theta_i|\mathcal{H}_t^{\text{CM only}})}{var(\theta_i)}$ is monotonically increasing in t. By Lemma 2, $\mathbb{E}[U_{i,t}^{\text{CM only}}|\mathcal{H}_t^{\text{CM only}}]$ is monotonically increasing in t. As rounds increase, the price history informativeness increases in CM as the history accumulates, the utility increases in CM with $\eta > 0$, and may exceed the utility in DM with $\eta = 0$ ($\mathcal{H}_t = \mathcal{H}_1 = \emptyset$, $\eta_t = \eta_1 = 0$ with parallel market). The difference in welfare in parallel markets and welfare in CM is,

$$W^{\text{CM only}} - W = I(\sum_{t=1}^{T} \mathbb{E}[U_{i,t}^{\text{CM only}}|\mathcal{H}_t^{\text{CM only}}] - T\mathbb{E}[U_{i,1}^{DM}])$$

Differentiate $W^{\text{CM only}} - W$ over ξ, we can see that it is monotonically decreasing in ξ,

$$\frac{dW^{\text{CM only}} - W}{d\xi} = I\sum_{t=1}^{T} \underbrace{\frac{d\mathbb{E}[U_{i,t}^{\text{CM only}}|\mathcal{H}_t^{\text{CM only}}]}{d\eta_t}}_{>0} \underbrace{\frac{d\eta_t}{d\xi}}_{<0} < 0 \tag{32}$$

where the first term in the summation is positive by Lemma 2, and the second term in the summation is negative given that $\frac{d\eta_t}{d\xi} = \frac{1}{\sigma_\theta^2}\frac{dcov(\theta_{i,t},(\bar{\mathbf{s}}_{\tau<t}^{CM})')cov(\bar{\mathbf{s}}_{\tau<t}^{CM},(\bar{\mathbf{s}}_{\tau<t}^{CM})')^{-1}cov(\bar{\mathbf{s}}_{\tau<t}^{CM},\theta_{i,t})}{d\xi} < 0$. Intuitively, when the asset sensitivity is higher, then CM price informativeness is lower and therefore the welfare with CM only is lower.

Similarly, Differentiate $W^{\text{CM only}} - W$ over κ, we can see that it is monotonically increasing in κ,

$$\frac{dW^{\text{CM only}} - W}{d\kappa} = I\sum_{t=1}^{T} \underbrace{\frac{d\mathbb{E}[U_{i,t}^{\text{CM only}}|\mathcal{H}_t^{\text{CM only}}]}{d\eta_t}}_{>0} \underbrace{\frac{d\eta_t}{d\kappa}}_{>0} > 0 \tag{33}$$

where the first term in the summation is positive by Lemma 2, and the second term in the summation is negative given that $\frac{d\eta_t}{d\kappa} = \frac{1}{\sigma_\theta^2}\frac{dcov(\theta_{i,t},(\bar{\mathbf{s}}_{\tau<t}^{CM})')cov(\bar{\mathbf{s}}_{\tau<t}^{CM},(\bar{\mathbf{s}}_{\tau<t}^{CM})')^{-1}cov(\bar{\mathbf{s}}_{\tau<t}^{CM},\theta_{i,t})}{d\kappa} > 0$. Intuitively, when the asset volatility is higher, then CM price informativeness is lower and therefore the welfare with CM only is lower.

When $\xi = \infty$ and $\kappa = 0$, then $\eta_t = 0$ for all t, $W^{\text{CM only}} - W < 0$ given Proposition 2. By equations (32) and (33), with finite rounds $T < \bar{T}$, there exists $\bar{\xi}^{\text{CM only}}$ and $\underline{\kappa}^{\text{CM only}}$, such that when the asset sensitivity is sufficiently high $\xi \in [\bar{\xi}^{\text{CM only}}, \infty)$ and the volatility is sufficiently high $\kappa \in [0, \underline{\kappa}^{\text{CM only}}]$, $W^{\text{CM only}} - W < 0$.

3. For sufficiently heterogeneous value $\rho_\ell < \underline{\rho}_\ell < 0$ and $\bar{\rho} > \bar{\bar{\rho}}$, by the proof of Proposition 2, traders will always choose DM if they have the choice of choosing either CM or DM and $\mathbb{E}[U_{i,1}^{CM}] < \mathbb{E}[U_{i,1}^{DM}]$.

   Given Lemma 2, $\mathbb{E}[U_{i,T}^{\text{CM only}}|\mathcal{H}_T^{\text{CM only}}]$ is increasing in the price history informativeness η. So for $T > 1$, $\mathbb{E}[U_{i,T}^{\text{CM only}} | \mathcal{H}_T^{\text{CM only}}] > \mathbb{E}[U_{i,1}^{\text{CM only}} | \mathcal{H}_1^{\text{CM only}}] = \mathbb{E}[U_{i,1}^{CM}]$.

Given that $\lim_{\rho \to \bar{\bar{\rho}}, \rho_\ell \to \underline{\rho}_\ell} \mathbb{E}[U_{i,1}^{CM}] = \lim_{\rho \to \bar{\bar{\rho}}, \rho_\ell \to \underline{\rho}_\ell} \mathbb{E}[U_{i,1}^{DM}]$ follows from the proof of Proposition 2, $\lim_{\rho \to \bar{\bar{\rho}}, \rho_\ell \to \underline{\rho}_\ell} \mathbb{E}[U_{i,\infty}^{\text{CM only}} | \mathcal{H}_\infty^{\text{CM only}}] > \lim_{\rho \to \bar{\bar{\rho}}, \rho_\ell \to \underline{\rho}_\ell} \mathbb{E}[U_{i,1}^{CM}] = \lim_{\rho \to \bar{\bar{\rho}}, \rho_\ell \to \underline{\rho}_\ell} \mathbb{E}[U_{i,1}^{DM}]$.

There exist correlation $\bar{\bar{\rho}} < \bar{\rho} < \tilde{\bar{\rho}}$ and $\underline{\underline{\rho}}_\ell < \rho_\ell < \underline{\rho}_\ell < 0$ and $\bar{T}$ such that for any $T \geq \bar{T}$,

$$\mathbb{E}[U_{i,T}^{\text{CM only}} | \mathcal{H}_T^{\text{CM only}}] > \mathbb{E}[U_{i,T}^{DM}] = \mathbb{E}[U_{i,1}^{DM}] \tag{34}$$

Given equation (34), for $T > \bar{T}$ the total welfare can be decomposed as

$$W^{\text{CM only}} - W = I\left(\sum_{t=1}^{\bar{T}} \mathbb{E}[U_{i,t}^{\text{CM only}} | \mathcal{H}_t^{\text{CM only}}] - \bar{T}\mathbb{E}[U_{i,1}^{DM}]\right) + I \sum_{\bar{T}}^{T} \underbrace{\left(\mathbb{E}[U_{i,t}^{\text{CM only}} | \mathcal{H}_t^{\text{CM only}}] - \mathbb{E}[U_{i,1}^{DM}]\right)}_{>0}$$

where the first part is finite, and the second part is positive. As T increases, $W^{\text{CM only}} - W$ increases. When $T \to \infty$, $\lim_{T \to \infty} W^{\text{CM only}} - W > I(\sum_{t=1}^{\bar{T}} \mathbb{E}[U_{i,t}^{\text{CM only}} | \mathcal{H}_t^{\text{CM only}}] - \bar{T}\mathbb{E}[U_{i,1}^{DM}]) + I \lim_{T \to \infty}(T - \bar{T}) \left(\mathbb{E}[U_{i,\bar{T}}^{\text{CM only}} | \mathcal{H}_t^{\text{CM only}}] - \mathbb{E}[U_{i,1}^{DM}]\right) \to \infty$.

Therefore, there exists $\tilde{T}$, such that when $T > \tilde{T}$, $\bar{\bar{\rho}} < \bar{\rho} < \tilde{\bar{\rho}}$ and $\underline{\underline{\rho}}_\ell < \rho_\ell < \underline{\rho}_\ell < 0$ and $\bar{T}$, centralizing the decentralized market improves the welfare $W^{\text{CM only}} - W > 0$.

■

D Evidence from the U.S. Equity Markets

The model provides us with testable predictions. It shows that a higher asset autocorrelation can lead to market fragmentation (Proposition 4). To test this prediction, we collected data for U.S. equities traded in exchanges, alternative trading systems (ATS), and over-the-counter (OTC) markets.

We obtain the ATS weekly summary of transaction volumes from FINRA and Exchange and OTC equity prices and transaction volumes from Wharton Research Data Service (WRDS). Our sample period is 2019-2022. We classify the lit exchanges as CM, e.g., Nasdaq, and NYSE. We classify ATS (e.g., Credit Suisse Crossfinder, Instinet) and OTC as DM.[22] We consider two samples for the regression and construct variables for each sample respectively. The first sample is the full sample that includes all equities traded in all venues. There is a concern that some equities may be restricted to be traded only in CM or DM due to regulations, preventing traders from changing their venues as the model assumption. To mitigate the influence of the market restrictions on our identification, we consider another sample, which includes those equities that have ever been traded in both CM and DM during 2019-2022. We drop singleton observations of equities with only one-week transactions in both samples.

We construct the dependent variable $DMshare_{i,t}$, which is the transaction volume of equity i in DM as a proportion of the total transaction volume of equity i in all venues in week t. Given that lower κ implies higher volatility in values, we use the price volatility in the last 100 days $Volatility^{[d-100,d]}$ as a proxy for κ, which is constructed as follows. We first calculate the standard deviation of the close price $p_{i,d}$ in the last 100 trading days $[d-100, d]$, and then take the weekly average of it for each equity i and week t.[23] We winsorize the top and bottom 1% to avoid the impact of extreme values.

We use the following regression to test the model prediction in Proposition 4 with both the full sample and a smaller sample of equities traded in both DM and CM,

$$DMshare_{i,t} = \beta Volatility_{i,t}^{[d-100,d]} + \delta_i + \gamma_t + \varepsilon_{i,t} \tag{35}$$

where δ_i are equity fixed effects, γ_t are week fixed effects, and $\varepsilon_{i,t}$ are robust standard errors.

One concern is that traders' market choices may affect the price fluctuations. It can cause reverse causality and weaken our identification results. Therefore, we construct the lagged price volatility $Volatility_{i,t}^{[d-200,d-101]}$ as an instrumental variable (IV). $Volatility_{i,t}^{[d-200,d-101]}$ is the weekly average of the standard deviation of the close price $p_{i,d}$ between trading day $[d-200, d-101]$ for

[22] Please refer to FINRA equity ATS Firms and SEC Form ATS-N Filings and Information for a complete list and more detailed information of current and past ATS for equities.

[23] As some OTC equities are not traded frequently, not all trading days have close prices. We use the midpoint of the best bid and ask prices on each trading day as the close price.

each equity i in week t. We winsorize the top and bottom 1% to avoid the extreme value.

Table 12 shows the summary of statistics of the variables. The average proportion traded in DM is 57.27% over the full sample and 10.94% for equities ever traded in both DM and CM. The price volatility and its IV on average are 4.190 and 4.164 respectively for the full sample. For equities traded in both DM and CM, the price volatility and its IV on average 4.652 and 4.709 respectively.

Figure 18 shows the average price volatility for each equity by their DM share. We can see that the average volatility is the highest for equities only traded in CM, lower for the equities traded in both DM and CM, and lowest for equities traded in DM only.

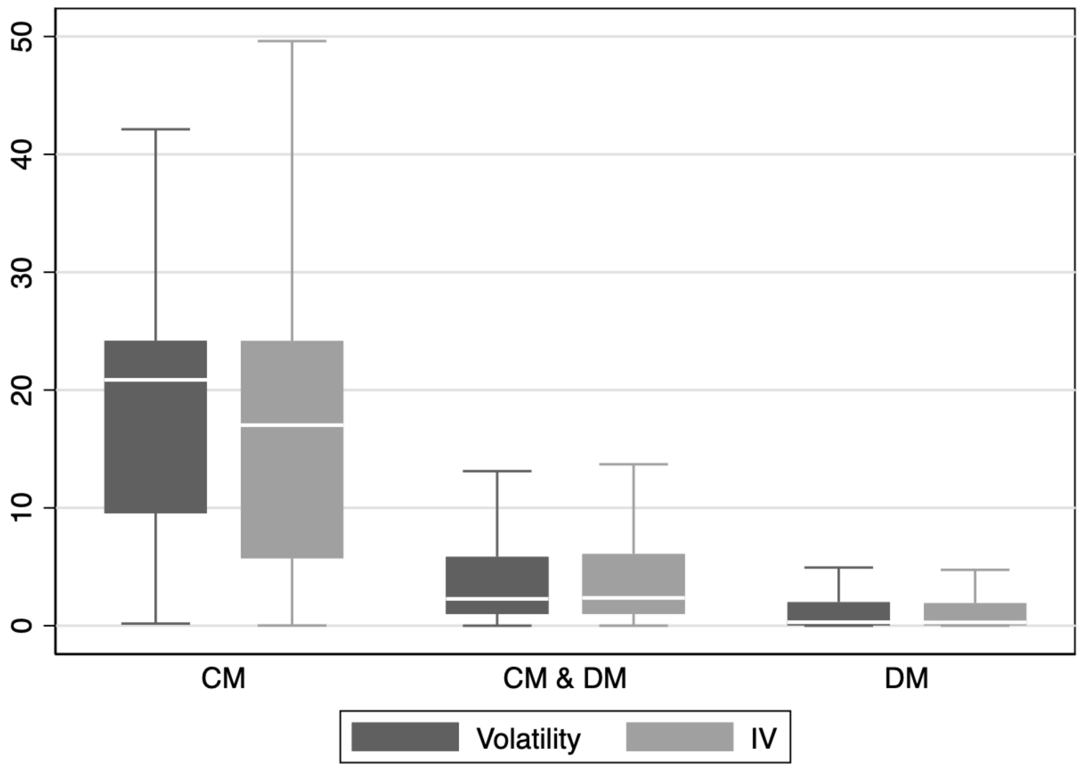
Table 13 shows the regression results of equation (35). Panel A shows the OLS regression results, where we can see that for both the full sample and the restricted sample with equities traded in CM and DM, the volatility is negatively correlated with the proportion of transaction volume in DM. Panel B-D shows the two-stage least-square (2SLS) regression results using the $Volatility^{[d-200,d-101]}$ as an IV. Panel B shows the reduced-form results with the IV as the independent variable. Panel C shows the first stage of 2SLS regression which indicates the IV is strongly correlated with $Volatility^{[d-100,d]}$. Panel D shows the second stage of 2SLS regression. We find that the volatility significantly decreases the proportion of transaction volume in DM, and the magnitude is larger than the OLS regression results.

Table 12: Summary of Statistics

Full Sample					
Variable	Obs	Mean	Std. Dev.	Min	Max
$DMshare(\%)$	3,451,675	57.27	45.39	0	100
$Volatility^{[d-100,d]}$	3,451,675	4.190	13.20	6.70e-05	111.7
$Volatility^{[d-200,d-101]}$	3,451,675	4.164	12.65	4.61e-05	105.0
Equities Traded in CM & DM					
Variable	Obs	Mean	Std. Dev.	Min	Max
$DMshare(\%)$	1,651,680	10.94	12.83	0	100
$Volatility^{[d-100,d]}$	1,651,680	4.652	9.033	6.70e-05	111.7
$Volatility^{[d-200,d-101]}$	1,651,680	4.709	9.038	4.61e-05	105.0

Note: This table presents the summary of statistics for U.S. equity traded in exchanges, ATS, and OTC market during 2019-2022.

Figure 18: Volatility of Each Equity by DM Share



Note: This figure shows the average volatility for each equity during 2019-2022 by their average DM share. We classified the lit exchanges as CM, and ATS or OTC as DM. The dark box plots the volatility between $[t-100, t]$. The lighter box plots the IV, volatility between $[t-200, t-100]$. The lower and the upper end of the box are values at the 25th and 75th percentile. The white line in the box indicates the median value. And the lower and upper end of whiskers are lower and upper adjacent values.

Table 13: The Impact of Equity Volatility on DM Volume Share

Panel A. OLS		
Dependent Variable: $DMshare$	Full	CM&DM
$Volatility^{[t-100,t]}$	-0.00372***	-0.0172***
	(0.000397)	(0.00189)
Week FE	Yes	Yes
Equity FE	Yes	Yes
Observations	3,451,675	1,651,680
R-squared	0.982	0.546
Panel B. Reduced		
Dependent Variable: $DMShare$	Full	CM&DM
$Volatility^{[t-200,t-101]}$	-0.00291***	-0.0129***
	(0.000419)	(0.00173)
Week FE	Yes	Yes
Equity FE	Yes	Yes
Observations	3,451,675	1,651,680
R-squared	0.982	0.546
Panel C. First Stage of 2SLS		
Dependent Variable: $Volatility^{[t-100,t]}$	Full	CM&DM
$Volatility^{[t-200,t-101]}$	0.154***	0.290***
	(0.00277)	(0.00344)
Week FE	Yes	Yes
Equity FE	Yes	Yes
Observations	3,451,675	1,651,680
R-squared	0.743	0.775
Panel D. Second Stage of 2SLS		
Dependent Variable: $DMshare$	Full	CM&DM
$Volatility^{[t-100,t]}$	-0.0189***	-0.0447***
	(0.00274)	(0.00599)
Week FE	Yes	Yes
Equity FE	Yes	Yes
Observations	3,451,675	1,651,680
R-squared	0.982	0.546
Cragg-Donald Wald F statistic	3100	7097

Note: This table shows the impact of equity volatility on the proportion of volume traded in the DM versus CM. Robust standard errors are included in parentheses. *** p<0.01, ** p<0.05, * p<0.1.