Dynamic Market Choice *

Xian Wu†

November 18, 2023

[Please click here for the latest version]

Abstract

In reality, we find assets traded in the transparent centralized market and opaque decentralized market. To explain the traders' choices of venues, we develop a model of dynamic learning and dynamic market choice between the centralized market and decentralized markets. We find that when traders' value correlation is moderately heterogeneous, and asset values are insensitive to shocks to fundamentals or shocks are predictable, switching between centralized and decentralized markets can be the optimal market choice. When asset values are sensitive to volatile fundamentals, assets are traded only in the centralized market. We provide empirical evidence in support of the model predictions. The model allows us to explore the impact of introducing transparency designs in the opaque decentralized market on traders' market choices and welfare. We find that post-trade transparency makes the choice of a decentralized market persistent. Regardless of its impact on market structure, post-trade transparency improves welfare. Surprisingly, pre-trade transparency may decrease welfare as it increases traders' incentives to choose a decentralized market earlier and hurts centralized market welfare.

Keywords: endogenous market structure, decentralized market, dynamic learning, transparency

JEL Classification: D47, D83, G10, G14

*I am grateful to Marzena Rostek and Briana Chang for their invaluable guidance and support. I thank Hengjie Ai and Sebastien Plante for their insightful comments.

†University of Wisconsin - Madison, Department of Economics. Email: xwu394@wisc.edu.

1 Introduction

In reality, we find assets traded in the transparent centralized and opaque decentralized market. For example, equities can be traded in lit exchanges such as Nasdaq or NYSE, and at the same time they can be traded in over 30 dark pools.[1] We also find that the proportion of assets traded in the decentralized market fluctuates over time. For example, the transaction volume of equities traded in the dark pools versus that in the exchanges is found to be lower during volatile times.[2] The existence of the opaque decentralized market, in particular the dark pools, has raised policy concerns on market fragmentation and its lack of transparency. The policy concern motivates us to explore the following question: What determines traders' market choices? To answer this question, this paper develops a dynamic model to study traders' endogenous market choices, and explores the impact of transparency policies in the decentralized market on market choices and welfare.

The model features short-lived traders arriving each period to trade a risky asset. Before trading, the traders choose between a centralized market with all traders or a decentralized market where traders are matched and trade bilaterally. Traders have heterogeneous correlations in their values. Traders can learn from the centralized market prices in past periods when they trade. In the baseline model, we assume the decentralized market is opaque, i.e. traders cannot see past decentralized market prices.

We find that different dynamic market choices can arise endogenously as a result of learning from price history. We show that the impact of past market choices on the current market choice can be summarized with a single sufficient statistic, the price history informativeness. It measures how much the traders can learn from price history. Higher price history informativeness improves liquidity and increases expected utility for traders. Such improvement is higher in the decentralized market. Therefore, traders have higher incentives to choose the decentralized market as centralized price history accumulates. However, once they've chosen the decentralized market, given its opaque nature, the price history informativeness decays as the price history gradually becomes stale and uninformative. As the price history informativeness decreases, the decentralized market becomes illiquid. This can push traders back to the centralized market. The evolution of price

[1] A dark pool is a type of alternative trading system (ATS) that allows institutional investors to trade securities without publicly revealing their intentions during the search for a buyer or seller. They emerged in the 1980s when the Securities and Exchange Commission (SEC) allowed brokers to transact large blocks of shares. See `https://www.investopedia.com/terms/d/dark-pool.asp`.

[2] See Investors Flee Dark Pools As Market Volatility Erupts, The Wall Street Journal, Sept. 2, 2011, and "Dark Pools" Draw More Trading Amid Low Volatility, The Wall Street Journal, May 3, 2019.

history informativeness leads to dynamic market choices.

The threshold of price history informativeness for traders to choose the decentralized market over the centralized market depends on traders' correlation heterogeneity. The growing and decay rate of price history informativeness depends on the asset properties. We will discuss these two types of determinants sequentially.

We find that when trader value correlations are sufficiently homogeneous or heterogeneous, traders will choose a constant market structure; otherwise, alternating between centralized and decentralized markets can be the optimal dynamic market structure. Specifically, if trader value correlations are sufficiently heterogeneous, traders will always choose the decentralized market. This is because each trader can benefit significantly from matching bilaterally with a counterparty that has the lowest correlation and therefore, the lowest adverse selection. This decrease in adverse selection dominates the loss of liquidity in a decentralized market. If trader value correlations are sufficiently homogeneous, traders will always choose the centralized market. Intuitively, when correlations are similar across traders, traders don't avoid much adverse selection in the decentralized market, so traders prefer the greater liquidity of the centralized market. For intermediate levels of correlation heterogeneity, traders can potentially alternate between the lit centralized market and opaque decentralized market depending on the asset properties.

We highlight two asset properties that affect the dynamic market choice. The first one is asset sensitivity to fundamentals. When the asset is insensitive to the fundamentals, its value changes less across time, making price history more informative and decaying slower. Therefore traders switch between the centralized and decentralized market or even stay in the decentralized market after one round in the centralized market if the sensitivity is sufficiently low. When the asset is more sensitive to fundamentals, its value changes more across rounds with the shocks to fundamentals. The price history informativeness is low and traders will stay in the centralized market.

The second asset property is the predictability of the fundamentals measured by autocorrelation of the shocks to fundamentals. When the predictability of the fundamentals is high, the past prices are more informative and traders tend to alternate between the centralized market and the decentralized market, or even stay in the decentralized market since the second round. When the predictability of the fundamentals is low, past prices are not informative and traders will stay in the centralized market.

These results are consistent with our real-life observations. Securities that are designed to be insensitive to issuers' fundamentals, like bonds, are firstly traded in the centralized primary market and then in the secondary over-the-counter market. Securities that are relatively more sensitive to issuers' fundamentals, like equities, are mostly traded in the centralized market, sometimes traded in dark pools. Securities that by design are most sensitive to issuers' fundamentals, like options, are predominantly traded in the centralized market.

So far we've assumed that the decentralized market operates in opacity. We may wonder if market choices change as we introduce transparency in the decentralized market. Discussing the effects of transparency on market structures is of policy interest. In practice, certain decentralized markets provide post-trade transparency—examples include TRACE in the bond market and blockchain technology in the crypto market. Some markets offer pre-trade transparency like request-for-quotes (RFQ). However, some decentralized markets, like dark pools for equities, remain relatively opaque. The lack of transparency in dark pools has sparked criticism and policy attention. Yet, the consequences of introducing trade transparency to dark pools are unclear. Our dynamic model enables us to investigate how transparency design affects traders' market choices and welfare.

We find that with post-trade transparency, traders stay in the decentralized market once they've chosen it. This contrasts with the alternating market structures without transparency. With an opaque decentralized market, the price informativeness decays, and traders opt for the centralized market once the price history becomes stale. In contrast, transparency makes decentralized market prices available to future traders, increases the price history informativeness, and therefore improves decentralized market liquidity. It attracts traders to remain in the decentralized market. We also find that regardless of its impact on market choices, post-trade transparency is shown to weakly increase overall welfare.

Surprisingly, pre-trade transparency does not necessarily improve welfare. Even though pre-trade transparency leads to higher utility in the decentralized market, there's a trade-off – the informativeness of price history may decrease because traders are more inclined to opt for the decentralized market earlier. In scenarios where the number of rounds is large and the asset value remains stable due to low sensitivity or high shock predictability (autocorrelation), the persistent and long-term effect of low price history informativeness becomes significant. Under these conditions, the loss in price history informativeness outweighs the utility gain in the decentralized market, ultimately leading to a decrease in welfare with pre-trade transparency.

In the last part of the paper, we empirically tested the model predictions of the relationship between value autocorrelation and market choices in the U.S. equity market. We collected data for equities traded in exchanges, alternative trading systems (ATS), and over-the-counter(OTC) markets during 2019-2022 from FINRA and Wharton Research Data Service(WRDS). We classify the lit exchanges such as Nasdaq and NYSE as centralized markets, and ATS/OTC as decentralized markets. As lower autocorrelation implies higher volatility in asset values, we use the price volatility in the last 100 trading days as a proxy for the value autocorrelation. We find a negative correlation between price volatility and the proportion of transaction volume traded in the ATS and OTC. There is a concern that the market choices may affect the price volatility and cause reverse causality. To address the concern, we use the price volatility from 200 trading days to 100 trading days ago as an instrumental variable. 2SLS regression results show a more significant negative impact of volatility on decentralized market share than the OLS regression results.

Literature: This paper is most closely related to the literature studying endogenous market choice between the decentralized market and the centralized market. Some studies delve into factors like search frictions (Pagano, 1989; Rust and Hall, 2003; Vogel, 2019) or limited trading capacity (Dugast et al., 2022) within OTC markets. Some papers examine cream skimming driven by price discrimination (Seppi, 1990; Desgranges and Foucault, 2005; Bolton et al., 2016; Lee and Wang, 2018). The papers most closely related to our paper focus on the trade-off between liquidity and adverse selection (Yoon, 2017). This paper extends the static set-up in Yoon (2017) to a dynamic model. While Yoon (2017) focuses on the heterogeneity of traders' value correlation and private signal precision, this paper highlights the impact of learning from price history on traders' market choices. We show that price history informativeness related to asset properties is a new mechanism for the decentralized market to emerge.

Second, this paper is related to the literature on decentralized trading mechanisms in a dynamic setting. Most literature concentrates on dark pools and size-discovery sessions. Zhu (2014) presents a two-period model of a dark pool, and shows that adding a dark pool alongside an exchange enhances price discovery in the centralized exchange. Duffie and Zhu (2017) show that starting with a work-up trading session and then moving to a price-discovery market improves welfare. However, Antill and Duffie (2021) find that allowing size-discovery sessions over time alongside continuous price discovery harms welfare. A more recent paper by Blonien (2023) shows whether the size discovery is beneficial or not depends on trading frequency. Existing literature takes the market structure as given, assuming the exogenous arrival rate of the decentralized trading mechanism, and discusses its impact on welfare. This paper contributes to the literature by relaxing such assumptions. The arrival of the decentralized market is endogenous and can give traders the highest welfare.

Finally, this paper is related to papers on transparency designs (Duffie et al., 2017; Asriyan et al., 2017; Ollar et al., 2021; Back et al., 2020; Kakhbod and Song, 2020, 2022; Glebkin et al., 2023; Cespa and Vives, 2023; Vairo and Dworczak, 2023). Existing literature considers the impact of transparency designs given exogenous market structure. We contribute to the literature by allowing traders to choose the venue as a response to transparency designs. Our model allows us to discuss the transparency designs on both dynamic market choice and welfare.

2 Model

Market Structure Consider a market of one divisible risky asset and one risk-free asset as a numéraire. The market has T rounds, and $I \geq 4$ even number of short-lived traders arrive each round. In each round before they trade, traders first choose the market structure $\mathcal{M} = \{CM, DM\}$ that gives them the higher expected utility conditional on the price history traders observe.[3] The market structure can either be a *centralized market* (CM) where all traders participate in the same exchange. The traders can also choose a *decentralized market* (DM) where traders are matched with a counterparty according to an algorithm a la Irving (1985). The matching is pairwise stable in the sense that no traders want to leave the current counterparties and form a new pair. We assume that the traders will choose CM if the DM and CM give them the same utility. This assumption ensures our following results of dynamic market structure do not arise from the indeterminacy of the tie-breaking rule. Traders choose DM only when they strictly prefer DM. [4]

Information structure Each trader i's value of the risky asset is $\theta_{i,t} \equiv d_t + e_{i,t}$. The common value part $d_t = u + \xi f_t$, where f_t comes from the shocks to the asset fundamentals, u comes from macro-level risks unrelated to the asset fundamentals such as interest rate risk, and ξ measures the asset's value sensitivity to the asset fundamentals relative to the macro-level risks. The higher ξ, the more sensitive the security to shocks to the asset fundamentals. Without loss of generality we normalized d_t to have a standard normal

[3] Please refer to the information structure for the details of the price history.

[4] This assumption also simplifies the analysis by avoiding the coexistence of both CM and DM. The coexistence of both market structures can be studied in this paper by having CM and DM coexist if traders in one market structure are worse off shifting to the other. See Section 7 for a discussion.

distribution, $d_t \sim \mathcal{N}(0,1)$, $u \sim \mathcal{N}(0, \frac{1}{1+\xi^2})$ and $f_t \sim \mathcal{N}(0, \frac{1}{1+\xi^2})$.[5] f_t is time-varying given the growth of the underlying asset, e.g. firm issuers. It follows an AR(1) process $f_t = \kappa f_{t-1} + y_t$, where $\kappa \in [0,1]$, $y_t \sim \mathcal{N}(0, (1-\kappa^2)\frac{1}{1+\xi^2})$ is the innovation independent of any other random variables. κ measures the autocorrelation of the shocks across rounds. $e_{i,t} \sim \mathcal{N}(0, \epsilon^2)$ captures the heterogeneity of traders' value. $e_{i,t}$ is independent of u and f_t. By assumption the mean of $\theta_{i,t}$ is normalized as $\mathbb{E}[\theta_{i,t}] = 0$. Denote the variance of $\theta_{i,t}$ as $\sigma_\theta^2 \equiv 1 + \epsilon^2$. We allow $e_{i,t}$ to be correlated across traders, such that $\{\theta_{i,t}\}_i$ has the joint correlation matrix at round t,

$$\mathcal{C}_t \equiv \begin{pmatrix} 1 & \rho_{1,2,t} & \dots & \rho_{1,I,t} \\ \rho_{2,1,t} & 1 & \dots & \rho_{2,I,t} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{I,1,t} & \rho_{I,2,t} & \dots & 1 \end{pmatrix}.$$

To simplify the analysis, we assume that for any trader i, there is only one trader $j \neq i$ whose value correlates ρ_ℓ with trader i, and any other traders $k \neq j, i$ has value correlation $\rho_{i,k} > \rho_\ell$ with trader i. Later in Section 3, we will see that this assumption ensures unique pairwise matching a la Irving (1985).

Following Rostek and Weretka (2012), the market is equicommonal by assumption, i.e. the average correlation between any trader i and the residual market is the same, $\frac{1}{I-1}\sum_{j\neq i}\rho_{i,j,t} = \bar{\rho}_t$.

Traders are uncertain about the asset value $\theta_{i,t}$ and cannot observe u, $\{f_t\}_t$ and $\{e_{i,t}\}_{i,t}$. After they choose the market structure and before their trading, each trader observes a private noisy signal about his true value $\theta_{i,t}$, $s_{i,t} = \theta_{i,t} + \varepsilon_{i,t}$, where $\varepsilon_{i,t} \sim (0, \sigma^2\sigma_\theta^2)$. σ^2 measures the relative importance of noise in the signal. Assume σ is sufficiently large, $\sigma \geq ((\frac{2(I-1)}{I})^{1/3} - 1)^{-1/2}$.[6]

Traders can observe the current market price and submit demand contingent on that. Besides the private signals and the current market price, traders can also observe the prices in the CM in the past rounds. Traders cannot observe prices in the DM other than the price in their current pair. We define the observed price history at round t as $\mathcal{H}_t \equiv \{p_s^{CM}\}_{s<t}$.

[5] This normalization just ensures the comparative statics with ξ does not change anything else other than the relative sensitivity with respect to two risks. In particular, it does not change the traders' value variances. The normalization is to ensure the comparative analysis is rigorous, but is not necessary to generate all the results in the paper.

[6] This assumption is a sufficient but not necessary condition to generate all the results in the paper. This is to avoid the nonmonotonicity of utility to σ and to simplify the proof of Lemma 3. It ensures the utility decreases with noise σ. (See Vives (2011) for a discussion of the nonmonotonic impact of σ.) Numerically, $\sigma \geq 2.63$ is sufficient for any $I \geq 4$. The bound can shrink significantly with large I and appropriate choice of $\bar{\rho}$ and ρ_ℓ. We are working on decreasing the bound for σ in later versions of this paper.

Given the symmetric market assumption, if the optimal market choice of all traders at round t is CM, then the price history $\mathcal{H}_{t+1} = \mathcal{H}_t \cup \{p_t\}$, otherwise, $\mathcal{H}_{t+1} = \mathcal{H}_t$.

Preferences: The market is a double auction in a linear-normal setting. After the traders choose the market structure, they submit a demand schedule $q_{i,t}$, to maximize the payoffs conditioning on the history of past round price information $\mathcal{H}_t$, current round signal $s_{i,t}$, and the current round price p_t,

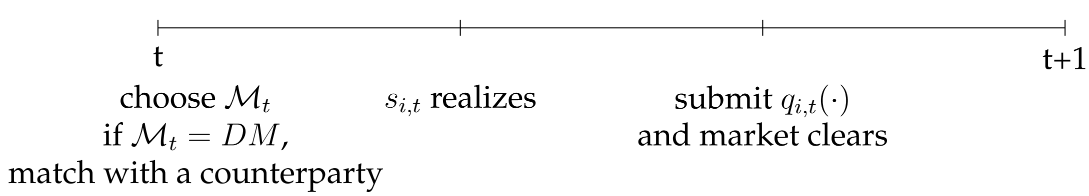
$$\mathbb{E}[U_{i,t}|\mathcal{H}_t, s_{i,t}, p_t] = \mathbb{E}[\theta_{i,t}q_{i,t} - \frac{\alpha}{2}(q_{i,t})^2 - p_t q_{i,t}|\mathcal{H}_t, s_{i,t}, p_t].$$

The linear-quadratic utility function form follows the literature(Kyle, 1989; Rostek and Weretka, 2012; Yoon, 2017), where α is trader's risk aversion.

The centralized market clears with p_t when $\sum_i q_{i,t}(p_t) = 0$. In the decentralized market, after the traders are matched bilaterally in $N = \frac{I}{2}$ pairs, each pair $n \in N$ clears independently with $p_{t,n}$ when $\sum_{i \in I(n)} q_{i,t}(p_{t,n}) = 0$. This set-up of decentralized market structure has a close mapping to markets in real life. In terms of the bond market, we use this decentralized trading mechanism to model the bilateral trade in the over-the-counter(OTC) market. In terms of the stock market, we use this decentralized market trading mechanism to model the dark pools operating as continuous non-displayed limit order books. This is the type of dark pool with the largest market share (around 70%) of total U.S. dark pool volumes in 2011 according to Tabb Group (2011). It includes many dark pools owned by major broker-dealers. [7].

Timing: We summarize the timing of each round with Figure 1.

Figure 1: Timing



3 Equilibrium

As the traders are short-lived, trading is static with a time-varying information set. Therefore, we are subject to solving the model round by round forwardly given price his-

[7] There are other two types of dark pools, one derives price from the lit venues, and the other acts like fast electronic market maker (Tabb Group, 2011; Zhu, 2014).

tory $\mathcal{H}_t$. In each round, the problem is solved with backward induction. First, we solve the trading strategy given the market structure. Then, we solve each trader's optimal market structure choice, by comparing each trader's expected utility in CM and DM. We apply the tie-breaking rule of choosing CM when CM and DM give the trader the same utility. By symmetry, each trader will have the same market choice and ex-ante expected utility. Given the optimal market choice, we can determine the evolution of the price history.

3.1 Second Stage Trading Equilibrium

Denote the chosen market structure as $\mathcal{M}^*$. By symmetry, choosing the market structure is equivalent to choosing the number of traders in the venue $I_{t,\mathcal{M}^*}$ and the average correlation across traders $\rho_{t,\mathcal{M}^*}$. It is easy to see that $\mathcal{M}^* = CM$, the number of traders in the exchange is $I_{t,\mathcal{M}^*} = I_t$ with an average correlation between any trader and the residual market $\rho_{t,M^*} = \bar{\rho}$. If $\mathcal{M}^* = DM$, the number of traders in each pair is $I_{t,\mathcal{M}^*} = 2$ and every pair clears independently. Without solving the ex-ante expected utility, we will not be able to know each trader's choice of counterparty. For now, let's assume that the correlation within each pair (i, j) is $\rho_{t,\mathcal{M}^*} = \rho_{i,j}$ and solve the bilateral equilibrium. With the equilibrium strategy solved in the second stage, we can write the ex-ante utility as a function of $\rho_{i,j}$ in the first stage, and the trader $j \neq i$ that gives the trader i the highest ex-ante utility will be the trader i's counterparty.

Given the market structure $\mathcal{M}_t^*$, at round t, traders submit a demand schedule $q_{i,t}$ to maximize the utility

$$\max_{q_{i,t}} \mathbb{E}[\theta_{i,t}q_{i,t} - \frac{\alpha}{2}(q_{i,t})^2 - p_t q_{i,t}|\mathcal{H}_t, s_{i,t}, p_t]$$

By taking first order condition with respect to $q_{i,t}$, we can solve the trader i's demand schedule,

$$q_{i,t} = \frac{\mathbb{E}[\theta_{i,t}|\mathcal{H}_t, s_{i,t}, p_t] - p_t}{\alpha + \lambda_{i,t}} \tag{1}$$

where $\lambda_{i,t} \equiv \frac{dp_t}{dq_{i,t}}$ is the price impact. By symmetry, the price impacts are the same for all traders in the same round $\lambda_{i,t} = \lambda_t, \forall i \in I_{t,\mathcal{M}^*}$. We can parameterize $\mathbb{E}[\theta_{i,t}|\mathcal{H}_t, s_{i,t}, p_t] = c_{\mathcal{H},i,t}\mathcal{H}_t + c_{s,i,t}s_{i,t} + c_{p,i,t}p_t$. By symmetry, the inference coefficients are the same for all traders in the same round, $c_{\mathcal{H},i,t} = c_{\mathcal{H},t}$, $c_{s,i,t} = c_{s,t}$ and $c_{p,i,t} = c_{p,t}$.

In equilibrium, by market clearing condition, λ_t is equal to the inverse of the slope of the residual demand,

$$\lambda_t = (-\sum_{j \neq i} \frac{dq_{j,t}}{dp_t}) = \frac{\alpha + \lambda_t}{(I_{t,\mathcal{M}^*} - 1)(1 - c_{p,t})}$$

Given the parameterization, the equilibrium price is,

$$p_t = (1 - c_{p,t})^{-1}(c_{\mathcal{H},t}\mathcal{H}_t + c_{s,t}\bar{s}_t) \tag{2}$$

where $\bar{s}_t = \frac{1}{I_{t,\mathcal{M}^*}}\sum_i s_{i,t}$ is the average signal in the exchange (for DM, it's the average signal in each pair).

The trader i's value $\theta_{i,t}$, the equilibrium price p_t given equation (2), the history $\mathcal{H}_t$ and the private signal $s_{i,t}$ are joint normally distributed. By projection theorem, the inference coefficients $c_{\mathcal{H},t}$, $c_{s,t}$, and $c_{p,t}$ can be determined given the joint distribution of $(\theta_{i,t}, s_{i,t}, \mathcal{H}_t, p_t)$.

We focus on linear Bayesian Nash equilibrium.

Theorem 1 (Trading Equilibrium). *Given the price history $\mathcal{H}_t$ and the market structure $\mathcal{M}_t^*$, the equilibrium at round t can be characterized by a fixed point of inference coefficients,*

$$c_{s,t} = \frac{1 - \rho_{t,\mathcal{M}^*}}{1 - \rho_{t,\mathcal{M}^*} + \sigma^2}$$

$$\boldsymbol{c}_{\mathcal{H},t} = \frac{(1 - \rho_{t,\mathcal{M}^*})\sigma^2}{(1 - \rho_{t,\mathcal{M}^*} + \sigma^2)\left(1 + (I_{t,\mathcal{M}^*} - 1)\rho_{t,\mathcal{M}^*} - I_{t,\mathcal{M}^*}\eta_t\right)}\boldsymbol{\tau}_t'\boldsymbol{\Upsilon}_t^{-1}$$

$$c_{p,t} = \frac{I_{t,\mathcal{M}^*}(\rho_{t,\mathcal{M}^*} - \eta_t)\sigma^2}{(1 - \rho_{t,\mathcal{M}^*} + \sigma^2)\left(1 + (I_{t,\mathcal{M}^*} - 1)\rho_{t,\mathcal{M}^*} - I_{t,\mathcal{M}^*}\eta_t\right)}$$

where $\eta_t = \frac{var(\theta_{i,t}) - var(\theta_{i,t}|\mathcal{H}_t)}{var(\theta_{i,t})} = \frac{\boldsymbol{\tau}_t'(\boldsymbol{\Upsilon}_t)^{-1}\boldsymbol{\tau}_t}{\sigma_\theta^2}$, $\boldsymbol{\tau}_t \equiv cov(\mathcal{H}_t, \theta_{i,t}) \in \mathbb{R}^{|\mathcal{H}|}$, and $\boldsymbol{\Upsilon}_t \equiv cov(\mathcal{H}_t, \mathcal{H}_t') \in \mathbb{R}^{|\mathcal{H}|\times|\mathcal{H}|}$.

The equilibrium price impact is

$$\lambda_t = \frac{\alpha}{(I_{t,\mathcal{M}^*} - 1)(1 - c_{p,t}) - 1}, \quad \forall i$$

The utility conditional on $\mathcal{H}_t$ for trader i is

$$\mathbb{E}[U_{i,t}^{\mathcal{M}^*}|\mathcal{H}_t] = \frac{\alpha + 2\lambda_t}{2(\alpha+\lambda_t)^2}\mathbb{E}[(\mathbb{E}[\theta_{i,t}|\mathcal{H}_t, s_{i,t}, p_t] - p_t)^2|\mathcal{H}_t] = \frac{\alpha + 2\lambda_t}{2(\alpha+\lambda_t)^2}\frac{I_{t,\mathcal{M}^*} - 1}{I_{t,\mathcal{M}^*}}\frac{(1 - \rho_{t,\mathcal{M}^*})^2}{1 - \rho_{t,\mathcal{M}^*} + \sigma^2}, \quad \forall i$$

3.2 First Stage Market Choice

Given the trading equilibrium in Theorem 1, we can obtain the ex-ante utility of the traders. By comparing the ex-ante utility of traders in DM and CM, we can determine the optimal market choice.

Ex-ante Utility in CM: If the market structure is CM, the ex-ante utility for trader i is

$$\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t] = \frac{\alpha + 2\lambda_t^{CM}}{2(\alpha + \lambda_t^{CM})^2}\frac{I-1}{I}\frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2} \quad \forall i \in I \tag{3}$$

where $\lambda_t^{CM} = \frac{\alpha}{(I_t-1)(1-c_{p,t})-1}$, $c_{p,t} = \frac{I_t(\bar{\rho}-\eta_t)\sigma^2}{(1-\bar{\rho}+\sigma^2)(1+(I_t-1)\bar{\rho}-I_t\eta_t)}$.

Ex-ante Utility in DM: For traders in the DM, we will need to first determine the trader i's counterparty a la Irving (1985). The trader j that gives trader i the highest utility is matched with trader i. Given that the traders $j \neq i$ are ex-ante identical except for their correlation with trader i, equivalently, this optimal choice of counterparty can be framed as the optimal choice of $\rho_{i,j}$ among the pairwise correlations $\{\rho_{i,j}\}_{j\neq i}$,

$$\max_{\rho_{i,j}|j\in I, j\neq i} \mathbb{E}[U_{i,t}^{DM}(\rho_{i,j})|\mathcal{H}_t]$$

Lemma 1 (Ex-ante Utility With Respect to Correlation Across Traders)**.** *Keep everything else constant, the ex-ante utility $\mathbb{E}[U_{i,t}^{\mathcal{M}}(\rho)|\mathcal{H}_t]$ is decreasing in the correlation ρ.*

By Lemma 1, the trader j with lowest correlation with i is matched as i's counterparty. By assumption, only one trader j has the lowest correlation ρ_ℓ with trader i, so the algorithm a la Irving (1985) generates a unique matching result. Given the matching result, trader i's ex-ante utility in DM is

$$\mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t] = \frac{\alpha + 2\lambda_t^{DM}}{4(\alpha + \lambda_t^{DM})^2}\frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2} \quad \forall i \in I \tag{4}$$

where $\lambda_t^{DM} = \frac{\alpha}{-c_{p,t}}$, $c_{p,t} = \frac{2(\rho_\ell-\eta_t)\sigma^2}{(1-\rho_\ell+\sigma^2)(1+\rho_\ell-2\eta_t)}$.

3.3 Dynamic Equilibrium

Given the expected utility in DM and CM characterized by equations (3) and (4), if $\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t] \geq \mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$, then the optimal market choice at round t is CM and the price history $\mathcal{H}_{t+1} = \mathcal{H}_t \cup \{p_t\}$, otherwise, the optimal market choice at round t is DM and $\mathcal{H}_{t+1} = \mathcal{H}_t$. We have the following recursive algorithm to generate the equilibrium of dynamic market choice through updates of $\mathcal{H}_t$,

Theorem 2 (Algorithm for Dynamic Market Choice Equilibrium)**.** *The Bayesian Nash equilibrium is a set of price history $\{\mathcal{H}_t\}_t$, a sequence of market choice $\{\mathcal{M}_t^*\}_t$, and a set of inference coefficients $\{c_{s,t}, c_{p,t}, \boldsymbol{c}_{\mathcal{H},t}\}$ that characterized forwardly recursively.*

1. *Initialize with* $t = 1$, $\mathcal{H}_1 = \emptyset$.

2. *Given* $\mathcal{H}_t$*, the equilibrium inference coefficients* $\{c_{s,t}, c_{p,t}, \boldsymbol{c}_{\mathcal{H},t}\}$ *is characterized in Theorem 1 with* $\rho_{t,\mathcal{M}^*} = \rho_\ell$ $I_{t,\mathcal{M}^*} = 2$ *if* $\mathcal{M}^* = DM$*, and* $\rho_{t,\mathcal{M}^*} = \bar{\rho}$ $I_{t,\mathcal{M}^*} = I$ *if* $\mathcal{M}^* = CM$.

3. *Given inference coefficients* $\{c_{s,t}, c_{p,t}, \boldsymbol{c}_{\mathcal{H},t}\}$*, If* $\frac{\alpha+2\lambda_t^{CM}}{2(\alpha+\lambda_t^{CM})^2}\frac{I_t-1}{I_t}\frac{(1-\bar{\rho}_t)^2}{1-\bar{\rho}_t+\sigma^2} \geq \frac{\alpha+2\lambda_t^{DM}}{4(\alpha+\lambda_t^{DM})^2}\frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2}$*, then* $\mathcal{M}_t^* = CM$*,* $\mathcal{H}_{t+1} = \mathcal{H}_t \cup \{p_t\}$*; otherwise,* $\mathcal{M}_t^* = DM$*,* $\mathcal{H}_{t+1} = \mathcal{H}_t$*. Repeat Steps 2-3 with the next* t*, until t=T.*

The proof of Theorem 2 is immediate from the above analysis.

3.4 Price History Informativeness

One observation from Theorem 1 and Theorem 2 is that, the impact of the price history $\mathcal{H}_t$ on the market choice can be summarized by a sufficient statistic, the informativeness of the price history to the traders, $\eta_t = \frac{var(\theta_{i,t})-var(\theta_{i,t}|\mathcal{H}_t)}{var(\theta_{i,t})}$.[8] It measures how much the price history reduces the variance of each trader's value. When the history has lower variance or higher covariance with $\theta_{i,t}$, η_t is higher.

Another observation is that the price history affects the current round utility only through price impact. This is because, as all traders equally have access to the same price history, their expected value updates by the same amount. The difference in the expected value and the price will stay constant with any price history. The price impact will be the only channel that the price history affects the current round demand schedule and utility.

Given the above observation that the price history informativeness η_t governs the impact of past market choice on the current market choice, we will first discuss the impact of η_t on the market choice before we analyze the dynamics. We have the following comparative statics results for η_t.

Lemma 2 (Comparative Statics With Price History Informativeness η)**.** *Keeping everything else constant, when* η *increases, the price impact* λ_t *decreases; and the ex-ante expected utility for any trader* i *increases.*

Lemma 2 shows the higher η improves liquidity and utility. When the price history is more informative, the trader values the price history by more and the current price by less. The residual market (the counterparty for the DM) requires a lower price increase to

[8] When $\mathcal{H}_t$ is a scalar, η_t is the square of the correlation between the price history $\mathcal{H}_t$ and any trader i's value $\theta_{i,t}$.

sell an additional unit to the trader given a more informative price history to calibrate the asset value, i.e. price impact decreases. Utility increases with lower price impact.

When the price history is sufficiently informative and the traders' value correlation is sufficiently heterogeneous, the traders' expected utility can be higher in the DM than in the CM.

Lemma 3 (Optimal Market Choice at Round t)**.** *Given $\mathcal{H}_t$, at round t,*

1. *$\mathbb{E}[u_{i,t}^{CM}|\mathcal{H}_t] - \mathbb{E}[u_{i,t}^{DM}|\mathcal{H}_t]$ is decreasing in η_t;*
2. *if $\bar{\rho} \leq \bar{\rho}^*(I, \rho_\ell, \sigma^2)$, then any trader i will choose CM;*
3. *if $\bar{\rho} > \bar{\rho}^*(I, \rho_\ell, \sigma^2)$, there exists $\tilde{\eta}(I, \bar{\rho}, \rho_\ell, \sigma^2)$ the any trader i will choose CM if $\eta_t \leq \tilde{\eta}$, and otherwise if $\eta_t > \tilde{\eta}$.*

Figure 2 serves as an example to Lemma 3. It shows the comparison of current round utility in CM versus DM with respect to trader value correlation and past price informativeness. We can see that when $\bar{\rho}$ is low, the utility in CM is always higher than DM. When $\bar{\rho}$ is high, there exists $\tilde{\eta}$ such that DM gives the traders higher utility than CM. As η increases, the utility difference between CM and DM decreases.

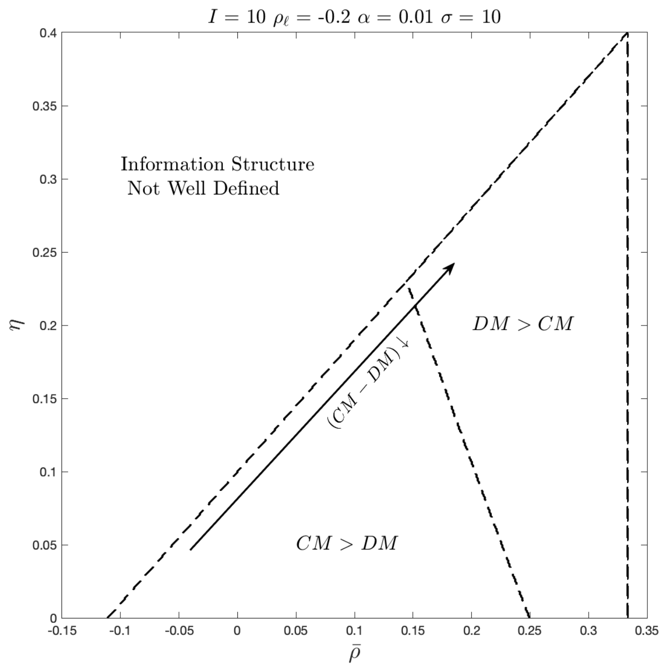
To understand the intuition of Lemma 3, we can decompose the trader i's utility into two parts, the liquidity effect and the learning-relative-to-market effect,

$$\mathbb{E}[U_{i,t}^{\mathcal{M}^*}|\mathcal{H}_t] = \underbrace{\frac{\alpha + 2\lambda_t}{2(\alpha+\lambda_t)^2}}_{\text{liquidity effect}} \underbrace{\mathbb{E}[(\mathbb{E}[\theta_{i,t}|\mathcal{H}_t, s_{i,t}, p_t] - p_t)^2|\mathcal{H}_t]}_{\text{learning-relative-to-market}}$$

The liquidity effect is inversely related to the price impacts. CM always has a higher liquidity effect than the bilateral DM given a larger market size. [9] The learning-relative-to-market effect is the ex-ante variance of the difference between the trader's expected value and the market price. It captures how much the trader's value differs from the market price, taking into account the adverse selection due to the learning of the residual market (or the counterparty in the DM) from the price. Hereafter for simplicity, we call it the learning effect. The learning effect decreases with the trader i's correlation with the

[9] People may mistakenly take liquidity here as the transaction volume. We want to clarify that the liquidity effect in this paper refers merely to the effect due to price impact but not the transaction volume. Equation (1) implies that the transaction volume is determined by both liquidity and learning-relative-to-market effect. Ex-ante, the variance of the transaction volume can be higher in DM than in CM when the learning-relative-to-market effect is sufficiently large. Therefore, we may see higher transaction volume ex-post with lower liquidity (high price impact) in the DM than in the CM.

Figure 2: Utility in CM v.s. DM With Trader Value Correlation and Price History Informativeness



residual market. When the correlation is low, the trader i's value is less correlated with the residual market, the residual market will not increase the price by much due to adverse selection, and therefore the learning effect is higher. When the correlation is sufficiently low, the learning effect is higher in the DM.

The intuition for DM to be the optimal market choice with sufficiently high η and ρ is as follows. The price history can decrease price impact both in the CM and DM. However, the improvement in the DM is larger than the CM, as the price impact is already very low in the CM, leaving less room for improvement. With sufficiently heterogeneous correlation, i.e., $\bar{\rho} > \bar{\rho}^* (I, \rho_\ell, \sigma^2)$, the enhancement in the learning effect becomes substantial. With high enough price history informativeness η, the loss of liquidity is marginal. In such instances, traders are inclined to opt for DM, prioritizing the valuable learning effect, even if it entails a sacrifice in liquidity.

Lemma 3 implies that, given heterogeneous value correlation and sufficiently informative price history, the traders have incentives to shift to DM. When traders stay in CM for a while, they may accumulate a long price history that has a high enough η for them to shift to DM in the next round. But when the traders stay in the DM for a while, given the opacity of DM, the price history informativeness η decays, incentivizing the traders to go back to CM.

Lemma 2 and Lemma 3 will be useful later in analyzing the dynamic market choice with respect to trader value correlations and asset properties.

4 Dynamic Market Choice

In this section, we will explore how market choices evolve dynamically. In particular, we are interested in how trader value correlations and asset properties affect the dynamic market choice.

4.1 Constant Market Choice

In this part, we discuss sufficient conditions for traders to choose only one market structure in all rounds.

Homogeneous Correlation: First, let's consider a simple case where the traders' value correlations are homogeneous. In this case, traders will always choose CM.

Proposition 1 (Homogeneous Correlation)**.** *When the traders value correlations are sufficiently homogeneous $\bar{\rho} \leq \bar{\rho}^* \left(I, \rho_\ell, \sigma^2\right)$, traders will always stay in the CM.*

The proof of Proposition 1 directly follows from Lemma 3, as no price history informativeness η will allow traders to choose DM. When the lowest correlation and the correlation with all other traders are similar, the benefit of trading with one counterparty with lower adverse selection in the DM is dominated by the loss of lower liquidity, regardless of the price history. Thus the traders have no incentive to choose DM in any rounds. This result is consistent with Yoon (2017), where the traders' heterogeneous value correlation is crucial for DM to exist.

Sufficiently Heterogeneous Correlation: On the opposite side, when the traders' value correlation is sufficiently heterogeneous, traders will always choose DM. If traders choose DM in the first round, they will choose DM for all rounds.

Lemma 4 (DM persisitency)**.** *Keep everything else constant, if $\mathcal{M}_1^* = DM$, then $\mathcal{M}_t^* = DM$ for all t.*

Proof. Suppose traders choose DM over CM in round 1. Easy to see that $\eta_1 = 0$. Given that in each round primitives $(I, \bar{\rho}, \rho_\ell, \sigma^2)$ are the same, this means they prefer DM over CM if $\eta_t = 0$. As DM is opaque, if $\eta_t = 0$ and traders choose DM at round t, then $\eta_{t+1} = 0$. This implies traders will always choose DM by forward induction. ■

Given Lemma 4, if we find sufficiently heterogeneous correlation makes traders choose DM in the first round, then they will stay in DM for all rounds.

Proposition 2 (Sufficiently Heterogeneous Correlation)**.** *There exists $\underline{\rho}_\ell < 0$ and $\bar{\bar{\rho}} > \bar{\rho}^*$ such that for any $\rho_\ell < \underline{\rho}_\ell$ and $\bar{\rho} > \bar{\bar{\rho}}$, traders will stay in the DM for all rounds.*

4.2 Alternating Market Choices

The equilibrium becomes more interesting when we the traders' value correlations are neither too homogenous nor too heterogeneous. Alternating between CM and DM can emerge endogenously as the optimal market choice. It's also worth mentioning that the optimal market choice generates the overall highest welfare, and Pareto dominates other market choices. We find that the asset properties, including asset sensitivity to shocks to fundamentals ξ, and the fundamental value predictability measured by autocorrelation κ, are crucial for the market choices.

Proposition 3 (Heterogeneous Correlation and Asset Sensitivity)**.** *With heterogeneous correlation $\bar{\rho} > \bar{\rho}^*(I, \rho_\ell, \sigma^2)$, $\rho_\ell \geq 0$, $\epsilon < \bar{\epsilon}(\sigma^2, I)$, $3 \leq T < \bar{T}(\sigma^2, I, \epsilon)$, and $\kappa < \bar{\kappa}(\sigma^2, I, \epsilon)$, there exists $\underline{\xi}$ and $\bar{\xi}$ such that such that traders will choose CM in the first round, and*

1. *When the asset sensitivity to shocks to fundamentals is sufficiently low $\xi \in [0, \underline{\xi})$, the traders shift to DM in the second round and stay there.*

2. *When the asset sensitivity to shocks to fundamentals is intermediate $\xi \in [\underline{\xi}, \bar{\xi})$, the traders will alternate between CM and DM.*

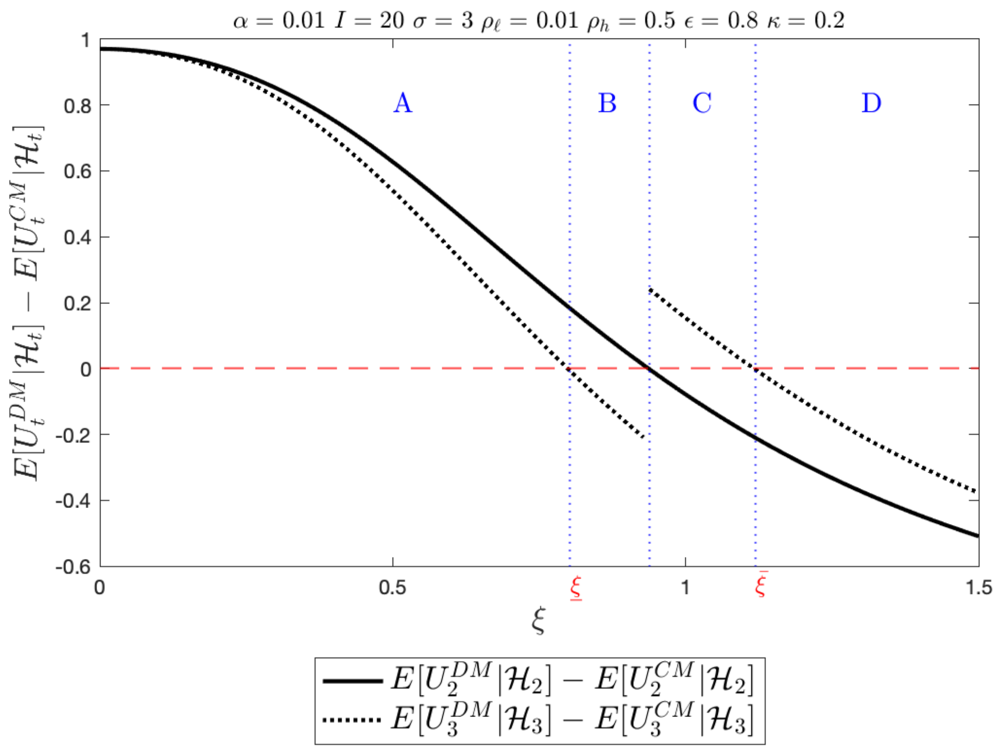
3. *When the asset sensitivity to shocks to fundamentals is sufficiently high $\xi \in [\bar{\xi}, \infty)$, the traders will always stay in the CM.*

Intuitively, when traders' value correlations are heterogeneous, i.e., when $\bar{\rho}$ and $\bar{\rho}_\ell$ are sufficiently different, the traders have incentives to shift to DM by our previous analysis. To further understand this result with respect to asset sensitivity, let's consider the following three-round example.

Example 1 (Three-round Market)**.** *We consider a market with $T = 3$. Assume that $\rho_\ell > 0$, such that in the 1st round the DM does not exist and the traders will always choose CM. And any pair of traders that don't have correlation ρ_ℓ have value correlation $\rho_h > \rho_\ell$.*

Figure 3 shows the market choice of the traders in the 2nd and 3rd round. In the Appendix, we also provided the price history informativeness in the 2nd and 3rd rounds with respect to ξ.

Figure 3: Dynamic Market Choice With Respect to Asset Sensitivity ξ in $T = 3$ Market



Note: The black solid line plots the difference between the ex-ante expected utility of DM and that of CM in the 2nd round, and the black dotted line plots that difference in the 3rd round. The red dashed line is a reference line of 0. When the black solid(dotted) line is above the reference line, then the traders choose DM in the 2nd round(3rd round), and if it is below the reference line, the traders choose CM in the 2nd round(3rd round). The jump in the difference of utility in CM v.s. DM in the third round comes from the difference in the second-round choice.

In region A, i.e. $\xi \in [0, \underline{\xi})$, traders choose DM in both 2nd and 3rd round. In region B, i.e. $\xi \in (\underline{\xi}, \bar{\xi}]$ and in the lower partition, traders choose DM in the 2nd round and CM in the 3rd round. In region C, $\xi \in (\underline{\xi}, \bar{\xi}]$ and in the higher partition, traders choose CM in the 2nd round and DM in the 3rd round. In region D, $\xi \in (\bar{\xi}, 1]$, traders choose CM in both the 2nd round and the 3rd round.

Region A: *When the asset sensitivity is low, this implies the asset value is less susceptible to shocks to fundamentals and more correlated across rounds. This also implies that the price history is more informative to the traders. A more informative price history can lower the price impact and increase liquidity effect and utility. Such an increase is higher for DM than CM, as CM is already very liquid thus leaving less room for liquidity improvement. The higher liquidity improvement in DM can decrease the loss of liquidity effect for choosing DM and be dominated by the gain in learning effect. This gives rise to a shift to DM in the 2nd round.*

When the asset sensitivity is sufficiently low, $\xi \in [0, \underline{\xi})$, traders will continue to stay in DM in the 3rd round, as the 1st round price is still informative enough for them to enjoy a higher learning effect at just a bit higher price impact in DM.

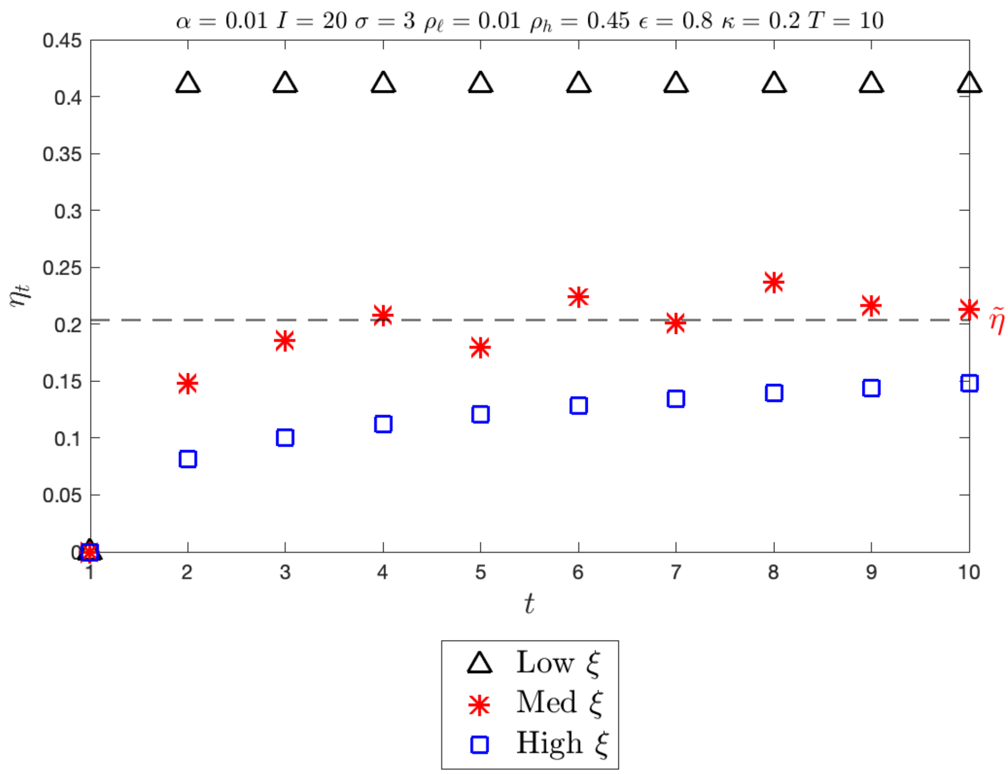
Region B: *However, when the asset sensitivity is not sufficiently low, i.e., $\xi \in [\underline{\xi}, \bar{\xi})$, traders will alternate between DM and CM. traders will choose DM in the second round and choose CM in the 3rd round. As the asset value is not that stable across time and traders don't know the DM price, the price in the 1st round becomes stale and not informative enough for the 3rd round values. The liquidity difference in CM and DM again becomes large, making traders shift back to CM for liquidity improvement at a loss of learning effect.*

Region C: *When the asset sensitivity is high but not high enough, i.e., $\xi \in [\underline{\xi}, \bar{\xi})$ and higher than that in Case 2, traders will still alternate between DM and CM. Traders will choose CM in the first two rounds and shift to DM in the last round. This is because the asset sensitivity is not low such that traders will choose CM in the second round for higher liquidity. However, the asset sensitivity is not sufficiently high, given traders' access to both the 1st round and 2nd round price in CM, the price history is sufficiently informative for the liquidity in the DM to be close enough to CM in the 3rd round. The traders shift to DM in the 3rd round for a better learning effect at a bit cost of liquidity effect.*

Region D: *When the asset sensitivity is sufficiently high, i.e., $\xi \in [\bar{\xi}, \infty)$, traders will stay in the CM for both the 2nd and 3rd round. This is because the value of assets changes frequently across time, making price history not informative enough to largely boost the liquidity in the DM. Therefore, the liquidity difference between the DM and CM remains large, preventing the traders from choosing DM for the benefit of the learning effect.*

To summarize, Example 1 shows how the asset sensitivity affects the informativeness of the price history η, and then affects the liquidity effect and therefore current market choice. Past prices in the CM increase η and lower price impact. A high learning effect (lower adverse selection) pushes traders to the DM. However, DM opacity can lower η and push traders back to the CM. These intuitions from Example 1 can be extended to more than 3 rounds. Figure 4 shows the evolution of price history informativeness and market choice with $T = 10$ with respect to different levels of asset sensitivities. When the marker is above(below) the reference line $\tilde{\eta}$ which is defined by Lemma 3 and calculated according to trader value correlations, then traders choose DM(CM).

Figure 4: Evolution of Price History Informativeness For Different Asset Sensitivities



Note: This figure shows the evolution of price history informativeness for different level of asset sensitivity ξ for $T = 10$. The black dashed line is a reference line of threshold $\tilde{\eta}$. When the marker is above the reference line, then the history informativeness in that round is higher than $\tilde{\eta}$ and traders choose DM. If the marker is below the reference line, then the history informativeness in that round is lower than $\tilde{\eta}$ and traders choose CM.

We would like to clarify that the mechanism for alternating market choice does not come from the tie-breaking rule which we do not impose any indeterminacy. It also differs from the mechanism as in Yoon (2017) where (i) traders in the DM do not access CM price; and (ii) marginal trader's (weak) indifference between DM and CM gives rise to coexistence. In this paper, traders choose DM when DM gives them a strictly higher utility than CM. DM emerges endogenously as a result of learning from price history, and fades endogenously when the price history becomes uninformative.

Proposition 3 is consistent with our real life observations. Securities that are designed to be insensitive to issuers' fundamentals, like bonds, are firstly traded in the centralized primary market and then mostly traded in the secondary over-the-counter market. Securities that are relatively more sensitive to issuers' fundamentals, like equities, are mostly traded in the centralized market, sometimes traded in dark pools. Securities that by design are most sensitive to issuers' fundamentals, like options, are only traded in the centralized market.

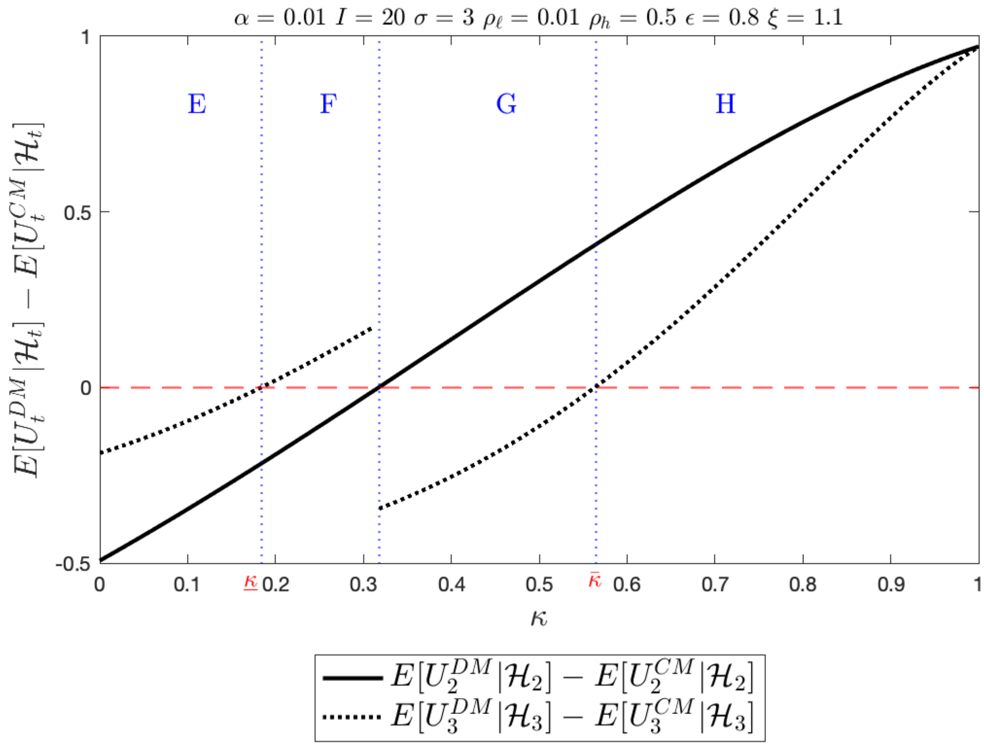
Proposition 4 (Heterogeneous Correlation and Autocorrelation)**.** *With heterogeneous correlation $\bar{\rho} > \bar{\rho}^*(I, \rho_\ell, \sigma^2)$, $\epsilon < \bar{\epsilon}(\sigma^2, I)$, $3 \leq T < \bar{T}(\sigma^2, I, \epsilon)$, and $\xi > \underline{\xi}(\sigma^2, I, \epsilon)$, there exists $\underline{\kappa}$ and $\bar{\kappa}$ such that traders will choose CM in the first round, and*

1. *When the autocorrelation is sufficiently low $\kappa \in [0, \underline{\kappa}]$, the traders will always stay in the CM.*
2. *When the autocorrelation is intermediate $\kappa \in (\underline{\kappa}, \bar{\kappa}]$, the traders will alternate between CM and DM.*
3. *When the autocorrelation is sufficiently high $\kappa \in (\bar{\kappa}, 1]$, the traders will choose DM over CM in the second round and never choose CM again.*

Figure 5 shows the market choice of the traders in the 2nd and 3rd round in Example 1 with respect to autocorrelation κ. In the Appendix, we also provided the price history informativeness in the 2nd and 3rd rounds with respect to κ. Similar to the analysis of Proposition 3, the intuition for Proposition 4 also works through the dynamics of the price history informativeness η. The price history informativeness η is increasing in autocorrelation κ. When autocorrelation is higher, this means the values are more stable across rounds, the price history is more informative, and the traders are more likely to shift to DM. The intuition of Example 1 also applies to a market with more rounds. Figure 6 shows the evolution of price history informativeness and market choice for a $T = 10$ round market with respect to different levels of autocorrelation. When the marker is above(below) the reference line $\tilde{\eta}$ which is defined by Lemma 3 and calculated according to trader value correlations, then traders choose DM(CM).

The autocorrelation κ captures the predictability of the fundamentals. Proposition 4 implies that when the shocks are more predictable, i.e. high κ, then the traders are more likely to trade in DM. The implication of Proposition 4 is consistent with some existing empirical literature. Both Menkveld et al. (2017) and Buti et al. (2022) find that the market share of the dark pools (corresponding to DM in our model) relative to the lit venues (corresponding to CM in our model) decreases when the market is more volatile.

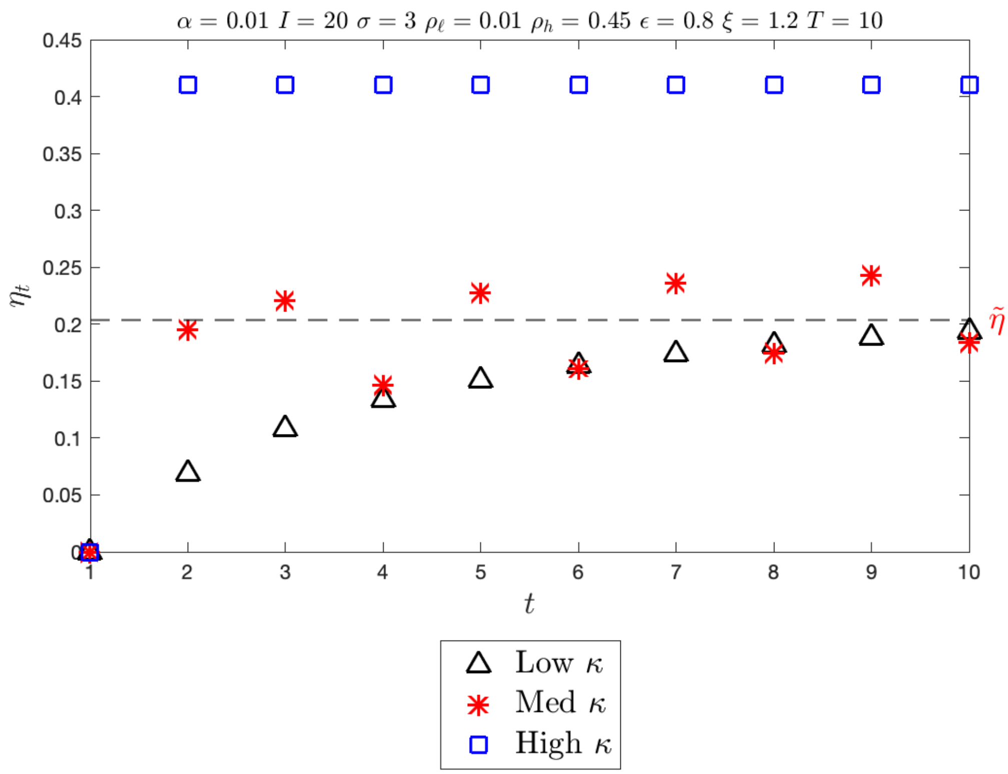
Figure 5: Dynamic Market Choices With Respect to Autocorrelation κ in $T = 3$ Market



Note: The black solid line plots the difference between the ex-ante expected utility of DM and that of CM in the 2nd round, and the black dotted line plots that difference in the 3rd round. The red dashed line is a reference line of 0. When the black solid(dotted) line is above the reference line, then the traders choose DM in the 2nd round(3rd round), and if it is below the reference line, the traders choose CM in the 2nd round(3rd round). The jump in the difference of utility in CM v.s. DM in the third round comes from the difference in the second-round choice.

In region E, i.e. $\kappa \in [0, \underline{\kappa}]$, traders choose CM in both 2nd and 3rd round. In region F, i.e. $\kappa \in (\underline{\kappa}, \bar{\kappa}]$ and in the lower partition, traders choose CM in the 2nd round and DM in the 3rd round. In region G, $\kappa \in (\underline{\kappa}, \bar{\kappa}]$ and in the higher partition, traders choose DM in the 2nd round and CM in the 3rd round. In region H, $\kappa \in (\bar{\kappa}, 1]$, traders choose DM in both the 2nd round and the 3rd round.

Figure 6: Evolution of Price History Informativeness For Different Autocorrelations



Note: This figure shows the evolution of price history informativeness for different levels of autocorrelation κ for $T = 10$. The black dashed line is a reference line of threshold $\tilde{\eta}$. When the marker is above the reference line, then the history informativeness in that round is higher than $\tilde{\eta}$ and traders choose DM. If the marker is below the reference line, then the history informativeness in that round is lower than $\tilde{\eta}$ and traders choose CM.

We also want to clarify the difference between asset sensitivity and value predictability. Even if the issuer's value has low predictability, it is possible for the issuer to design securities that have low asset sensitivity to be traded in the DM.

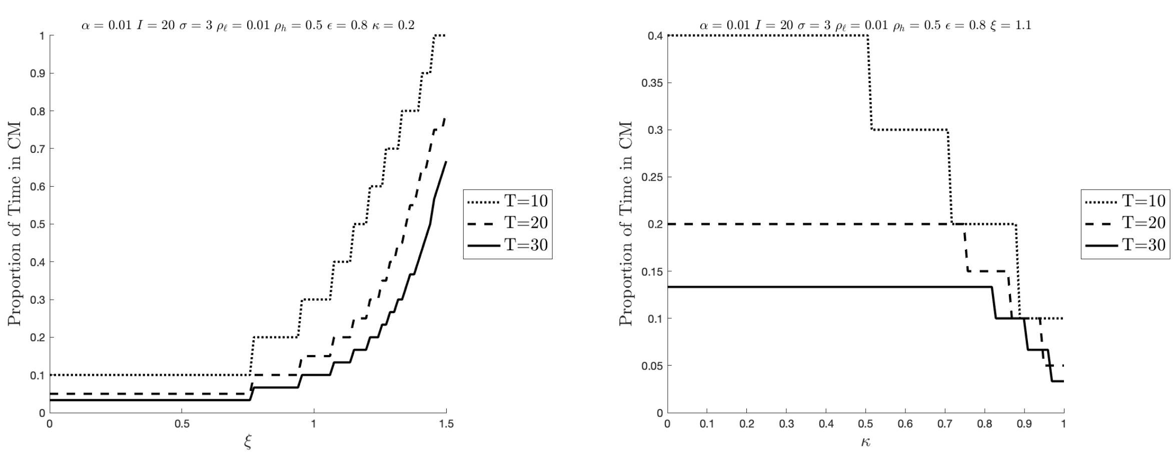
4.3 Proportion of Time in CM

Figure 7 shows the proportion of time when traders choose CM with respect to asset sensitivity ξ and autocorrelation (or shock predictability) κ. Unsurprisingly, we find that the proportion of time in CM increases with ξ, and decreases with κ. The patterns are consistent with our intuition for Propositions 3 and 4.

Figure 7 also shows the proportion of time when traders in CM with respect to rounds T. Numerically, we find that alternating market choice between DM and CM is in general more prevalent as trading round T increases. Note that with a small probability the proportion of time in CM with smaller T can be lower than that with larger T, this is because the last round can end at different stages of an alternating cycle.

Intuitively, with longer T the price history informativeness η increases as its length accumulates, and it is more likely for traders to choose DM over CM. This implies assets with shorter terms are more likely to be traded in the centralized market, e.g. most options are less than 90 days. Assets with the longer term are more likely to be traded in the decentralized market or alternating market structure, e.g. bonds have maturities as long as 30 years, and equities usually don't have maturity.

Figure 7: Proportion of Time in CM



(a) Asset sensitivity ξ

(b) Autocorrelation κ

5 Transparency and Dynamic Market Choice

So far, we have assumed that DM is opaque, i.e. future traders cannot see prices in DM and traders in DM cannot see prices in other pairs. In this section, we will consider introducing transparency designs in DM.

It is of policy interest to discuss the impact of transparency on the market structures and welfare. In reality, traders have post-trade transparency in some decentralized markets, e.g. TRACE in the bond market, and blockchain technology in the crypto market. Some decentralized trading mechanism allows pre-trade transparency, e.g. request-for-quote. However, some decentralized markets are relatively opaque, e.g. dark pools for equities. The lack of transparency in dark pools has received critique and policy attention. However, the impact of introducing transparency to dark pools remains unclear. Our dynamic model allows us to explore the impact of transparency designs on traders' market choices and welfare.

5.1 Post-trade Transparency

In this section, we will consider introducing post-trade transparency to DM, i.e., prices in DM will enter the price history and affect future market choices.

It is easy to see that Theorem 1 still applies to equilibrium with post-trade transparency. Denote the number of trading pairs in the DM as $N = \frac{I}{2}$, and each trading pair as n. We can slightly modify the price updating rule in Theorem 2 to characterize the new equilibrium.

Theorem 3 (Algorithm for Dynamic Market Choice Equilibrium with Post-trade Transparency)**.** *The Bayesian Nash equilibrium is a set of price history $\{\mathcal{H}_t\}_t$, a sequence of market choice $\{\mathcal{M}_t^*\}_t$, and a set of inference coefficients $\{c_{s,t}, c_{p,t}, \boldsymbol{c}_{\mathcal{H},t}\}$ that characterized forwardly recursively.*

1. *Initialize with $t = 1$, $\mathcal{H}_1 = \emptyset$.*

2. *Given $\mathcal{H}_t$, the equilibrium inference coefficients $\{c_{s,t}, c_{p,t}, \boldsymbol{c}_{\mathcal{H},t}\}$ is characterized in Theorem 1 with $\rho_{t,\mathcal{M}^*} = \rho_\ell$ $I_{t,\mathcal{M}^*} = 2$ if $\mathcal{M}^* = DM$, and $\rho_{t,\mathcal{M}^*} = \bar{\rho}$ $I_{t,\mathcal{M}^*} = I$ if $\mathcal{M}^* = CM$.*

3. *Given inference coefficients $\{c_{s,t}, c_{p,t}, \boldsymbol{c}_{\mathcal{H},t}\}$, If $\frac{\alpha+2\lambda_t^{CM}}{2(\alpha+\lambda_t^{CM})^2} \frac{I_t-1}{I_t} \frac{(1-\bar{\rho}_t)^2}{1-\bar{\rho}_t+\sigma^2} \geq \frac{\alpha+2\lambda_t^{DM}}{4(\alpha+\lambda_t^{DM})^2} \frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2}$, then $\mathcal{M}_t^* = CM$, $\mathcal{H}_{t+1} = \mathcal{H}_t \cup \{p_t\}$; otherwise, $\mathcal{M}_t^* = DM$, $\mathcal{H}_{t+1} = \mathcal{H}_t \cup \{p_{n,t}\}_n$, where $p_{n,t}$ is the equilibrium price of bilateral trading pair n. Repeat Steps 2-3 with the next t, until t=T.*

The proof of Theorem 3 follows the analysis in Section 3.

First, we explore the impact of post-trade transparency on traders' optimal market choice. Perhaps surprisingly, we find that with post-trade transparency, traders will stay in DM once they have chosen it. This is because the price history informativeness η never decays, attracting traders to stay in DM.

Proposition 5 (Post-trade Transparency: Once DM, Always DM)**.** *With post-trade transparency, if $\mathcal{M}_t^* = DM$, then $\mathcal{M}_\tau^* = DM$, $\forall \tau \geq t$.*

By Proposition 5, the potential dynamic market choices will be (i) choosing DM for all rounds; (ii) choosing CM at first and DM thereafter; and (iii) choosing CM for all rounds. Alternating back and forth between DM and CM is no longer an optimal dynamic market choice. Note that this result is different from Lemma 4 which only describes one possible market choice, i.e., DM persists when traders choose DM in the first round. Proposition 5 implies, if we introduce post-trade transparency in dark pools, the traders will not return to the centralized market.

Still, regardless of its impact on the market choice, post-trade transparency in DM weakly increases the overall welfare.

Proposition 6 (Post-trade Transparency Improves Welfare)**.** *Post-trade transparency weakly improves welfare regardless of market choices.*

Post-trade transparency in DM does not affect the utility of traders when they choose CM, but can weakly increase welfare when they choose DM. The intuition is as follows. η_t^{post} with post-trade transparency will always be weakly higher than η_t without post-trade transparency, as the DM prices are informationally equivalent to the average signal of each bilateral pair, which is at least as informative as the centralized market price in the same round if traders choose CM without post-trade transparency. Given $\eta_t^{post} \geq \eta_t$, any market choice without post-trade transparency will not give traders higher utility than DM with post-trade transparency. Given that traders are ex-ante identical, the welfare improvement is Pareto.

5.2 Pre-trade Transparency

In this section, we will consider introducing pre-trade transparency in DM. We allow traders in each pair to not only submit demand schedules contingent on their price but also the prices in other pairs. Their demand schedule in DM at round t will be $q_{i,t}(\boldsymbol{p}_t) : \mathbb{R}^N \to \mathbb{R}$, where $\boldsymbol{p}_t \in \mathbb{R}^N$ is the vector all prices in all pairs whose n^{th} element is the price in pair n at round t, $p_{n,t}$. For tractability, besides that each trader will have a correlation ρ_ℓ with only one trader, we further assume that each trader has a correlation ρ_h with all other traders in the same round.

5.2.1 Equilibrium Characterization with Pre-trade Transparency

It's easy to see that given history $\mathcal{H}_{i,t}$, the trading equilibrium in CM will not be affected by the pre-trade transparency in DM. We can still apply Theorem 1 to characterize CM equilibrium. We need to solve for the new trading equilibrium for DM.

With pre-trade transparency, traders in the DM will have access to prices from other pairs and submit demand schedules contingent on them. Trader $i \in I(n)$ submit demand schedule $q_{i,t}(\boldsymbol{p}_t) : \mathbb{R}^N \to \mathbb{R}$ to maximize the expected utility conditional on the history $\mathcal{H}_t$, private signal $s_{i,t}$, and

$$\max_{q_{i,t}(\boldsymbol{p}_t)} \mathbb{E}[\theta_{i,t} q_{i,t} - \frac{1}{2}\alpha q_{i,t}^2 - p_{n,t} q_t | \boldsymbol{p}_t, \mathcal{H}_t, s_{i,t}]$$

trader i 's first-order condition as

$$q^i(\boldsymbol{p}_t) = \frac{\mathbb{E}[\theta_{i,t} | \boldsymbol{p}_t, \mathcal{H}_t, s_{i,t}] - p_{n,t}}{\alpha + \lambda_{i,t}}$$

where $\lambda_{i,t}$ is the trader i's price impact within pair n. Trader i also has cross-pair price impact as traders from other pairs will change their bids when price p_n changes with i's bid. Trader i's price impact over all pairs can be described with a price impact matrix $\boldsymbol{\Lambda}_{i,t} = (\frac{d\boldsymbol{p}}{d\boldsymbol{q}_{i,t}}) \in \mathbb{R}^{N \times N}$, where the n^{th} diagonal elements is $\lambda_{i,t}$. Each trader i 's price impact matrix is equal to the transpose of the Jacobian of trader i's inverse residual supply:

$$(\boldsymbol{\Lambda}_{i,t})' = \left(-\sum_{j \neq i} \frac{d\boldsymbol{q}_{j,t}}{d\boldsymbol{p}_t} \right)^{-1}$$

We can parameterize $\mathbb{E}[\theta_{i,t} | \boldsymbol{p}_t, \mathcal{H}_t, s_{i,t}] = \boldsymbol{c}_{\mathcal{H},i,t} \mathcal{H}_t + c_{s,i,t} s_{i,t} + \boldsymbol{c}_{p,i,t} \boldsymbol{p}_t$. $\boldsymbol{c}_{\mathcal{H},i,t} \in \mathbb{R}^{1 \times |\mathcal{H}_t|}$, $c_{s,i,t} \in \mathbb{R}$, and $\boldsymbol{c}_{p,i,t} \in \mathbb{R}^{1 \times N}$. Given symmetry within each pair, $\boldsymbol{c}_{\mathcal{H},i,t} = \boldsymbol{c}_{\mathcal{H},n,t}$, $c_{s,i,t} = c_{c,n,t}$, $\boldsymbol{c}_{p,i,t} = \boldsymbol{c}_{p,n,t}$ and $\lambda_{i,t} = \lambda_{n,t}$.

Given the market clearing condition, $\sum_{i \in I(n)} q_{i,t}(\boldsymbol{p}_t) = 0$, and trader symmetry within exchanges, we have the equilibrium price in all pairs in vector form,

$$\boldsymbol{p}_t = \left(\boldsymbol{Id} - \boldsymbol{C}_{p,t}\right)^{-1}\left(\boldsymbol{C}_{\mathcal{H},t}\mathcal{H}_t + \boldsymbol{C}_{s,t}\bar{\boldsymbol{s}}_t\right),$$

where $\boldsymbol{C}_{s,t} = \operatorname{diag}\left(c_{s,n,t}\right)_n \in \mathbb{R}^{N\times N}, \boldsymbol{C}_{\mathcal{H},t} = \left(\boldsymbol{c}_{\mathcal{H},n,t}\right)_n \in \mathbb{R}^{N\times|\mathcal{H}_t|}, \boldsymbol{C}_{p,t} = \left(\boldsymbol{c}_{p,n,t}\right)_n \in \mathbb{R}^{N\times N}$. $\bar{\boldsymbol{s}}_t \in \mathbb{R}^N$ is the average signals for all pairs, where the n^{th} element is the average signal in pair n.

Given that value $\theta_{i,t}$, private signal $s_{i,t}$, prices $\boldsymbol{p}_t$, and price history $\mathcal{H}_t$ are jointly normally distributed, we can solve the inference coefficients through projection theorem.

Theorem 4 (DM Trading Equilibrium with Pre-trade Transparency)**.** *The equilibrium in DM with pre-trade transparency can be characterized by the inference coefficients* $\boldsymbol{C}_{s,t} = \operatorname{diag}\left(c_{s,n,t}\right)_n \in \mathbb{R}^{N\times N}, \boldsymbol{C}_{\mathcal{H},t} = \left(\boldsymbol{c}_{\mathcal{H},n,t}\right)_n \in \mathbb{R}^{N\times|\mathcal{H}_t|}, \boldsymbol{C}_{p,t} = \left(\boldsymbol{c}_{p,n,t}\right)_n \in \mathbb{R}^{N\times N}$*,*

$$\begin{aligned}
\boldsymbol{C}_{s,t} &= \operatorname{diag}\left(\frac{1-\rho_{n,t}}{1-\rho_{n,t}+\sigma^2}\right)_n \\
\boldsymbol{C}_{\mathcal{H},t} &= \operatorname{diag}\left(\frac{(1-\rho_{n,t})\sigma^2}{2(1-\rho_{n,t}+\sigma^2)}\right)_n (\bar{\mathcal{C}}_t - \mathbf{1}\mathbf{1}'\eta)^{-1}\mathbf{1}\boldsymbol{\tau}_t'\boldsymbol{\Upsilon}_t^{-1} \\
\boldsymbol{C}_{p,t} &= \operatorname{diag}\left(\frac{\sigma^2}{1-\rho_{n,t}+\sigma^2}\right)_n \left(\boldsymbol{Id} - \operatorname{diag}\left(\frac{1-\rho_{n,t}}{2}\right)(\bar{\mathcal{C}} - \mathbf{1}\mathbf{1}'\eta)^{-1}\right)
\end{aligned}$$

$\eta_t = \frac{\boldsymbol{\tau}_t'\boldsymbol{\Upsilon}_t^{-1}\boldsymbol{\tau}_t}{\sigma_\theta^2}$ *is price history informativeness.* $\bar{\mathcal{C}} = \frac{cov(\bar{\boldsymbol{\theta}}_t, \bar{\boldsymbol{\theta}}_t')}{\sigma_\theta^2} \in \mathbb{R}^{N\times N}$ *is the correlation of pairwise average values across all pairs, where* $\bar{\boldsymbol{\theta}}_t \in \mathbb{R}^N$ *is the vector of average value per trading pair where the* n^{th} *value is* $\bar{\theta}_{n,t} = \sum_{i\in I(n)}\theta_{i,t}$*.*

The price impact for trader i *in pair* n *is*

$$\lambda_{n,t} = \left(\left(\left(\left(\boldsymbol{Id} - \boldsymbol{C}_{p,t}\right)^{-1}\right)_{nn}\right)^{-1} - 1\right)^{-1}\alpha.$$

where $(A)_{nn}$ *is an operator that gives the* n^{th} *diagonal element of matrix* A*.*

The expect utility for trader i *in pair* n *conditional on the price history is*

$$\mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t] = \frac{\alpha+2\lambda_t}{2(\alpha+\lambda_t)^2}\mathbb{E}[(\mathbb{E}[\theta_{i,t}|\mathcal{H}_t, s_{i,t}, \boldsymbol{p}_t] - p_{t,n})^2|\mathcal{H}_t] = \frac{\alpha+2\lambda_{n,t}}{2(\alpha+\lambda_{n,t})^2}\frac{1}{2}\frac{(1-\rho_{n,t})^2}{1-\rho_{n,t}+\sigma^2}$$

Thoerem 4 shows that the price history's impact on the current round utility is still through price impact, and can be summarized by the sufficient statistic, price history informativeness η_t.

Our next question is, will pre-trade transparency change the matching results in DM?

We find that the expected utility is still monotonic in the correlation $\rho_{n,t}$ (see Lemma 5). Therefore, each trader will be matched with the counterparty that has the lowest correlation ρ_ℓ, same as the matching results in Section 3.

Lemma 5 (Monotonicity of Utility with Pre-trade Transparency)**.** *With pre-trade transparency, $\mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$ is monotonically decreasing in $\rho_{n,t}$.*

Given that introducing pre-trade transparency does not change the matching results in DM, and the price history update rule remains the same, we can still apply Theorem 2 to characterize the equilibrium.

5.2.2 Pre-trade Transprarency's Impact on Market Choice and Welfare

With the equilibrium characterization, we would be able to discuss the impact of pre-trade transparency on market choice and welfare.

First, we find that given price history $\mathcal{H}_t$, introducing pre-trade transparency always weakly increases the utility for all traders in DM.

Lemma 6 (Pre-trade Transparency Increases DM Utility)**.** *Given price history $\mathcal{H}_t$, introducing pre-trade transparency weakly increases the utility for all traders in DM.*

Given Lemma 6, it's intuitive that everything else constant, it's more likely for traders to choose DM over CM as the threshold history informativeness $\tilde{\eta}$ for traders to opt for DM is weakly lower.

Proposition 7 (Pre-trade Transparency Precipitates DM)**.** *With pre-trade transparency, (i) the first time for traders to choose DM is no later than without transparency; (ii) if the round traders firstly choose DM is the same as the round traders firstly choose DM without pre-trade transparency, then they stay in DM for weakly longer.*

The fact that pre-trade transparency can make traders choose DM earlier creates nuances in terms of welfare. By Lemma 6 we know that transparency increases utility for traders in DM given the price history. However, choosing DM earlier and staying longer can potentially decrease the price history informativness and welfare in later rounds. Pre-trade transparency can bring down welfare when the loss of history informativeness dominates the benefit in DM.

Proposition 8 (Pre-trade Transparency and Welfare)**.**

1. *For sufficiently heterogeneous trader value $\rho_\ell < \underline{\rho}_\ell < 0$ and $\bar{\rho} > \bar{\bar{\rho}}$, pre-trade transparency weakly improves welfare.*

2. *For sufficiently homogenous trader value, $\bar{\rho} < \bar{\rho}^{*,pre}(I, \rho_\ell, \sigma^2)$, pre-trade transparency does not change welfare.*

3. *When traders' value correlations are neither sufficiently heterogeneous nor sufficiently homogenous, pre-trade transparency can decrease welfare when the number of rounds T is sufficiently large, asset sensitivity ξ is low, or autocorrelation κ is high.*

Intuitively, Proposition 8.1 corresponds to the constant DM choices both with and without pre-trade transparency, and given Lemma 6, pre-trade transparency should always weakly increase welfare. Proposition 8.2 corresponds to the constant CM choices both with and without pre-trade transparency. As traders don't choose DM, pre-trade transparency does not change welfare.

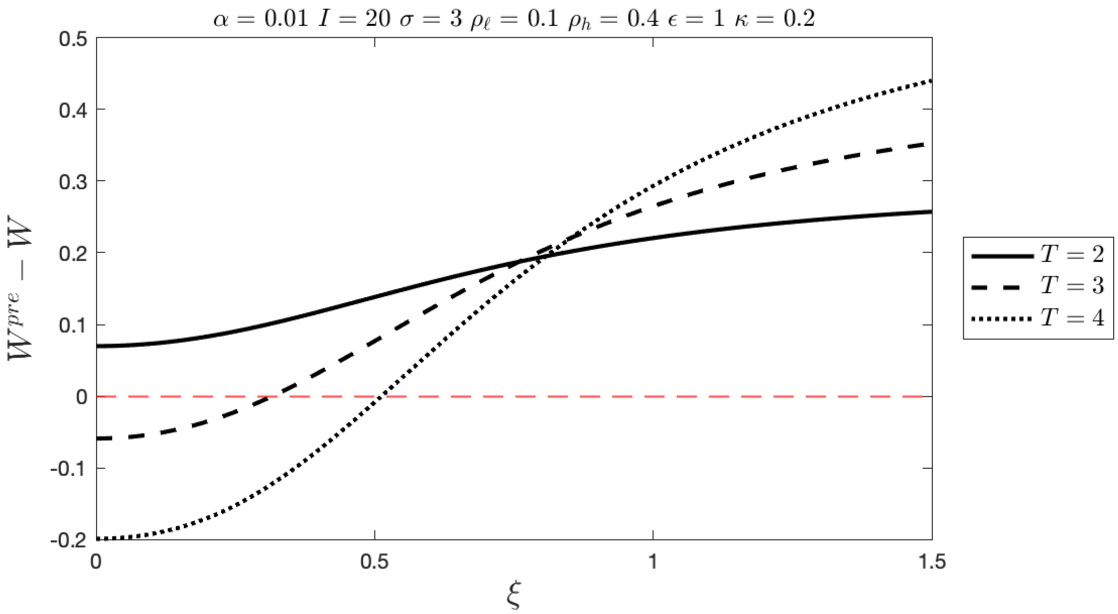
The impact of pre-trade transparency on welfare is ambiguous when traders' value correlation heterogeneity is of intermediate level (Proposition 8.3). This is when traders can have alternating market choices in the benchmark model. Despite that DM utility is higher with pre-trade transparency, price history informativeness may be lower as traders are more likely to choose DM earlier with pre-trade transparency. Figure 8 shows the difference between welfare with pre-trade transparency and welfare with opaque DM with respect to asset sensitivity ξ, autocorrelation κ and the number of rounds T. When the number of rounds T is large, and the asset value is stable either due to low sensitivity ξ or high shock predictability (autocorrelation) κ, low price history informativeness has a persistent and long-run impact. With these conditions, the loss of price history informativeness dominates the utility gain in DM, making pre-trade transparency welfare-decreasing.

This welfare result contrasts Vairo and Dworczak (2023) where they find pre-trade transparency always improves welfare. The key difference is that they focus on the impact of transparency given the decentralized market structure, but we endogenize the impact of pre-trade transparency on dynamic market choice and highlight the loss in price history informativeness.

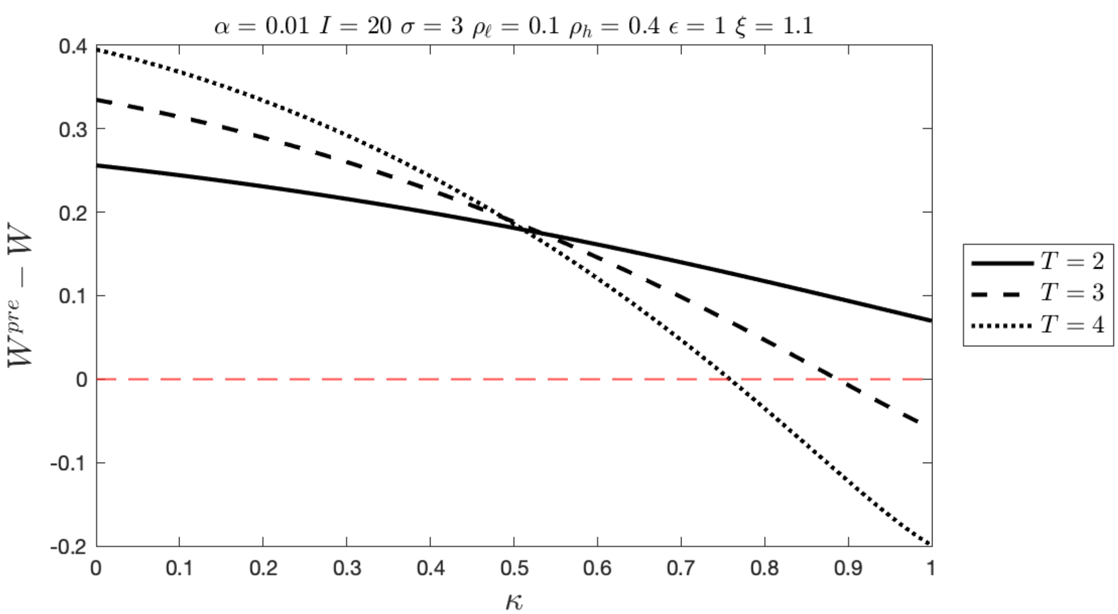
6 Empirical Evidence

The model provides us with testable predictions. It shows that a higher asset autocorrelation can lead to market fragmentation (Proposition 4). To test this prediction, we collected data for equities traded in exchanges, alternative trading systems (ATS), and over-the-counter(OTC) markets.

Figure 8: The Difference Between Welfare With Pre-trade Transparency and Welfare With Opaque DM



(a) Asset sensitivity ξ



(b) Autocorrelation κ

Note: Each black line plots the welfare with pre-trade transparency minus the welfare with opaque DM. The red dash line is a reference line of 0. If the black line is higher than (or at) the reference line, then pre-trade transparency (weakly) improves welfare, otherwise, it decreases welfare.

We obtain the ATS weekly summary of transaction volumes from FINRA and Exchange and OTC equity prices and transaction volumes from Wharton Research Data Service(WRDS). Our sample period is 2019-2022. We classify the lit exchanges as CM, e.g., Nasdaq, and NYSE. We classify ATS (e.g., Credit Suisse Crossfinder, Instinet) and OTC as DM.[10] We consider two samples for the regression and construct variables for each sample respectively. The first sample is the full sample that includes all equities traded in all venues. There is a concern that some equities may be restricted to be traded only in CM or DM due to regulations, preventing traders from changing their venues as the model assumption. To mitigate the influence of the venue restrictions on our identification, we consider another sample, which includes those equities that have ever been traded in both CM and DM during 2019-2022. We drop singleton observations of equities with only one-week transaction in both samples.

We construct the dependent variable $DMshare_{i,t}$, which is the transaction volume of equity i in DM as a proportion of the total transaction volume of equity i in all venues in week t. Given that lower κ implies higher volatility in values, we use the price volatility in the last 100 days $Volatility^{[d-100,d]}$ as a proxy for κ, which is constructed as follows. We first calculate the standard deviation of the close price $p_{i,d}$ in the last 100 trading days $[d-100, d]$, and then take the weekly average of it for each equity i and week t.[11] We winsor the top and bottom 1% to avoid the impact of extreme values.

We use the following regression to test the model prediction in Proposition 4 with both the full sample and a smaller sample of equities traded in both DM and CM,

$$DMshare_{i,t} = \beta Volatility_{i,t}^{[d-100,d]} + \delta_i + \gamma_t + \varepsilon_{i,t} \tag{5}$$

where δ_i are equity fixed effects, γ_t are week fixed effects, and $\varepsilon_{i,t}$ are robust standard errors.

One concern is that traders' market choices may affect the price fluctuations. It can cause reverse causality and weaken our identification results. Therefore, we construct the lagged price volatility $Volatility_{i,t}^{[d-200,d-101]}$ as an instrumental variable (IV). $Volatility_{i,t}^{[d-200,d-101]}$ is the weekly average of the standard deviation of the close price $p_{i,d}$ between trading day $[d-200, d-101]$ for each equity i in week t. We winsor the top and bottom 1% to avoid the extreme value.

[10] Please refer to FINRA equity ATS Firms and SEC Form ATS-N Filings and Information for a complete list and more detailed information of current and past ATS for equities.

[11] As some OTC equities are not traded frequently, not all trading days have close prices. We use the midpoint of the best bid and ask prices on each trading day as the close price.

Table 1 shows the summary of statistics of the variables. The average proportion traded in DM is 57.27% over the full sample and 10.94% for equities ever traded in both DM and CM. The price volatility and its IV on average are 4.190 and 4.164 respectively for the full sample. For equities traded in both DM and CM, the price volatility and its IV on average 4.652 and 4.709 respectively.

Figure 9 shows the average price volatility and DM share weighted by each equity's total transaction volume in 2019-2022. We find that the proportion of volume traded in DM is inversely related to the volatility.

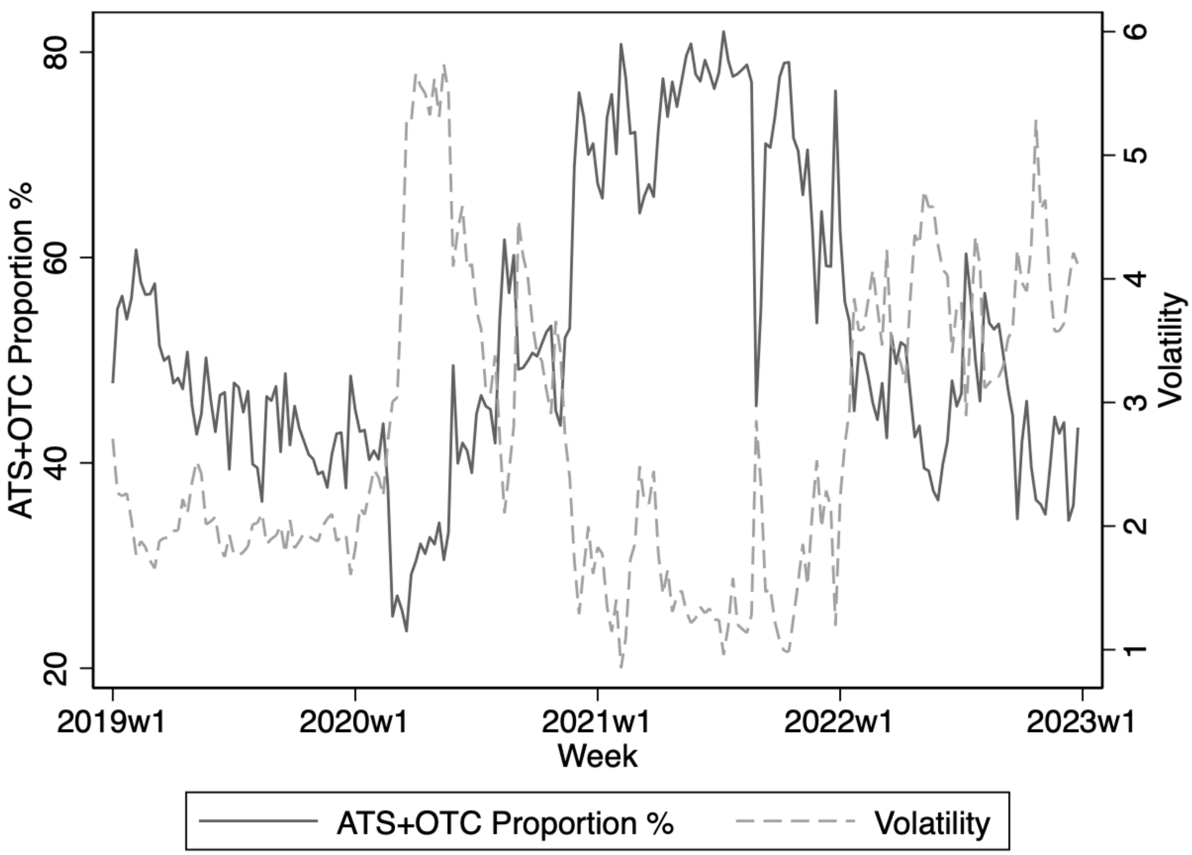
Figure 10 shows the average price volatility for each equity by their DM share. We can see that the average volatility is the highest for equities only traded in CM, lower for the equities traded in both DM and CM, and lowest for equities traded in DM only.

Table 2 shows the regression results of equation (5). Panel A shows the OLS regression results, where we can see that for both the full sample and the restricted sample with equities traded in CM and DM, the volatility is negatively correlated with the proportion of transaction volume in DM. Panel B-D shows the two-stage least-square (2SLS) regression results using the $Volatility^{[d-200,d-101]}$ as an IV. Panel B shows the reduced-form results with the IV as the independent variable. Panel C shows the first stage of 2SLS regression which indicates the IV is strongly correlated with $Volatility^{[d-100,d]}$. Panel D shows the second stage of 2SLS regression. We find that the volatility significantly decreases the proportion of transaction volume in DM, and the magnitude is larger than the OLS regression results.

Table 1: Summary of Statistics

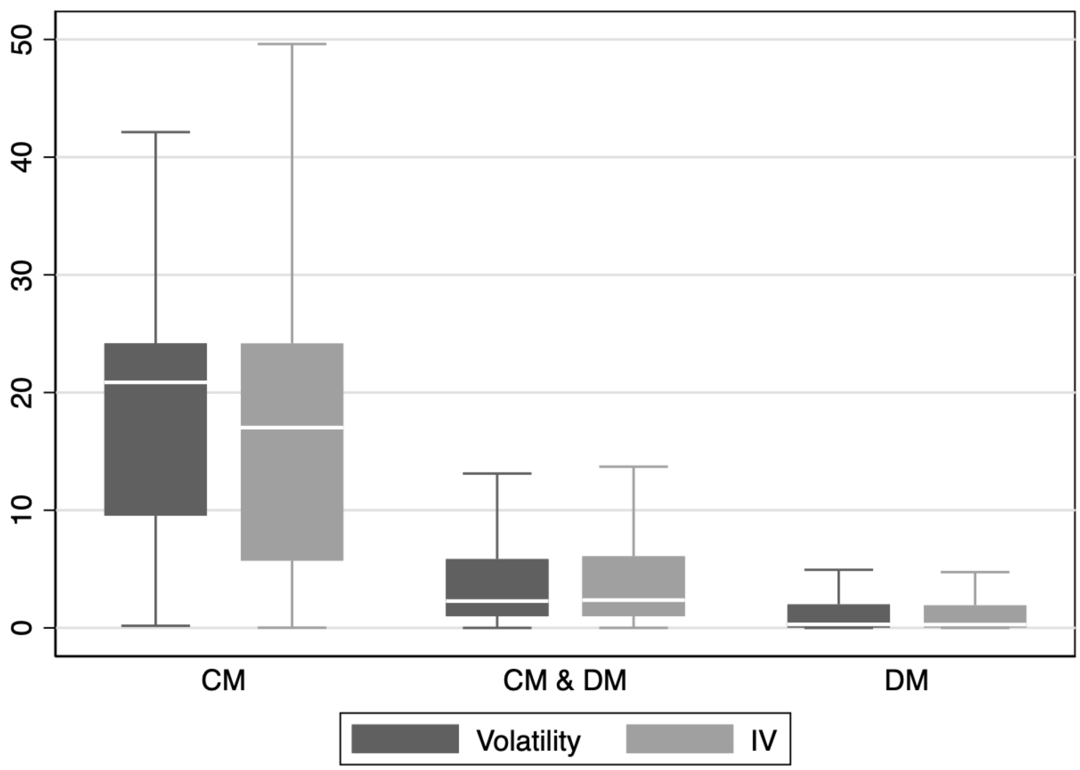
Full Sample					
Variable	Obs	Mean	Std. Dev.	Min	Max
$DMshare(\%)$	3,451,675	57.27	45.39	0	100
$Volatility^{[d-100,d]}$	3,451,675	4.190	13.20	6.70e-05	111.7
$Volatility^{[d-200,d-101]}$	3,451,675	4.164	12.65	4.61e-05	105.0
Equities Traded in CM & DM					
Variable	Obs	Mean	Std. Dev.	Min	Max
$DMshare(\%)$	1,651,680	10.94	12.83	0	100
$Volatility^{[d-100,d]}$	1,651,680	4.652	9.033	6.70e-05	111.7
$Volatility^{[d-200,d-101]}$	1,651,680	4.709	9.038	4.61e-05	105.0

Figure 9: Volatility and DM Share in 2019-2022



Note: This figure shows the average volatility and weekly DM share (ATS+OTC transaction volume share) weighted by each equity's total transaction volume in CM and DM during 2019-2022.

Figure 10: Volatility of Each Equity by DM Share



Note: This figure shows the average volatility for each equity during 2019-2022 by their average DM share. We classified the lit exchanges as CM, and ATS or OTC as DM. The dark box plots the volatility between $[t-100, t]$. The lighter box plots the IV, volatility between $[t-200, t-100]$. The lower and the upper end of the box are values at 25% and 75% percentile. The white line in the box indicates median value. And the lower and upper end of whiskers are lower and upper adjacent values.

Table 2: The Impact of Equity Volatility on DM Volume Share

Panel A. OLS		
Dependent Variable: $DMshare$	Full	CM&DM
$Volatility^{[t-100,t]}$	-0.00372***	-0.0172***
	(0.000397)	(0.00189)
Week FE	Yes	Yes
Equity FE	Yes	Yes
Observations	3,451,675	1,651,680
R-squared	0.982	0.546
Panel B. Reduced		
Dependent Variable: $DMShare$	Full	CM&DM
$Volatility^{[t-200,t-101]}$	-0.00291***	-0.0129***
	(0.000419)	(0.00173)
Week FE	Yes	Yes
Equity FE	Yes	Yes
Observations	3,451,675	1,651,680
R-squared	0.982	0.546
Panel C. First Stage of 2SLS		
Dependent Variable: $Volatility^{[t-100,t]}$	Full	CM&DM
$Volatility^{[t-200,t-101]}$	0.154***	0.290***
	(0.00277)	(0.00344)
Week FE	Yes	Yes
Equity FE	Yes	Yes
Observations	3,451,675	1,651,680
R-squared	0.743	0.775
Panel D. Second Stage of 2SLS		
Dependent Variable: $DMshare$	Full	CM&DM
$Volatility^{[t-100,t]}$	-0.0189***	-0.0447***
	(0.00274)	(0.00599)
Week FE	Yes	Yes
Equity FE	Yes	Yes
Observations	3,451,675	1,651,680
R-squared	0.982	0.546
Cragg-Donald Wald F statistic	3100	7097

Note: This table shows the impact of equity volatility on the proportion of volume traded in the DM versus CM. Robust standard errors are included in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

7 Conclusion and Discussions

This paper presents a model examining the dynamic market choice between centralized and decentralized markets, where arriving traders must decide between a centralized market and a bilaterally matched decentralized market in each period. The emergence of dynamic market choice is observed as a consequence of learning from the centralized market price history. Optimal market choices, influenced by asset properties, include switching between centralized and decentralized markets when traders' value correlation is moderately heterogeneous. In cases where asset values are insensitive to shocks or shocks are predictable, traders alternate between centralized and decentralized markets or remain in the decentralized market after one round in the centralized market. Conversely, when asset values are sensitive to unpredictable fundamentals, traders choose to stay in the centralized market.

Additionally, we explore the impact of introducing transparency in the opaque decentralized market on traders' market choices. Our findings indicate that post-trade transparency encourages traders to stick with the decentralized market once chosen. Despite its influence on market choices, post-trade transparency improves welfare. We find that pre-trade transparency will make the traders choose DM earlier. However, the welfare effect is ambiguous. Pre-trade transparency can decrease welfare when the number of rounds is large and when the asset value is stable due to insensitivity or high predictability of the shocks.

In this paper, we do not allow for the coexistence of DM and CM in the same round. However, coexistence can be studied by revising the tie-breaking rule. If we allow the traders to choose DM or CM until they don't want to deviate to the other venue, we will have coexistence and endogenous CM size when the DM and CM utility are close enough to each other.[12] Note that with coexistence, price history informativeness η can still decay. Either η decays immediately after coexistence as the coexisted CM price has a larger variance with a smaller CM size. Or with an appropriate choice of parameters price informativeness may increase but it will disappear soon as higher η makes traders strictly prefer DM and coexistence is no longer possible. And then η starts to decay as DM is opaque. However, price history informativeness decays at a lower rate as traders in future rounds see coexisted CM prices. The intuitions from this paper still apply, but we may see DM with coexistence persist longer than DM without coexistence.

[12] See Yoon (2017) for a discussion of coexistence of CM and DM.

It is interesting to see that learning from price history alone generates rich dynamic market choices. It is also important to recognize that we abstract away from the inventory held by traders by assuming short-lived traders. Adding dynamic inventory significantly reduces tractability in the linear-quadratic double auction setting like this paper. Inventory management across rounds is also an important aspect of trading strategies. We believe dynamic market choice with both dynamic inventory and learning effect warrants future research.

References

Antill, S. and D. Duffie (2021). Augmenting markets with mechanisms. *The Review of Economic Studies 88*(4), 1665–1719.

Asriyan, V., W. Fuchs, and B. Green (2017). Information spillovers in asset markets with correlated values. *American Economic Review 107*(7), 2007–2040.

Back, K., R. Liu, and A. Teguia (2020). Signaling in otc markets: benefits and costs of transparency. *Journal of Financial and Quantitative Analysis 55*(1), 47–75.

Blonien, P. (2023). Size discovery in slow markets. *Available at SSRN 4484749*.

Bolton, P., T. Santos, and J. A. Scheinkman (2016). Cream-skimming in financial markets. *The Journal of Finance 71*(2), 709–736.

Buti, S., B. Rindi, and I. M. Werner (2022). Diving into dark pools. *Financial Management 51*(4), 961–994.

Cespa, G. and X. Vives (2023). Market opacity and fragility. *Available at SSRN 4339568*.

Desgranges, G. and T. Foucault (2005). Reputation-based pricing and price improvements. *Journal of Economics and Business 57*(6), 493–527.

Duffie, D., P. Dworczak, and H. Zhu (2017). Benchmarks in search markets. *The Journal of Finance 72*(5), 1983–2044.

Duffie, D. and H. Zhu (2017). Size discovery. *The Review of Financial Studies 30*(4), 1095–1150.

Dugast, J., S. Üslü, and P.-O. Weill (2022). A theory of participation in otc and centralized markets. *The Review of Economic Studies 89*(6), 3223–3266.

Glebkin, S., B. Z. Yueshen, and J. Shen (2023). Simultaneous multilateral search. *The Review of Financial Studies 36*(2), 571–614.

Irving, R. W. (1985). An efficient algorithm for the "stable roommates" problem. *Journal of Algorithms 6*(4), 577–595.

Kakhbod, A. and F. Song (2020). Dynamic price discovery: Transparency vs. information design. *Games and Economic Behavior 122*, 203–232.

Kakhbod, A. and F. Song (2022). Public vs. private offers with informed and forward-looking dealers. Technical report, Working Paper.

Kyle, A. S. (1989). Informed speculation with imperfect competition. *The Review of Economic Studies 56*(3), 317–355.

Lee, T. and C. Wang (2018). Why trade over-the-counter? *Jacobs Levy Equity Management Center for Quantitative Financial Research Paper*.

Menkveld, A. J., B. Z. Yueshen, and H. Zhu (2017). Shades of darkness: A pecking order of trading venues. *Journal of Financial Economics 124*(3), 503–534.

Ollar, M., M. J. Rostek, and J. H. Yoon (2021). Privacy in markets. *Available at SSRN 3071374*.

Pagano, M. (1989). Trading volume and asset liquidity. *The Quarterly Journal of Economics 104*(2), 255–274.

Rostek, M. and M. Weretka (2012). Price inference in small markets. *Econometrica 80*(2), 687–711.

Rust, J. and G. Hall (2003). Middlemen versus market makers: A theory of competitive exchange. *Journal of Political Economy 111*(2), 353–403.

Seppi, D. J. (1990). Equilibrium block trading and asymmetric information. *the Journal of Finance 45*(1), 73–94.

Tabb Group (2011). Liquidity matrix. *Technical report*.

Vairo, M. and P. Dworczak (2023). What type of transparency in otc markets? *Available at SSRN 4431190*.

Vives, X. (2011). Strategic supply function competition with private information. *Econometrica 79*(6), 1919–1966.

Vogel, S. (2019). When to introduce electronic trading platforms in over-the-counter markets? *Available at SSRN 2895222*.

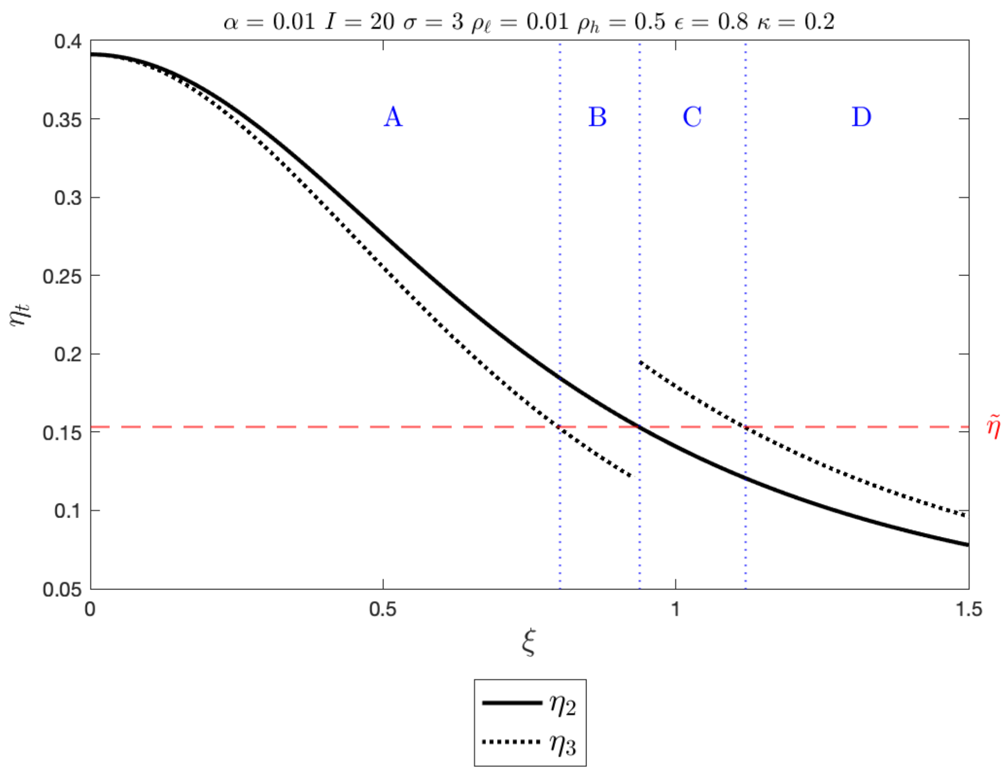
Yoon, J. H. (2017). Endogenous market structure:over-the-counter versus exchange trading. *Working Paper*.

Zhu, H. (2014). Do dark pools harm price discovery? *The Review of Financial Studies 27*(3), 747–789.

Appendices

A Additional Figures

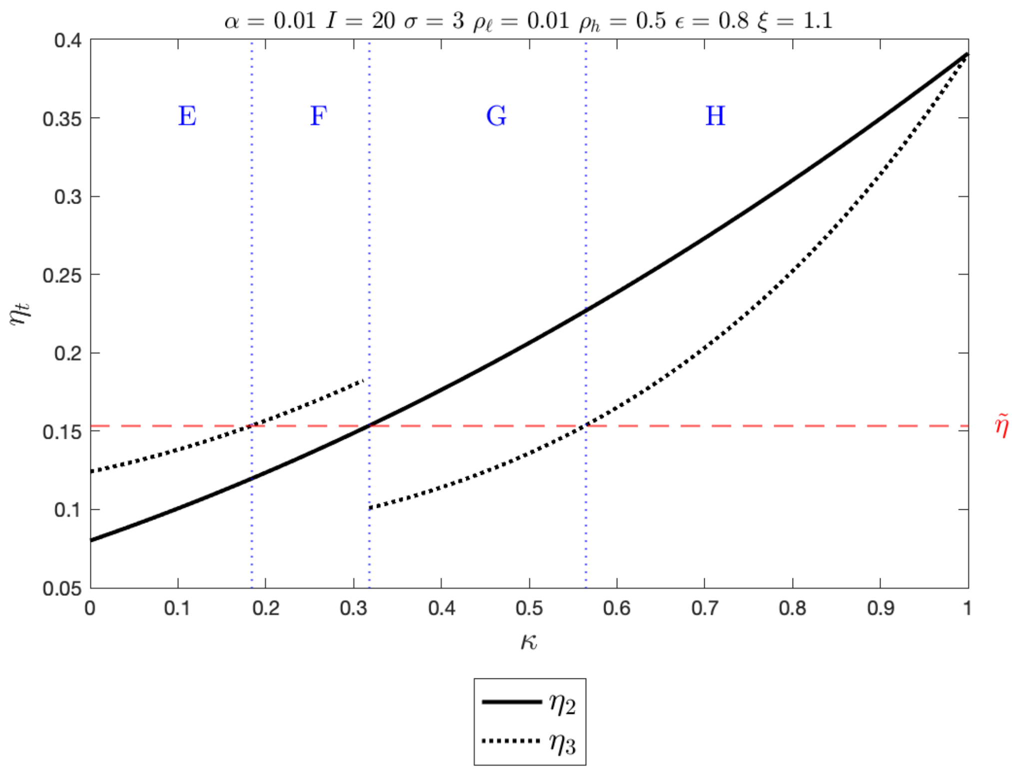
Figure 11: Price History Informativeness With Respect to Asset Sensitivity ξ in $T = 3$ Market



Note: $\eta_1 = 0$. The black solid line plots η_2, and the black dotted line plots η_3. The red dashed line is a reference line of $\tilde{\eta}$. When the black solid(dotted) line is above the reference line, then the traders choose DM in the 2nd round(3rd round), and if it is below the reference line, the traders choose CM in the 2nd round(3rd round). The jump in the difference of utility in CM v.s. DM in the third round comes from the difference in the second-round choice.

In region A, i.e. $\xi \in [0, \underline{\xi})$, traders choose DM in both 2nd and 3rd round. In region B, i.e. $\xi \in (\underline{\xi}, \bar{\xi}]$ and in the lower partition, traders choose DM in the 2nd round and CM in the 3rd round. In region C, $\xi \in (\underline{\xi}, \bar{\xi}]$ and in the higher partition, traders choose CM in the 2nd round and DM in the 3rd round. In region D, $\xi \in (\bar{\xi}, 1]$, traders choose CM in both the 2nd round and the 3rd round.

Figure 12: Price History Informativeness With Respect to Autocorrelation κ in $T = 3$ Market



Note: $\eta_1 = 0$. The black solid line plots η_2, and the black dotted line plots η_3. The red dashed line is a reference line of $\tilde{\eta}$. When the black solid(dotted) line is above the reference line, then the traders choose DM in the 2nd round(3rd round), and if it is below the reference line, the traders choose CM in the 2nd round(3rd round). The jump in the difference of utility in CM v.s. DM in the third round comes from the difference in the second-round choice.

In region E, i.e. $\kappa \in [0, \underline{\kappa}]$, traders choose CM in both 2nd and 3rd round. In region F, i.e. $\kappa \in (\underline{\kappa}, \bar{\kappa}]$ and in the lower partition, traders choose CM in the 2nd round and DM in the 3rd round. In region G, $\kappa \in (\underline{\kappa}, \bar{\kappa}]$ and in the higher partition, traders choose DM in the 2nd round and CM in the 3rd round. In region H, $\kappa \in (\bar{\kappa}, 1]$, traders choose DM in both the 2nd round and the 3rd round.

B Proofs

Proof of Theorem 1. Given the market structure $\mathcal{M}^*$, at round t, traders submit a demand schedule $q_{i,t}$ to maximize the utility

$$\max_{q_{i,t}} \mathbb{E}[\theta_{i,t} q_{i,t} - \frac{\alpha}{2}(q_{i,t})^2 - p_t q_{i,t} | \mathcal{H}_t, s_{i,t}, p_t]$$

By taking first order condition with respect to $q_{i,t}$, we can solve the trader i's demand schedule,

$$q_{i,t} = \frac{\mathbb{E}[\theta_{i,t}|\mathcal{H}_t, s_{i,t}, p_t] - p_t}{\alpha + \lambda_{i,t}}$$

where $\lambda_{i,t} \equiv \frac{dp_t}{dq_{i,t}}$ is the price impact. By symmetry, the price impacts are the same for all traders in the same round $\lambda_{i,t} = \lambda_t, \forall i \in I_{t,\mathcal{M}^*}$. We can parameterize $\mathbb{E}[\theta_{i,t}|\mathcal{H}_t, s_{i,t}, p_t] = \boldsymbol{c}_{\mathcal{H},i,t}\mathcal{H}_t + c_{s,i,t}s_{i,t} + c_{p,i,t}p_t$, where $\boldsymbol{c}_{\mathcal{H},i,t} \in \mathbb{R}^{1\times|\mathcal{H}_t|}$, $c_{s,i,t} \in \mathbb{R}$, and $c_{p,i,t} \in \mathbb{R}$. By symmetry, the inference coefficients are the same for all traders in the same round, $\boldsymbol{c}_{\mathcal{H},i,t} = \boldsymbol{c}_{\mathcal{H},t}$, $c_{s,i,t} = c_{s,t}$ and $c_{p,i,t} = c_{p,t}$.

In equilibrium, by market clearing condition, λ_t is equal to the inverse of the slope of the residual demand,

$$\lambda_t = (-\sum_{j\neq i} \frac{dq_{j,t}}{dp_t}) = \frac{\alpha}{(I_t - 1)(1 - c_{p,t}) - 1}$$

Given the parameterization, the equilibrium price is,

$$p_t = (1 - c_{p,t})^{-1}(\boldsymbol{c}_{\mathcal{H},t}\mathcal{H}_t + c_{s,t}\bar{s}_t) \tag{6}$$

where $\bar{s}_t = \frac{1}{I_t}\sum_i s_{i,t}$ is the average signal in the exchange (for DM, it's the average signal in each pair).

(Step 1: Inference Coefficients) The trader i's value $\theta_{i,t}$, the equilibrium price p_t given equation (6), the history $\mathcal{H}_t$ and the private signal $s_{i,t}$ are joint normally distributed. By projection theorem, the inference coefficients $c_{\mathcal{H},t}$, $c_{s,t}$, and $c_{p,t}$ can be determined given the joint distribution of $(\theta_{i,t}, s_{i,t}, \mathcal{H}_t, p_t)$,

$$\begin{pmatrix} \theta_{i,t} \\ s_{i,t} \\ \mathcal{H}_t \\ p_t \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} \mathbb{E}[\theta] \\ \mathbb{E}[\theta] \\ \mathbb{E}[\boldsymbol{\theta}] \\ \mathbb{E}[\theta] \end{pmatrix}, \begin{pmatrix} var(\theta_{i,t}) & cov(\theta_{i,t}, s_{i,t}) & cov(\theta_{i,t}, \mathcal{H}'_t) & cov(\theta_{i,t}, p'_t) \\ cov(s_{i,t}, \theta_{i,t}) & var(s_{i,t}) & cov(s_{i,t}, \mathcal{H}'_t) & cov(s_{i,t}, p'_t) \\ cov(\mathcal{H}_t, \theta_{i,t}) & cov(\mathcal{H}_t, s_{i,t}) & cov(\mathcal{H}_t, \mathcal{H}'_t) & cov(\mathcal{H}_t, p'_t) \\ cov(p_t, \theta_{i,t}) & cov(p_t, s_{i,t}) & cov(p_t, \mathcal{H}'_t) & cov(p_t, p'_t) \end{pmatrix}\right]$$

where

$$\begin{aligned}
cov\left(p_t,\theta_{i,t}\right) &= \left(1-c_{p,t}\right)^{-1}\left(c_{s,t}cov\left(\bar{s}_t,\theta_{i,t}\right)+\boldsymbol{c}_{\mathcal{H},t}cov(\mathcal{H}_t,\theta_{i,t})\right)\\
cov\left(p_t,s_{i,t}\right) &= \left(1-c_{p,t}\right)^{-1}\left(c_{s,t}cov\left(\bar{s}_t,s_{i,t}\right)+\boldsymbol{c}_{\mathcal{H},t}cov(\mathcal{H}_t,s_{i,t})\right)\\
cov(p_t,\mathcal{H}_t') &= \left(1-c_{p,t}\right)^{-1}\left(c_{s,t}cov\left(\bar{s}_t,\mathcal{H}_t'\right)+\boldsymbol{c}_{\mathcal{H},t}cov(\mathcal{H}_t,\mathcal{H}_t')\right)\\
cov\left(p_t,p_t'\right) &= \left(1-c_{p,t}\right)^{-1}\left(c_{s,t}var(\bar{s}_t)+c_{s,t}cov\left(\bar{s}_t,\mathcal{H}_t'\right)\boldsymbol{c}_{\mathcal{H},t}'+\boldsymbol{c}_{\mathcal{H}}cov(\mathcal{H}_t,\mathcal{H}_t')\,\boldsymbol{c}_{\mathcal{H}}'\right)
\end{aligned}$$

By projection theorem, we have

$$[c_{s,t},\boldsymbol{c}_{\mathcal{H},t},c_{p,t}]\begin{pmatrix} cov(s_{i,t}) & cov(s_{i,t},\mathcal{H}_t') & cov\left(s_{i,t},p_t\right)\\ cov\left(\mathcal{H}_t,s_{i,t}\right) & cov\left(\mathcal{H}_t,\mathcal{H}_t'\right) & cov\left(\mathcal{H}_t,p_t\right)\\ cov\left(p_t,s_{i,t}\right) & cov\left(p_t,\mathcal{H}_t'\right) & cov\left(p_t,p_t'\right)\end{pmatrix}=[cov(\theta_{i,t},s_{i,t}),cov(\theta_{i,t},\mathcal{H}_t'),cov(\theta_{i,t},p_t)] \tag{7}$$

From equation (20), we have the following equations,

$$cov(c_{s,t}s_{i,t}+\boldsymbol{c}_{\mathcal{H},t}\mathcal{H}_t+c_{p,t}p_t,s_{i,t})=cov(\theta_{i,t},s_{i,t}) \tag{8}$$

$$cov(c_{s,t}s_{i,t}+\boldsymbol{c}_{\mathcal{H}_t,t}\mathcal{H}_t+c_{p,t}p_t,\mathcal{H}_t')=cov(\theta_{i,t},\mathcal{H}_t') \tag{9}$$

$$cov(c_{s,t}s_{i,t}+\boldsymbol{c}_{\mathcal{H}_t,t}\mathcal{H}_t+c_{p,t}p_t,p_t')=cov(\theta_{i,t},p_t) \tag{10}$$

Given that $p_t=(1-c_{p,t})^{-1}(\boldsymbol{c}_{\mathcal{H},t}\mathcal{H}_t+c_{s,t}\bar{s}_t)$, subtracting $\boldsymbol{c}_{\mathcal{H}}$ times equation (22) from equation (23) gives us

$$cov(c_{s,t}s_{i,t}+\boldsymbol{c}_{\mathcal{H},t}\mathcal{H}_t+c_{p,t}p_t,\bar{s}_t)=cov(\theta_{i,t},\bar{s}_t) \tag{11}$$

Averaging equation (21) over i in the same exchange gives

$$c_{s,t}(1+\sigma^2)\sigma_\theta^2+cov(\boldsymbol{c}_{\mathcal{H},t}\mathcal{H}_t+c_{p,t}p_t,\bar{s}_t)=\sigma_\theta^2 \tag{12}$$

Comparing equation (24) and (25), we have

$$c_{s,t}=\frac{cov(\theta_{i,t},\bar{s}_t)-\sigma_\theta^2}{cov(s_{i,t},\bar{s}_t)-(1+\sigma^2)\sigma_\theta^2}=\frac{1-\rho_{t,\mathcal{M}^*}}{1-\rho_{t,\mathcal{M}^*}+\sigma^2} \tag{13}$$

where $\rho_{t,\mathcal{M}^*}$ is the correlation of traders given market structure $\mathcal{M}^*$.

Given equation (26), we can rewrite equation (22) as

$$\left(1-c_{p,t}\right)^{-1}\left(c_{s,t}cov(s_{i,t},\mathcal{H}_t')+\boldsymbol{c}_{\mathcal{H},t}cov(\mathcal{H}_t,\mathcal{H}_t')\right)=cov(\theta_{i,t},\mathcal{H}_t') \tag{14}$$

and equation (24) as

$$(1-c_{p,t})^{-1}\left(c_{s,t}var(\bar{s}_t)+\boldsymbol{c}_{\mathcal{H}}cov(\mathcal{H}_t,\bar{s}_t)\right)=cov(\theta_{i,t},\bar{s}_t) \tag{15}$$

Given that $cov(\mathcal{H},\theta_i)=cov(\mathcal{H},s_i)=cov(\mathcal{H},s_j), \forall j\neq i$, and c_s in equation (26) , we can solve the term $\boldsymbol{c}_{\mathcal{H},t}$ and $c_{p,t}$ by equation (14) and equation (15),

$$\boldsymbol{c}_{\mathcal{H},t}=\frac{(1-\rho_{t,\mathcal{M}^*})\sigma^2}{(1-\rho_{t,\mathcal{M}^*}+\sigma^2)\left(1+(I_{t,\mathcal{M}^*}-1)\rho_{t,\mathcal{M}^*}-I_{t,\mathcal{M}^*}\eta\right)}\boldsymbol{\tau}_t'\boldsymbol{\Upsilon}_t^{-1}$$

$$c_{p,t}=\frac{I_{t,\mathcal{M}^*}(\rho_{t,\mathcal{M}^*}-\eta)\sigma^2}{(1-\rho_{t,\mathcal{M}^*}+\sigma^2)\left(1+(I_{t,\mathcal{M}^*}-1)\rho_{t,\mathcal{M}^*}-I_{t,\mathcal{M}^*}\eta\right)}$$

where $\eta=\boldsymbol{\tau}_t'(\boldsymbol{\Upsilon}_t)^{-1}\boldsymbol{\tau}_t$, $\boldsymbol{\tau}_t\equiv\frac{cov(\mathcal{H}_t,\theta_{i,t})}{\sigma_\theta^2}\in\mathbb{R}^{|\mathcal{H}|}$, and $\boldsymbol{\Upsilon}_t\equiv\frac{cov(\mathcal{H}_t,\mathcal{H}_t')}{\sigma_\theta^2}\in\mathbb{R}^{|\mathcal{H}|\times|\mathcal{H}|}$.

The eqilibirum price impact is

$$\lambda_t=\frac{\alpha}{(I_{t,\mathcal{M}^*}-1)(1-c_{p,t})-1}, \quad \forall i$$

The ex-ante utility for trader i is

$$\mathbb{E}[U_{i,t}|\mathcal{H}_t]=\frac{\alpha+2\lambda_t}{2(\alpha+\lambda_t)^2}\mathbb{E}[(\mathbb{E}[\theta_{i,t}|\mathcal{H}_t,s_{i,t},p_t]-p_t)^2|\mathcal{H}_t]=\frac{\alpha+2\lambda_t}{2(\alpha+\lambda_t)^2}\frac{I_{t,\mathcal{M}^*}-1}{I_{t,\mathcal{M}^*}}\frac{(1-\rho_{t,\mathcal{M}^*})^2}{1-\rho_{t,\mathcal{M}^*}+\sigma^2}, \quad \forall i$$

■

Proof of Lemma 1. We leave out the subscripts t and $\mathcal{M}^*$ to ease the notation. Taking the derivative of welfare over ρ, we have

$$\frac{d\mathbb{E}[U_i|\mathcal{H}]}{d\rho}=-\frac{\alpha+2\lambda}{2(\alpha+\lambda)^2}\frac{I-1}{I}\frac{(1-\rho)\,(1-\rho+2\,\sigma^2)}{(1-\rho+\sigma^2)^2}-\frac{\lambda}{(\alpha+\lambda)^3}\frac{I-1}{I}\frac{(1-\rho)^2}{1-\rho+\sigma^2}\frac{d\lambda}{d\rho}<0.$$

Keep everything else constant,

$$\frac{d\lambda}{d\rho}=\lambda^2\frac{I\,\sigma^2\,(I-1)\,\left(I(\eta-\frac{(I-1)\rho+1}{I})^2+\frac{I-1}{I}(1-\rho)^2+(1-\eta)\sigma^2\right)}{\alpha\,(1-\rho+\sigma^2)^2\,(1+(I-1)\rho-I\eta)^2}>0.$$

Thus $\frac{d\mathbb{E}[U_i|\mathcal{H}]}{d\rho}<0$. The traders' welfare decreases with trader value correlation ρ. ■

Proof of Lemma 2. We leave out the subscripts t and $\mathcal{M}^*$ to ease the notation. Keep everything else constant,

$$\frac{d\lambda}{d\eta} = -\lambda^2 \frac{I\,\sigma^2\ (I-1)\ (1-\rho)}{\alpha\ (1-\rho+\sigma^2)\ (1+(I-1)\rho - I\eta)^2} < 0.$$

Therefore the price impact decreases with price history informativeness η.

The expected utility of any trader i is

$$\mathbb{E}[U_i|\mathcal{H}] = \frac{\alpha+2\lambda}{2(\alpha+\lambda)^2}\frac{I-1}{I}\frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2}.$$

Taking the derivative of welfare over η, we have

$$\frac{d\mathbb{E}[U_i|\mathcal{H}]}{d\eta} = \frac{\sigma^2}{\alpha}\frac{(1-\rho+\sigma^2)\ (1+(I-1)\rho - I\,\eta)}{(I-1)\ (1+(I-1)\rho+\sigma^2 - I\,\eta)^3} > 0.$$

Therefore the traders' welfare increases with price history informativeness η. ■

Proof of Lemma 3. We leave out the subscript t to ease the notation.

Monotonicity: The difference between trader i's utility in the CM and DM is

$$\mathbb{E}[U_i^{CM}|\mathcal{H}] - \mathbb{E}[U_i^{DM}|\mathcal{H}] = \frac{\alpha+2\lambda^{CM}}{2(\alpha+\lambda^{CM})^2}\frac{I-1}{I}\frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2} - \frac{\alpha+2\lambda^{DM}}{2(\alpha+\lambda^{DM})^2}\frac{1}{2}\frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2}.$$

Taking its derivative over the public informativeness η, we have

$$\frac{d(\mathbb{E}[U_i^{CM}|\mathcal{H}] - \mathbb{E}[U_i^{DM}|\mathcal{H}])}{d\eta} = \frac{\sigma^2}{\alpha}\left(\frac{(1-\bar{\rho}+\sigma^2)\ (1+(I-1)\bar{\rho} - I\,\eta)}{(I-1)\ (1+(I-1)\bar{\rho}+\sigma^2 - I\,\eta)^3} - \frac{(1-\rho_\ell+\sigma^2)\ (1+\rho_\ell - 2\,\eta)}{(1+\rho_\ell+\sigma^2-2\,\eta)^3}\right) < 0.$$

given that $\sigma \geq ((\frac{2(I-1)}{I})^{1/3}-1)^{-1/2}$, $\bar{\rho} > \rho_\ell$, and $\eta \leq \frac{1+(I-1)\bar{\rho}}{I} \leq \frac{1+\rho_\ell}{2}$, for the joint correlation matrix of values to be positive semidefinite.

CM v.s. DM: The lowest possible η is ρ_ℓ for equilibrium existence in the DM. $\lim_{\eta\to\rho_\ell}\lambda^{DM} = \infty$ and $\lim_{\eta\to\rho_\ell}\mathbb{E}[U_i^{DM}|\mathcal{H}] = 0$, therefore

$$\lim_{\eta\to\rho_\ell}(\mathbb{E}[U_i^{CM}|\mathcal{H}] - \mathbb{E}[U_i^{DM}|\mathcal{H}]) = \lim_{\eta\to\rho_\ell}\mathbb{E}[U_i^{CM}|\mathcal{H}] > 0. \tag{16}$$

Given $\frac{(\rho_\ell+1)}{2} - \frac{1+(I-1)\bar{\rho}}{I} \geq 0$ for the joint correlation matrix of values to be positive semidefinite, the maximum η is $\frac{1+(I-1)\bar{\rho}}{I}$,

$$\lim_{\eta\to\frac{1+(I-1)\bar{\rho}}{I}}(\mathbb{E}[U_i^{CM}|\mathcal{H}]-\mathbb{E}[U_i^{DM}|\mathcal{H}]) = \frac{1}{2\alpha}\frac{I-1}{I}\frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2}-\frac{\alpha+2\lim_{\eta\to\frac{1+(I-1)\bar{\rho}}{I}}\lambda_n}{4(\alpha+\lim_{\eta\to\frac{1+(I-1)\bar{\rho}}{I}}\lambda_n)^2}\frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2}. \tag{17}$$

where $\lim_{\eta\to\frac{1+(I-1)\bar{\rho}}{I}}\lambda_n = \frac{\alpha\left(1-\rho_\ell+\sigma^2\right)\left(\frac{1+\rho_\ell}{2}-\frac{1+(I-1)\bar{\rho}}{I}\right)}{(\rho_\ell-\frac{1+(I-1)\bar{\rho}}{I})\sigma^2}$. There exists unique $\bar{\rho}^*$ as a function of (I,ρ_ℓ,σ^2) such that $\lim_{\eta\to\frac{1+(I-1)\bar{\rho}}{I}}(\mathbb{E}[U_i^{CM}|\mathcal{H}]-\mathbb{E}[U_i^{DM}|\mathcal{H}])=0$ if $\bar{\rho}=\bar{\rho}^*(I,\rho_\ell,\sigma^2)$. If $\bar{\rho}>\bar{\rho}^*$,

$$\lim_{\eta\to\frac{1+(I-1)\bar{\rho}}{I}}(\mathbb{E}[U_i^{CM}|\mathcal{H}]-\mathbb{E}[U_i^{DM}|\mathcal{H}])<0. \tag{18}$$

Given that the difference between the ex-ante utility of the centralized market and that of the decentralized market is continuous and monotonically decreasing in η, by equations (16) and (18), if $\bar{\rho}>\bar{\rho}^*(I,\rho_\ell,\sigma^2)$, there exist $\tilde{\eta}(I,\bar{\rho},\rho_\ell,\sigma^2)$ such that the centralized market has equal welfare as the decentralized market if $\eta=\tilde{\eta}$, the centralized market has higher welfare than the decentralized market if $\eta<\tilde{\eta}$, and otherwise if $\eta\geq\tilde{\eta}$.

If $\bar{\rho}\leq\bar{\rho}^*(I,\rho_\ell,\sigma^2)$,

$$\lim_{\eta\to\frac{1+(I-1)\bar{\rho}}{I}}(\mathbb{E}[U_i^{CM}|\mathcal{H}]-\mathbb{E}[U_i^{DM}|\mathcal{H}])>0. \tag{19}$$

Given that the difference between the utility of the centralized market and that of the decentralized market is continuous and monotonically decreasing in η, by equation (19) the utility in the centralized market is always higher than the utility in the decentralized market regardless of η. ■

Proof of Proposition 1. The proof of Proposition 1 directly follows from Lemma 3, as no price history informativeness η will allow traders to choose DM. ■

Proof of Proposition 2. By Lemma 1, the expected utility $\mathbb{E}[U_i^{CM}|\mathcal{H}]$ decreases with $\bar{\rho}$, $\mathbb{E}[U_i^{DM}|\mathcal{H}]$ decreases with ρ_ℓ, if $\mathbb{E}[U_i^{CM}(\bar{\bar{\rho}})|\mathcal{H}]-\mathbb{E}[U_i^{DM}(\underline{\rho}_\ell)|\mathcal{H}]<0$, then $\mathbb{E}[U_i^{CM}(\bar{\rho})|\mathcal{H}]-\mathbb{E}[U_i^{DM}(\rho_\ell)|\mathcal{H}]$ for any $\rho_\ell<\underline{\rho}_\ell$ and $\bar{\rho}>\bar{\bar{\rho}}$.

By Lemma 4 we are subject to find $\underline{\rho}_\ell$ and $\bar{\bar{\rho}}$ that makes $\mathbb{E}[U_i^{CM}(\bar{\bar{\rho}})|\mathcal{H}]-\mathbb{E}[U_i^{DM}(\underline{\rho}_\ell)|\mathcal{H}]<0$ when $\eta=0$. It's easy to see $\underline{\rho}_\ell<0$ for DM to exist. And by Lemma 3, $\bar{\bar{\rho}}>\bar{\rho}^*$ given there exists η for traders to choose DM over CM.

When $\eta=0$, the trader's utility in the CM is

$$\mathbb{E}[U_i^{CM}|\mathcal{H}] = \frac{\alpha+2\lambda^{CM}}{2(\alpha+\lambda^{CM})^2}\frac{I-1}{I}\frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2} \quad \forall i\in I$$

where $\lambda^{CM} = \frac{\alpha}{(I-1)(1-c_p)-1}$, $c_p = \frac{I\bar{\rho}\sigma^2}{(1-\bar{\rho}+\sigma^2)(1+(I-1)\bar{\rho})}$.

The trader's utility in the DM is

$$\mathbb{E}[U_i^{DM}|\mathcal{H}] = \frac{\alpha + 2\lambda^{DM}}{4(\alpha + \lambda_1^{DM})^2} \frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2} \quad \forall i \in I$$

where $\lambda^{DM} = \frac{\alpha}{-c_p}$, $c_p = \frac{2\rho_\ell\sigma^2}{(1-\rho_\ell+\sigma^2)(1+\rho_\ell)}$.

For the correlation matrix to be well-defined (positive-semidefinite), the maximum $\bar{\rho}$ as a function of ρ_ℓ is $\frac{\frac{I(1+\rho_\ell)}{2}-1}{I-1}$.

$$\lim_{\rho_\ell \to -1} \lim_{\bar{\rho} \to \frac{\frac{I(1+\rho_\ell)}{2}-1}{I-1}} \mathbb{E}[U_i^{DM}|\mathcal{H}] - \mathbb{E}[U_i^{CM}|\mathcal{H}] = \frac{1}{\alpha(2+\sigma^2)} - \frac{1}{2\alpha}\frac{I}{(I+(I-1)\sigma^2)} > 0$$

Given $\mathbb{E}[U_i^{CM}|\mathcal{H}]$ decreases with $\bar{\rho}$, $\mathbb{E}[U_i^{DM}|\mathcal{H}]$ decreases with ρ_ℓ, and $\mathbb{E}[U_i^{DM}|\mathcal{H}] - \mathbb{E}[U_{i,1}^{CM}|\mathcal{H}]$ is continuous in $\bar{\rho}$ and ρ_ℓ, there exists $\underline{\rho}_\ell < 0$ and $\bar{\bar{\rho}} > \bar{\rho}^*$ such that for any $\rho_\ell < \underline{\rho}_\ell$ and $\bar{\rho} > \bar{\bar{\rho}}$, $\mathbb{E}[U_i^{DM}|\mathcal{H}] - \mathbb{E}[U_i^{CM}|\mathcal{H}] > 0$. ■

Proof of Proposition 3. With $\rho_\ell \geq 0$, the DM equilibrium does not exist due to extreme adverse selection. Traders will choose CM in the first round.

Step 1. Less (More) history, lower (higher) η_t: $\eta_t = \frac{var(\theta_{i,t}) - var(\theta_{i,t}|\mathcal{H}_t)}{var(\theta_{i,t})}$. To see this point, consider $\tilde{\eta}_t$ derived from $\tilde{\mathcal{H}}_t$. $\tilde{\mathcal{H}}_t$ is a strict subset of the price history $\tilde{\mathcal{H}}_t \subset \mathcal{H}_t$. $\tilde{\eta}_t = \frac{var(\theta_{i,t}) - var(\theta_{i,t}|\tilde{\mathcal{H}}_t)}{var(\theta_{i,t})}$. As $\tilde{\mathcal{H}}_t$ is a sub-sigma-algebra of $\mathcal{H}_t$, $var(\theta_{i,t}|\mathcal{H}_t) \leq var(\theta_{i,t}|\tilde{\mathcal{H}}_t)$. Thus $\tilde{\eta}_t \leq \eta_t$. This result tells us to keep everything else including the market choices in other rounds constant, if the trader chooses DM(CM) instead at round t, the informativeness in any round $\tau > t$ decreases(increases).

Step 2. Higher ξ, lower η_t: With symmetric market assumption, the price history is a linear combination of the past average signals in the CM. Let $\mathcal{H}_t = \boldsymbol{L}\bar{\boldsymbol{s}}_{\tau<t}^{CM}$, where $\bar{\boldsymbol{s}}_{\tau<t}^{CM} \in \mathbb{R}^{|\mathcal{H}_t|}$ is the vector of the average signals in past rounds where CM is the optimal market choice, and $\boldsymbol{L} \in \mathbb{R}^{|\mathcal{H}_t| \times |\mathcal{H}_t|}$ is a linear operator. We have the following equivalence:

$$\begin{aligned}
\eta_t &= \frac{cov(\theta_{i,t}, \mathcal{H}_t)cov(\mathcal{H}_t, \mathcal{H}_t')^{-1}cov(\mathcal{H}_t, \theta_{i,t})}{\sigma_\theta^2} \\
&= \frac{cov(\theta_{i,t}, \boldsymbol{L}'(\bar{\boldsymbol{s}}_{\tau<t}^{CM})')cov(\boldsymbol{L}\bar{\boldsymbol{s}}_{\tau<t}^{CM}, (\bar{\boldsymbol{s}}_{\tau<t}^{CM})'\boldsymbol{L}')^{-1}cov(\boldsymbol{L}\bar{\boldsymbol{s}}_{\tau<t}^{CM}, \theta_{i,t})}{\sigma_\theta^2} \\
&= \frac{cov(\theta_{i,t}, (\bar{\boldsymbol{s}}_{\tau<t}^{CM})')cov(\bar{\boldsymbol{s}}_{\tau<t}^{CM}, (\bar{\boldsymbol{s}}_{\tau<t}^{CM})')^{-1}cov(\bar{\boldsymbol{s}}_{\tau<t}^{CM}, \theta_{i,t})}{\sigma_\theta^2}
\end{aligned}$$

We only need to compute the joint distribution of $\{\bar{s}_\tau\}_{\tau<t}$ and $\{\theta_{i,t}\}_i$ to obtain the η_t

given the above equivalence.

Given the primitive, we have $cov(\bar{s}_\tau^{CM}, \bar{s}_\tau^{CM}) = \frac{1+(I-1)\bar{\rho}+\sigma^2}{I}\sigma_\theta^2$, $cov(\bar{s}_\tau^{CM}, \bar{s}_t^{CM}) = \frac{1+\xi^2\kappa^{t-\tau}}{(1+\xi^2)(1+\epsilon^2)}\sigma_\theta^2$, $cov(\bar{s}_\tau^{CM}, \theta_{i,t}) = \frac{1+\xi^2\kappa^{t-\tau}}{(1+\xi^2)(1+\epsilon^2)}\sigma_\theta^2$ for $\tau < t$. Fixing the past market choice, we have the following comparative static:

$$\frac{d\eta_t}{d\xi} < 0$$

which means, the price history informativeness is decreasing in asset sensitivity ξ given past market choice.

Step 3. Existence of $\underline{\xi}$: By Lemma 3, to show the existence of $\underline{\xi}$, we will need to check if there exists ξ that $\eta_t \geq \tilde{\eta}, \forall t$. By Step 1 the lowest possible η_t over t and all possible market choices is the $\underline{\eta}_T$ with price history set including only p_1^{CM}. Given $\frac{d\eta_t}{d\xi} < 0$, we are subject to check if the smallest ξ makes $\underline{\eta}_T \geq \tilde{\eta}$.

$$\lim_{\xi \to 0} \underline{\eta}_T = \frac{1}{(1+\epsilon^2)^2} \frac{I}{1+(I-1)\bar{\rho}+\sigma^2}$$

To show that $\underline{\eta}_T \geq \tilde{\eta}$, we are subject to show $\lim_{\xi\to 0} \mathbb{E}[U_i^{CM}(\underline{\eta}_T)|\mathcal{H}] - \mathbb{E}[U_i^{DM}(\underline{\eta}_T)|\mathcal{H}] < 0$.

$$\lim_{\xi\to 0} \mathbb{E}[U_i^{CM}(\underline{\eta}_T)|\mathcal{H}] - \mathbb{E}[U_i^{DM}(\underline{\eta}_T)|\mathcal{H}] = \frac{\alpha+2\lambda^{CM}}{2(\alpha+\lambda^{CM})^2}\frac{I-1}{I}\frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2} - \frac{\alpha+2\lambda^{DM}}{2(\alpha+\lambda^{DM})^2}\frac{1}{2}\frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2}.$$

$\frac{d \lim_{\xi\to 0} \mathbb{E}[U_i^{CM}(\underline{\eta}_T)|\mathcal{H}] - \mathbb{E}[U_i^{DM}(\underline{\eta}_T)|\mathcal{H}]}{d\epsilon} < 0$. There exist $\bar{\epsilon}(\sigma^2, I)$ such that for any $\epsilon < \bar{\epsilon}(\sigma^2, I)$, $\lim_{\xi\to 0} \mathbb{E}[U_i^{CM}(\underline{\eta}_T)|\mathcal{H}] - \mathbb{E}[U_i^{DM}(\underline{\eta}_T)|\mathcal{H}] < 0$. Given $\frac{d\eta_t}{d\xi} < 0$ and η_t is continuous in ξ, and $\epsilon < \bar{\epsilon}(\sigma^2, I)$, there exists $\underline{\xi}$, such that for any $\xi \in [0, \underline{\xi})$, traders will stay in the DM since the 2nd round.

Step 4. Existence of $\bar{\xi}$: By Lemma 3, to show the existence of $\bar{\xi}$, we will need to check if there exists ξ that $\eta_t \leq \tilde{\eta}, \forall t$. By Step 1 the highest possible η_t over t and all possible market choices is $\bar{\eta}_T$ when all past market choices are CMs and all past prices are available. Therefore, we are subject to check a hypothetical $\bar{\eta}_T$ that is generated with the history of all past CM prices. Given Step 2, $\frac{d\eta_t}{d\xi} < 0$ and η_t is continuous in ξ, we are subject to check if the highest ξ makes $\bar{\eta}_T \leq \tilde{\eta}$.

$$\lim_{\xi\to\infty} \bar{\eta}_T < (\frac{\kappa}{1+\epsilon^2})^2 \frac{I(T-1)}{1+(I-1)\bar{\rho}+\sigma^2}$$

There exists $\bar{\kappa}$ such that for $\kappa < \bar{\kappa}(\sigma^2, I, \epsilon)$, $3 \leq T < \bar{T}(\sigma^2, I, \epsilon)$, $\frac{d \lim_{\xi\to 0} \mathbb{E}[U_i^{CM}(\bar{\eta}_T)|\mathcal{H}] - \mathbb{E}[U_i^{DM}(\bar{\eta}_T)|\mathcal{H}]}{dI} > 0$. Given $\frac{d\eta_t}{d\xi} < 0$, η_t is continuous in ξ, and $\kappa < \bar{\kappa}(\sigma^2, I, \epsilon)$, there exist $\bar{\xi}$, for any $\xi \in [\bar{\xi}, \infty)$, traders will stay in the CM.

Step 5. Summarize: Given $3 \leq T < \bar{T}(\sigma^2, I, \epsilon)$, $\epsilon < \bar{\epsilon}(\sigma^2, I)$ and $\kappa < \bar{\kappa}(\sigma^2, I, \epsilon)$, there exists $\underline{\xi}$ and $\bar{\xi}$ such that traders will choose CM in the first round, and

1. When the asset sensitivity to shocks to fundamentals is sufficiently low $\xi \in [0, \underline{\xi})$, the traders shift to DM in the second round and stay there.

2. When the asset sensitivity to shocks to fundamentals is intermediate $\xi \in [\underline{\xi}, \bar{\xi})$, the traders will alternate between CM and DM. This is because, for $\xi \in [\underline{\xi}, \bar{\xi})$, there exists t such that $\eta_t > \tilde{\eta}$, and there also exists t such that $\eta_t < \tilde{\eta}$.

3. When the asset sensitivity to shocks to fundamentals is sufficiently high $\xi \in [\bar{\xi}, \infty)$, the traders will always stay in the CM.

■

Proof of Proposition 4. With $\rho_\ell \geq 0$, the DM equilibrium does not exist due to extreme adverse selection. Traders will choose CM in the first round.

Step 1. Less (More) history, lower (higher) η_t: See proof of Proposition 3.

Step 2. Higher ξ, lower η_t: The derivation of η as a function of the joint distribution of signals and values follows from the proof of Proposition 3. Fixing the past market choice, we have the following comparative static:

$$\frac{d\eta_t}{d\kappa} > 0$$

which means, the price history informativeness is decreasing in autocorrelation κ given past market choice.

Step 3. Existence of $\bar{\kappa}$: By Lemma 3, to show the existence of $\bar{\kappa}$, we will need to check if there exists κ that $\eta_t \geq \tilde{\eta}$, $\forall t$. By Step 1 the lowest possible η_t over t and all possible market choices is the $\underline{\eta}_T$ with price history set including only p_1^{CM}. Given $\frac{d\eta_t}{d\kappa} > 0$, we are subject to check if the highest κ makes $\underline{\eta}_T \geq \tilde{\eta}$.

$$\lim_{\kappa \to 1} \underline{\eta}_T = \frac{1}{(1+\epsilon^2)^2} \frac{I}{1 + (I-1)\bar{\rho} + \sigma^2}$$

There exist $\bar{\epsilon}(\sigma^2, I)$ such that for any $\epsilon < \bar{\epsilon}(\sigma^2, I)$, $\lim_{\kappa \to 0} \mathbb{E}[U_i^{CM}(\underline{\eta}_T)|\mathcal{H}] - \mathbb{E}[U_i^{DM}(\underline{\eta}_T)|\mathcal{H}] < 0$. Given $\frac{d\eta_t}{d\kappa} > 0$ and η_t is continuous in κ, and $\epsilon < \bar{\epsilon}(\sigma^2, I)$, there exists $\bar{\kappa}$, such that for any $\kappa \in (\bar{\kappa}, 1]$, traders will stay in the DM since the 2nd round.

Step 4. Existence of $\underline{\kappa}$: By Lemma 3, to show the existence of $\underline{\kappa}$, we will need to check if there exists κ that $\eta_t \leq \tilde{\eta}$, $\forall t$. By Step 1 the highest possible η_t over t and all possible market choices is the $\bar{\eta}_T$ when all past market choices are CMs and all past prices are available. Therefore, we are subject to check a hypothetical $\bar{\eta}_T$ that is generated with the history of all past CM prices. Given Step 2, $\frac{d\eta_t}{d\kappa} > 0$ and η_t is continuous in κ, we are subject to check if the highest κ makes $\bar{\eta}_T \leq \tilde{\eta}$.

$$\lim_{\kappa \to 0} \bar{\eta}_T < (\frac{1}{(1+\xi^2)(1+\epsilon^2)})^2 \frac{I(T-1)}{1+(I-1)\bar{\rho}+\sigma^2}$$

There exists $\underline{\xi}$ such that for $\xi > \underline{\xi}(\sigma^2, I, \epsilon)$, $3 \leq T < \bar{T}(\sigma^2, I, \epsilon)$, $\frac{d \lim_{\xi \to 0} \mathbb{E}[U_i^{CM}(\bar{\eta}_T)|\mathcal{H}] - \mathbb{E}[U_i^{DM}(\bar{\eta}_T)|\mathcal{H}]}{dI} > 0$. Given $\frac{d\eta_t}{d\kappa} > 0$, η_t is continuous in κ, and $3 \leq T < \bar{T}(\sigma^2, I, \epsilon)$, $\xi > \underline{\xi}(\sigma^2, I, \epsilon)$, there exist $\underline{\kappa}$, for any $\kappa \in [0, \underline{\kappa}]$, traders will stay in the CM.

Step 5. Summarize: Given $\epsilon < \bar{\epsilon}(\sigma^2, I)$ and $\xi > \underline{\xi}(\sigma^2, I, \epsilon)$, $3 \leq T < \bar{T}(\sigma^2, I, \epsilon)$, there exists $\underline{\kappa}$ and $\bar{\kappa}$ such that traders will choose CM in the first round, and

1. When the autocorrelation is sufficiently low $\kappa \in [0, \underline{\kappa}]$, the traders will always stay in the CM.

2. When the autocorrelation is intermediate $\kappa \in (\underline{\kappa}, \bar{\kappa}]$, the traders will alternate between CM and DM, as there exists t such that $\eta_t > \tilde{\eta}$, and there also exists t such that $\eta_t < \tilde{\eta}$.

3. When the autocorrelation is sufficiently high $\kappa \in (\bar{\kappa}, 1]$, the traders will choose DM over CM in the second round and never choose CM again.

■

Proof of Proposition 5. The proof of Proposition 5 is simple and intuitive. By the first monotonicity result in Lemma 3, if $\mathcal{M}_t^* = DM$ for η_t, and price history informativeness increases $\eta_{t+1} \geq \eta_t$, then $\mathcal{M}_{t+1}^* = DM$. We are subject to show that $\eta_{t+1} \geq \eta_t$ if traders choose DM at round t. $\eta_t = \frac{var(\theta_i) - var(\theta_i|\mathcal{H}_t)}{var(\theta_i)}$. If traders choose DM at round t, then $\mathcal{H}_{t+1} = \mathcal{H}_t \cup \{p_{n,t}\}_n$, and $\eta_{t+1} = \frac{var(\theta_i) - var(\theta_i|\mathcal{H}_{t+1})}{var(\theta_i)}$. Given that $\mathcal{H}_t \subset \mathcal{H}_{t+1}$, $var(\theta_i|\mathcal{H}_{t+1}) \leq var(\theta_i|\mathcal{H}_t)$, and therefore $\eta_{t+1} \geq \eta_t$. ■

Proof of Proposition 6. If traders always stay in CM. Post-trade transparency has no impact on welfare.

If traders have ever chosen DM, denote the round that traders first choose DM as t^*. For $t \leq t^*$, post-trade transparency has no impact on traders' utility. For $t > t^*$, denote the price history informativeness as $\eta_t^{post} = \frac{var(\theta_i) - var(\theta_i|\mathcal{H}_t^{post})}{var(\theta_i)}$. By symmetry, the price history is a linear combination of the average signal in the market and is informationally equivalent to the average signal per exchange. Thus $\eta_t^{post} = \frac{var(\theta_i)-var(\theta_i|\mathcal{S}_t^{post})}{var(\theta_i)}$, where $\mathcal{S}_t^{post} \equiv \{\bar{s}_\tau\}_{\tau<t^*}, \{\bar{s}_{n,\tau}\}_{n,t^*\le\tau<t})$. Without post-trade transparency in DM, $\eta_t = \frac{var(\theta_i)-var(\theta_i|\mathcal{S}_t)}{var(\theta_i)}$, where $\mathcal{S}_t \subset \{\bar{s}_\tau\}_{\tau<t}$. filtration generated by $\mathcal{S}_t$ is a sub σ-algebra of filtration generated by $\mathcal{S}_t^{post}$, therefore, $var(\theta_i|\mathcal{S}_t^{post}) \le var(\theta_i|\mathcal{S}_t)$, and $\eta_t^{post} \ge \eta_t$, $\forall t$.

If the traders choose DM at round t without post trade transparency, given that $\eta_t^{post} \ge \eta_t$, $\mathbb{E}[U_{i,t}^{DM,post}|\mathcal{H}_t^{post}] \ge \mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$.

If the traders choose CM at round t without post-trade transparency, $\mathbb{E}[U_{i,t}^{DM,post}] > \mathbb{E}[U_{i,t}^{CM,post}|\mathcal{H}_t^{post}] \ge \mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]$, the first inequality follows from the fact that traders prefer DM over CM at round t given proof of Proposition 5, the second equality follows from $\eta_t^{post} > \eta_t$. ∎

Proof of Theorem 4. **(Step 1: Optimization)** Let the cross pair price information be $\boldsymbol{p}_t \in \mathbb{R}^N$, whose n^{th} element is the price in pair n at round t, $p_{n,t}$. Trader $i \in I(n)$ submit demand schedule $q_{i,t}(\boldsymbol{p}_t) : \mathbb{R}^N \to \mathbb{R}$ to maximize the expected utility conditional on the history $\mathcal{H}_t$, private signal $s_{i,t}$, and

$$\max_{q_{i,t}(\boldsymbol{p}_t)} \mathbb{E}[\theta_{i,t}q_{i,t} - \frac{1}{2}\alpha q_{i,t}^2 - p_{n,t}q_t|\boldsymbol{p}_t, \mathcal{H}_t, s_{i,t}]$$

trader i 's first-order condition as

$$q^i(\boldsymbol{p}_t) = \frac{\mathbb{E}[\theta_{i,t}|\boldsymbol{p}_t, \mathcal{H}_t, s_{i,t}] - p_{n,t}}{\alpha + \lambda_{i,t}}$$

where $\lambda_{i,t}$ is the trader i's price impact within pair n. Trader i also have cross-pair price impact as traders from other pairs will change their bids when price p_n change with i's bid. Trader i's price impact over all pairs can be describe with a price impact matrix $\boldsymbol{\Lambda}_{i,t} = (\frac{d\boldsymbol{p}}{d\boldsymbol{q}_{i,t}}) \in \mathbb{R}^{N\times N}$, where the n^{th} diagonal elements is $\lambda_{i,t}$. Each trader i 's price impact matrix is equal to the transpose of the Jacobian of trader i 's inverse residual supply:

$$(\boldsymbol{\Lambda}_{i,t})' = \left(-\sum_{j\neq i}\frac{d\boldsymbol{q}_{j,t}}{d\boldsymbol{p}_t}\right)^{-1}$$

We can parameterize $\mathbb{E}[\theta_{i,t}|\boldsymbol{p}_t, \mathcal{H}_t, s_{i,t}] = \boldsymbol{c}_{\mathcal{H},i,t}\mathcal{H}_t + c_{s,i,t}s_{i,t} + \boldsymbol{c}_{p,i,t}\boldsymbol{p}_t$. $\boldsymbol{c}_{\mathcal{H},i,t} \in \mathbb{R}^{1\times|\mathcal{H}_t|}$, $c_{s,i,t} \in \mathbb{R}$, and $\boldsymbol{c}_{p,i,t} \in \mathbb{R}^{1\times N}$. Given symmetry within each pair, $\boldsymbol{c}_{\mathcal{H},i,t} = \boldsymbol{c}_{\mathcal{H},n,t}$, $c_{s,i,t} = c_{c,n,t}$, $\boldsymbol{c}_{p,i,t} = \boldsymbol{c}_{p,n,t}$ and $\lambda_{i,t} = \lambda_{n,t}$.

Given the market clearing condition, $\sum_{i\in I(n)} q_{i,t}(\boldsymbol{p}_t) = 0$, and trader symmetry within exchanges, we have the equilibrium price in all pairs in vector form,

$$\boldsymbol{p}_t = (\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} (\boldsymbol{C}_{\mathcal{H},t}\mathcal{H}_t + \boldsymbol{C}_{s,t}\bar{\boldsymbol{s}}_t),$$

where $\boldsymbol{C}_{s,t} = \text{diag}\,(c_{s,n,t})_n \in \mathbb{R}^{N\times N}, \boldsymbol{C}_{\mathcal{H},t} = (\boldsymbol{c}_{\mathcal{H},n,t})_n \in \mathbb{R}^{N\times|\mathcal{H}_t|}, \boldsymbol{C}_{p,t} = (\boldsymbol{c}_{p,n,t})_n \in \mathbb{R}^{N\times N}$. $\bar{\boldsymbol{s}}_t \in \mathbb{R}^N$ is the average signals for all pairs, where the n^{th} element is the average signal in pair n.

(Step 2: Inference Coefficients) We determine the inference coefficients as a function of the primitives (and in closed form). Random vector $(\theta_{i,t}, s_{i,t}, \mathcal{H}_t, \boldsymbol{p}_t)$ is jointly normally distributed:

$$\begin{pmatrix} \theta_{i,t} \\ s_{i,t} \\ \mathcal{H}_t \\ \boldsymbol{p}_t \end{pmatrix} \sim \mathcal{N} \left[\begin{pmatrix} \mathbb{E}[\theta] \\ \mathbb{E}[\theta] \\ \mathbb{E}[\boldsymbol{\theta}] \\ \mathbb{E}[\boldsymbol{\theta}] \end{pmatrix}, \begin{pmatrix} var(\theta_{i,t}) & cov(\theta_{i,t}, s_{i,t}) & cov(\theta_{i,t}, \mathcal{H}'_t) & cov(\theta_{i,t}, \boldsymbol{p}'_t) \\ cov(s_{i,t}, \theta_{i,t}) & var(s_{i,t}) & cov(s_{i,t}, \mathcal{H}'_t) & cov(s_{i,t}, \boldsymbol{p}'_t) \\ cov(\mathcal{H}_t, \theta_{i,t}) & cov(\mathcal{H}_t, s_{i,t}) & cov(\mathcal{H}_t, \mathcal{H}'_t) & cov(\mathcal{H}_t, \boldsymbol{p}'_t) \\ cov(\boldsymbol{p}_t, \theta_{i,t}) & cov(\boldsymbol{p}_t, s_{i,t}) & cov(\boldsymbol{p}_t, \mathcal{H}'_t) & cov(\boldsymbol{p}_t, \boldsymbol{p}'_t) \end{pmatrix} \right]$$

where

$$\begin{aligned}
&cov(\boldsymbol{p}_t, \theta_{i,t}) = (\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} (\boldsymbol{C}_{s,t} cov(\bar{\boldsymbol{s}}_t, \theta_{i,t}) + \boldsymbol{C}_{\mathcal{H},t} cov(\mathcal{H}_t, \theta_{i,t})) \\
&cov(\boldsymbol{p}_t, s_{i,t}) = (\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} (\boldsymbol{C}_{s,t} cov(\bar{\boldsymbol{s}}_t, s_{i,t}) + \boldsymbol{C}_{\mathcal{H},t} cov(\mathcal{H}_t, s_{i,t})) \\
&cov(\boldsymbol{p}_t, \mathcal{H}'_t) = (\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} (\boldsymbol{C}_{s,t} cov(\bar{\boldsymbol{s}}_t, \mathcal{H}'_t) + \boldsymbol{C}_{\mathcal{H},t} cov(\mathcal{H}_t, \mathcal{H}'_t)) \\
&cov(\boldsymbol{p}_t, \boldsymbol{p}'_t) = (\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} \big(\boldsymbol{C}_{s,t} cov(\bar{\boldsymbol{s}}_t, \bar{\boldsymbol{s}}'_t) (\boldsymbol{C}_{s,t})' + \boldsymbol{C}_{\mathcal{H},t} cov(\mathcal{H}_t, \mathcal{H}'_t)\boldsymbol{C}'_{\mathcal{H},t} + \boldsymbol{C}_{\mathcal{H},t} cov(\mathcal{H}_t, \bar{\boldsymbol{s}}'_t)\boldsymbol{C}'_{s,t} \\
&+\boldsymbol{C}_{s,t} cov(\bar{\boldsymbol{s}}_t, \mathcal{H}'_t)\boldsymbol{C}'_{\mathcal{H},t}\big) \left((\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1}\right)'
\end{aligned}$$

By projection theorem, we have

$$[c_{s,n,t}, \boldsymbol{c}_{\mathcal{H},n,t}, \boldsymbol{c}_{p,n,t}] \begin{pmatrix} var(s_{i,t}) & cov(s_{i,t}, \mathcal{H}'_t) & cov(s_{i,t}, \boldsymbol{p}'_t) \\ cov(\mathcal{H}_t, s_{i,t}) & cov(\mathcal{H}_t, \mathcal{H}'_t) & cov(\mathcal{H}_t, \boldsymbol{p}'_t) \\ cov(\boldsymbol{p}_t, s_{i,t}) & cov(\boldsymbol{p}_t, \mathcal{H}'_t) & cov(\boldsymbol{p}_t, \boldsymbol{p}'_t) \end{pmatrix} = [cov(\theta_{i,t}, s_{i,t}), cov(\theta_{i,t}, \mathcal{H}'_t), cov(\theta_{i,t}, \boldsymbol{p}'_t)] \tag{20}$$

From equation (20), we have the following equations,

$$cov(c_{s,n,t}s_{i,t} + \boldsymbol{c}_{\mathcal{H},n,t}\mathcal{H}_t + \boldsymbol{c}_{p,n,t}\boldsymbol{p}_t, s_{i,t}) = \sigma^2_\theta \tag{21}$$

$$cov(c_{s,n,t}s_{i,t} + \boldsymbol{c}_{\mathcal{H},n,t}\mathcal{H}_t + \boldsymbol{c}_{p,n,t}\boldsymbol{p}_t, \mathcal{H}_t) = cov(\theta_{i,t}, \mathcal{H}'_t) \tag{22}$$

$$cov(c_{s,n,t}s_{i,t} + \boldsymbol{c}_{\mathcal{H},n,t}\mathcal{H}_t + \boldsymbol{c}_{p,n,t}\boldsymbol{p}_t, \boldsymbol{p}'_t) = cov(\theta_{i,t}, \boldsymbol{p}'_t) \tag{23}$$

Given that $\boldsymbol{p}_t = (\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} (\boldsymbol{C}_{\mathcal{H},t}\mathcal{H} + \boldsymbol{C}_{s,t}\bar{\boldsymbol{s}}_t)$, subtracting $\boldsymbol{C}_{\mathcal{H},t}$ times equation (22) from equation (23) gives us

$$cov(c_{s,n,t}s_{i,t} + \boldsymbol{c}_{\mathcal{H},n,t}\mathcal{H}_t + \boldsymbol{c}_{p,n,t}\boldsymbol{p}_t, \bar{\boldsymbol{s}}'_t) = cov(\theta_{i,t}, \bar{\boldsymbol{s}}'_t). \tag{24}$$

Averaging equation (21) over $i \in I(n)$ gives

$$c_{s,n,t}(1+\sigma^2)\sigma_\theta^2 + cov(\boldsymbol{c}_{\mathcal{H},n,t}\mathcal{H}_t + \boldsymbol{c}_{p,n,t}\boldsymbol{p}_t, \bar{s}_n) = \sigma_\theta^2, \quad \forall n. \tag{25}$$

Comparing equation (24) and (25), we have

$$c_{s,n,t} = \frac{cov(\theta_{i,t}, \bar{s}_n) - \sigma_\theta^2}{cov(s_{i,t}, \bar{s}_n) - (1+\sigma^2)\sigma_\theta^2} = \frac{1-\rho_{n,t}}{1-\rho_{n,t}+\sigma^2}. \tag{26}$$

where $\rho_{n,t}$ is the correlation for traders in pair n.

Given $\boldsymbol{C}_{s,t} = \mathrm{diag}(c_{s,n,t})$ solved in equation (26), we can rewrite equation (22) in matrix form,

$$(\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} (\boldsymbol{C}_{s,t}\boldsymbol{1}\boldsymbol{\tau}'_t + \boldsymbol{C}_{\mathcal{H},t}\boldsymbol{\Upsilon}_t) = \boldsymbol{1}\boldsymbol{\tau}'_t. \tag{27}$$

and equation (24) as

$$(\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} (\boldsymbol{C}_{s,t}cov(\bar{\boldsymbol{s}}_t, \bar{\boldsymbol{s}}'_t) + \boldsymbol{C}_{\mathcal{H},t}\boldsymbol{\tau}_t\boldsymbol{1}') = cov(\bar{\boldsymbol{\theta}}_t, \bar{\boldsymbol{s}}'_t) \tag{28}$$

where $\boldsymbol{\tau}_t = cov(\mathcal{H}_t, \theta_{i,t})$, $\boldsymbol{\Upsilon}_t = cov(\mathcal{H}_t, \mathcal{H}'_t)$.

We can solve the term $\boldsymbol{C}_{\mathcal{H},t}$ and $\boldsymbol{C}_{p,t}$ by the above two equations,

$$\boldsymbol{C}_{p,t} = \boldsymbol{Id} - \boldsymbol{C}_{s,t} - \boldsymbol{C}_{s,t}\,\mathrm{diag}\left(\tfrac{\sigma^2}{I_n}\right)_n (\bar{\mathcal{C}} - \boldsymbol{1}\boldsymbol{1}'\eta_t)^{-1} = \mathrm{diag}\left(\tfrac{\sigma^2}{1-\rho_{n,t}+\sigma^2}\right)_n \left(\boldsymbol{Id} - \mathrm{diag}\left(\tfrac{1-\rho_{n,t}}{2}\right)(\bar{\mathcal{C}} - \boldsymbol{1}\boldsymbol{1}'\eta_t)^{-1}\right)$$

$$\boldsymbol{C}_{\mathcal{H},t} = (\boldsymbol{Id} - \boldsymbol{C}_{p,t} - \boldsymbol{C}_{s,t})\boldsymbol{1}\boldsymbol{\tau}'_t\boldsymbol{\Upsilon}_t^{-1} = \mathrm{diag}\left(\frac{(1-\rho_{n,t})\sigma^2}{2(1-\rho_{n,t}+\sigma^2)}\right)_n (\bar{\mathcal{C}} - \boldsymbol{1}\boldsymbol{1}'\eta_t)^{-1}\boldsymbol{1}\boldsymbol{\tau}'_t\boldsymbol{\Upsilon}_t^{-1}$$

$\eta_t = \frac{\boldsymbol{\tau}'_t\boldsymbol{\Upsilon}_t^{-1}\boldsymbol{\tau}_t}{\sigma_\theta^2}$ is price history informativeness. $\bar{\mathcal{C}} = \frac{cov(\bar{\boldsymbol{\theta}}_t, \bar{\boldsymbol{\theta}}'_t)}{\sigma_\theta^2} \in \mathbb{R}^{N\times N}$ is the correlation of pairwise average values across all pairs, where $\bar{\boldsymbol{\theta}}_t \in \mathbb{R}^N$ is the vector of average value per trading pair where the n^{th} value is $\bar{\theta}_{n,t} = \sum_{i\in I(n)} \theta_{i,t}$.

(Step 3: Price impacts) In equilibrium, each trader i 's price impact is equal to the transpose of the Jacobian of trader i 's inverse residual supply:

$$(\boldsymbol{\Lambda}_{i,t})' = \left(-\sum_{j\neq i}\frac{d\boldsymbol{q}_{j,t}}{d\boldsymbol{p}_t}\right)^{-1} = (\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1}\,\mathrm{diag}\left(\frac{\alpha + \lambda_{n,t}}{2 - \boldsymbol{1}_{i\in I(n)}}\right)_n.$$

From the last equation, we can solve for the within-exchange price impact for all $i \in I(n)$,

$$\lambda_{n,t} = \left(\left(((\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1})_{nn} \right)^{-1} - 1 \right)^{-1} \alpha.$$

where $(A)_{nn}$ is an operator that gives the n^{th} diagonal element of matrix A. Denote the matrix of within-exchange price impacts by $\hat{\boldsymbol{\Lambda}}_t \equiv \operatorname{diag}(\lambda_{n,t})_n$. In equilibrium,

$$\hat{\boldsymbol{\Lambda}}_t = \left(\left(\left[(\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} \right]_{nn} \right)^{-1} - \boldsymbol{Id} \right)^{-1} \alpha,$$

where $[A]_{nn}$ is an operator that gives the diagonal elements of matrix A while setting all off-diagonal elements to zero.

In this paper, we focus on nonnegative price impacts such that the residual supply curve is downward-sloping, i.e., $\lambda_n \geq 0$, for all n. This is satisfied under the following condition:

$$((\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1})_{nn} \leq 1$$

$\left((\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1}\right)_{nn} = (1 + \frac{\sigma^2}{1-\rho_{n,t}}) \left(1 - \frac{\sigma^2}{2}(\frac{1+\rho_{n,t}+\sigma^2}{2} - \eta_t - A_t)^{-1}\right)$, where $A_t = (\frac{cov(\bar{\theta}_{n,t}, \bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2} - \boldsymbol{1}'\eta_t)(\frac{cov(\bar{\boldsymbol{\theta}}_{-n,t}, \bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2} - \boldsymbol{1}\boldsymbol{1}'\eta_t)^{-1}(\frac{cov(\bar{\boldsymbol{\theta}}_{-n,t}, \bar{\theta}_{n,t})}{\sigma_\theta^2} - \boldsymbol{1}\eta_t)$. Therefore, the following condition are needed for equilibrium existence,

$$\eta_t + A_t \geq \rho_{n,t} \quad \forall n$$

The second-order condition for the trader i 's optimization problem is, $\lambda_n \geq -\frac{1}{2}\alpha$, and is trivially satisfied with nonnegative price impacts.

(Step 4: Utility) Given the inference coefficients and price impacts solved in previous section, the expected utility conditional on price history is

$$\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t] = \frac{\alpha + 2\lambda_t}{2(\alpha + \lambda_t)^2}\mathbb{E}[(\mathbb{E}[\theta_{i,t}|\mathcal{H}_t, s_{i,t}, \boldsymbol{p}_t] - p_{t,n})^2|\mathcal{H}_t] = \frac{\alpha + 2\lambda_{n,t}}{2(\alpha + \lambda_{n,t})^2}\frac{1}{2}\frac{(1-\rho_{n,t})^2}{1 - \rho_{n,t} + \sigma^2}$$

■

Proof of Lemma 5. Taking derivative of $U_{i,t}^{DM}$ with respect to $\rho_{n,t}$, we have

$$\frac{d\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]}{\rho_{n,t}} = -\underbrace{\frac{\alpha + 2\lambda_{n,t}}{4(\alpha + \lambda_{n,t})^2}\frac{(1-\rho_{n,t})(1-\rho_{n,t}+2\sigma^2)}{(1-\rho_{n,t}+\sigma^2)^2}}_{>0} - \underbrace{\frac{\lambda_{n,t}}{2(\alpha+\lambda_{n,t})^3}\frac{(1-\rho_{n,t})^2}{1-\rho_{n,t}+\sigma^2}}_{>0}\frac{d\lambda_{n,t}}{d\rho_{n,t}}$$

.

The derivate of price impact to correlation $\frac{d\lambda_{n,t}}{d\rho_{n,t}}$ is

$$\begin{aligned}\frac{d\lambda_{n,t}}{d\rho_{n,t}} &= \frac{\lambda_{n,t}^2}{\alpha}\left(\left((\boldsymbol{Id}-\boldsymbol{C}_{p,t})^{-1}\right)_{nn}\right)^{-2}\frac{d\left((\boldsymbol{Id}-\boldsymbol{C}_{p,t})^{-1}\right)_{nn}}{d\rho_{n,t}} \\ &= \frac{\lambda_{n,t}^2}{\alpha\left(\left((\boldsymbol{Id}-\boldsymbol{C}_{p,t})^{-1}\right)_{nn}\right)^2}\left(\frac{\sigma^2\left((\boldsymbol{Id}-\boldsymbol{C}_{p,t})^{-1}\right)_{nn}}{(1-\rho_{n,t})(1-\rho_{n,t}+\sigma^2)}+(1+\frac{\sigma^2}{1-\rho_{n,t}})\frac{\sigma^2}{4}(\frac{1+\rho_{n,t}+\sigma^2}{2}-\eta_t-A_t)^{-2}\right) \\ &> 0\end{aligned}$$

given that $\left((\boldsymbol{Id}-\boldsymbol{C}_{p,t})^{-1}\right)_{nn} = (1+\frac{\sigma^2}{1-\rho_{n,t}})\left(1-\frac{\sigma^2}{2}(\frac{1+\rho_{n,t}+\sigma^2}{2}-\eta_t-A_t)^{-1}\right)$, where $A_t = (\frac{cov(\bar{\theta}_{n,t},\bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2}-\boldsymbol{1}'\eta)(\frac{cov(\bar{\boldsymbol{\theta}}_{-n,t},\bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2}-\boldsymbol{1}\boldsymbol{1}'\eta)^{-1}(\frac{cov(\bar{\boldsymbol{\theta}}_{-n,t},\bar{\theta}_{n,t})}{\sigma_\theta^2}-\boldsymbol{1}\eta)$, and $\bar{\boldsymbol{\theta}}_{-n,t}\in\mathbb{R}^{N-1}$ is the vector of average values in pairs $m\neq n$. The last inequalify follows from the fact that $\frac{1+\rho_{n,t}}{2}-\eta_t-A_t>0$ give positive-semidefinite joint correlation matrix of $\bar{\theta}_{n,t}$, $\bar{\boldsymbol{\theta}}_{-n,t}$ and history $\mathcal{H}_t$.

Therefore, $\frac{d\mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]}{\rho_{n,t}}<0$. Expected utility in DM is decreasing in $\rho_{n,t}$. ■

Proof of Lemma 6. To show that given $\mathcal{H}_t$ (and therefore given η_t) the utility for any trader i weakly increases, we are subject to show that the expected utiltity $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]\geq\mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$. Comparing $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]$ in Theorem 4 and $\mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$ in Theorem 1, we find if and only if $\lambda_{n,t}^{DM,pre}\leq\lambda_{n,t}^{DM}$, then $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]\geq\mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$.

$\lambda_{n,t}^{DM,pre}\leq\lambda_{n,t}^{DM}$ if and only if

$$\left((\boldsymbol{Id}-\boldsymbol{C}_{p,t})^{-1}\right)_{nn}\leq\frac{1}{1-c_p^{DM}} \tag{29}$$

Following proof of Lemma 5, $\left((\boldsymbol{Id}-\boldsymbol{C}_{p,t})^{-1}\right)_{nn}=(1+\frac{\sigma^2}{1-\rho_{n,t}})\left(1-\frac{\sigma^2}{2}(\frac{1+\rho_{n,t}+\sigma^2}{2}-\eta_t-A_t)^{-1}\right)\leq(1+\frac{\sigma^2}{1-\rho_{n,t}})\frac{\frac{1+\rho_{n,t}}{2}-\eta_t}{\frac{1+\rho_{n,t}+\sigma^2}{2}-\eta_t}$ as $A_t=(\frac{cov(\bar{\theta}_{n,t},\bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2}-\boldsymbol{1}'\eta_t)(\frac{cov(\bar{\boldsymbol{\theta}}_{-n,t},\bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2}-\boldsymbol{1}\boldsymbol{1}'\eta_t)^{-1}(\frac{cov(\bar{\boldsymbol{\theta}}_{-n,t},\bar{\theta}_{n,t})}{\sigma_\theta^2}-\boldsymbol{1}\eta_t)\geq 0$ given it has a quardratic form and $\frac{cov(\bar{\boldsymbol{\theta}}'_{-n,t},\bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2}-\boldsymbol{1}\boldsymbol{1}'\eta_t=\frac{cov(\bar{\boldsymbol{\theta}}_{-n,t},\bar{\boldsymbol{\theta}}'_{-n,t}|\mathcal{H}_t)}{\sigma_\theta^2}$ is positive semidefinite. By Theorem 1, $\frac{1}{1-c_p^{DM}}=(1+\frac{\sigma^2}{1-\rho_{n,t}})\frac{\frac{1+\rho_{n,t}}{2}-\eta_t}{\frac{1+\rho_{n,t}+\sigma^2}{2}-\eta_t}$. Therefore, equation 29 holds, $\lambda_{n,t}^{DM,pre}\leq\lambda_{n,t}^{DM}$ and $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]\geq\mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$. ■

Proof of Proposition 7. **First Time Choosing DM:** Let the threshold to choose DM without pre-trade transparency by $\tilde{\eta}$ (see Lemma 3), and the threshold to choose DM with pre-trade transparency by $\tilde{\eta}^{pre}$. Suppose $\bar{\rho}>\bar{\rho}^*$, for any $\mathcal{H}_t$ generating $\eta_t\geq\tilde{\eta}$, given results of Lemma 6, $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]\geq\mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]\geq\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]$. This implies (i) the threshold to choose DM without pre-trade transparency is at least as low as $\tilde{\eta}$, $\tilde{\eta}^{pre}\leq\tilde{\eta}$; (ii) and the first round that traders choose DM with pre-trade transparency is no later than without pre-trade transparency, i.e. if $t_1^{DM} \equiv \min_t\{\mathcal{M}_t^* = DM\}$, then $t_1^{DM,pre} = \min_t\{\mathcal{M}_t^{*,pre} = DM\} \leq t_1^{DM}$.

First Time Stay in DM: If traders choose DM with pre-trade transparency in the same round as with opaque DM, i.e., $t_1^{DM} = t_1^{DM,pre}$, then the length of stay in DM when traders first choose DM is (weakly) longer with pre-trade transparency. This is because, given that they enter the DM at the same round, the evolution of η_t is the same before they firstly exit the DM after the first time they choose DM. And $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t] \geq \mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$ implies, the first time traders exit DM with pre-trade transparency is no earlier than the first time when they exit the opaque DM. Thus, the length of stay in DM when traders first choose DM is (weakly) longer with pre-trade transparency.

We are not sure about the following rounds of choosing DM, as the evolution of η_t will not be the same with and without pre-trade transparency, except for the $\tilde{\eta} = 0$ special case. If $\tilde{\eta} = 0$ then $\tilde{\eta}^{pre} = 0$, trader will always choose DM.

■

Lemma 7. *If A and $A + B$ are invertible, and B has rank 1, then let $g = \operatorname{trace}(BA^{-1})$. Then $g \neq -1$ and*

$$(A+B)^{-1} = A^{-1} - \frac{1}{1+g} A^{-1} B A^{-1}.$$

Proof of Proposition 8. **Constant CM regardless of pre-trade transparency:** It is trivial that when traders choose CM for all rounds with or without pre-trade transparency, then pre-trade transparency should not have any impact on the welfare. Given that $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t] > \mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$ for any η, choosing CM constantly implies $\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t] - \mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t] \geq 0$ for any η_t. We know that if $\bar{\rho} = \rho_h = \rho_\ell$,

$$\begin{aligned}
\left((\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1}\right)_{nn} &= (1 + \frac{\sigma^2}{1-\rho_\ell}) \left(1 - \frac{\sigma^2}{2}(\frac{1+\rho_\ell+\sigma^2}{2} - \eta_t - A_t)^{-1}\right) \\
&> \frac{1}{(I-1)(1-c_{p,t}^{CM})} = \frac{1}{I-1}(1+\frac{\sigma^2}{1-\rho_\ell})(\frac{1+(I-1)\rho_\ell}{I} - \eta_t)(\frac{1+(I-1)\rho_\ell+\sigma^2}{I} - \eta_t)^{-1}
\end{aligned}$$

where $A_t = (\frac{cov(\bar{\theta}_{n,t}, \bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2} - \mathbf{1}'\eta_t)(\frac{cov(\bar{\boldsymbol{\theta}}'_{-n,t}, \bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2} - \mathbf{11}'\eta_t)^{-1}(\frac{cov(\bar{\boldsymbol{\theta}}_{-n,t}, \bar{\theta}_{n,t})}{\sigma_\theta^2} - \mathbf{1}\eta_t) = (\rho_h - \eta_t)\frac{g}{1+g}$, $g = N\frac{\rho_h - \eta_t}{\frac{1+\rho_\ell}{2} - \rho_h}$ by Lemma 7, and $\bar{\boldsymbol{\theta}}_{-n,t} \in \mathbb{R}^{N-1}$ is the vector of average values in pairs $m \neq n$. Therefore, $\lambda_{n,t}^{CM} < \lambda_{n,t}^{DM,pre}$ and

$$\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t] = \frac{\alpha + 2\lambda_{n,t}^{CM}}{2(\alpha+\lambda_{n,t}^{CM})^2}\frac{I-1}{I}\frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2} > \mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t] = \frac{\alpha+2\lambda_{n,t}^{DM,pre}}{4(\alpha+\lambda_{n,t}^{DM,pre})^2}\frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2}$$

Given that $\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]$ is decreasing in $\bar{\rho}$, there exists $\bar{\rho}^{*,pre}(\rho_\ell, I, \sigma^2)$ such that if $\bar{\rho} <$

$\bar{\rho}^{*,pre}(\rho_\ell, I, \sigma^2)$, $\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t] - \mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t] \geq 0$ for any η. So if $\bar{\rho} < \bar{\rho}^{*,pre}(\rho_\ell, I, \sigma^2)$, traders always choose CM with and without pre-trade transparency.

Transparency changes market choice: Without pre-trade transparency, when traders' value correlations are neither sufficiently heterogeneous nor sufficiently homogenous, alternating market choice can be optimal according to Proposition 3 and Proposition 4. With pre-trade transparency, as the utility in DM is higher given the same parameters, traders are more likely to choose DM (weakly) earlier (see Proposition 7). And this can potentially decrease the price history informativeness and the welfare. We can find a non-trivial set of parameters such that the pre-trade transparency can decrease welfare. A most intuitive case is a set of paramters such that (i) traders always choose CM or alternate between CM and DM with $\eta_t > 0$ for $t > 1$ without pre-trade transparency; (ii) traders always choose DM with pre-trade transparency, resulting $\eta_t = 0$ for all t; (iii) the total welfare over all rounds is higher without pre-trade transparancy. We provide proof of existence of such parameters below.

To satisfy condition (i), the traders' expected utility in CM is higher than the expected utility in opaque DM when $\eta_t = 0$, i.e. $\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]|_{\eta_t=0} \geq \mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]|_{\eta_t=0}$.

To satisfy condition (ii), we require the traders in DM with pre-trade transparency to have higher utility than in CM given $\eta_t = 0$. When $\eta_t = 0$, the expected utility in CM is

$$\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]|_{\eta_t=0} = \frac{\alpha + 2\lambda^{CM}}{2(\alpha + \lambda^{CM})^2} \frac{I-1}{I} \frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2} \quad \forall i \in I$$

where $\lambda^{CM} = \frac{\alpha}{(I-1)(1-c_p)-1}$, $c_p = \frac{I\bar{\rho}\sigma^2}{(1-\bar{\rho}+\sigma^2)(1+(I-1)\bar{\rho})}$. And when $\eta_t = 0$, the expected utility in DM with pre-trade transparency is

$$\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0} = \frac{\alpha + 2\lambda^{DM,pre}}{2(\alpha + \lambda^{DM,pre})^2} \frac{1}{2} \frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2}$$

where $\lambda^{DM,pre} = (1 + \frac{\sigma^2}{1-\rho_\ell})(\frac{1+\rho_\ell}{2} - A_0)\left(\frac{\sigma^2}{2} - (\frac{\sigma^2}{1-\rho_\ell})(\frac{1+\rho_\ell}{2} - A_0)\right)^{-1}\alpha$,
$A_0 = \frac{cov(\bar{\theta}_{n,t}, \bar{\boldsymbol{\theta}}'_{-n,t}) cov(\bar{\boldsymbol{\theta}}_{-n,t}, \bar{\boldsymbol{\theta}}'_{-n,t})^{-1} cov(\bar{\boldsymbol{\theta}}_{-n,t}, \bar{\theta}_{n,t})}{\sigma_\theta^2} \geq 0$.

Given that $\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]|_{\eta_t=0}$ is decreasing in $\bar{\rho}$ and $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0}$ is decreasing in ρ_ℓ given Lemma 1, and $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0} > \mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]|_{\eta_t=0}$ when $\rho_\ell = 0$ and $\bar{\rho} = \frac{\frac{I}{2}-1}{I-1}$, there exists $0 \leq \rho_\ell < \bar{\rho}_\ell$ and $\bar{\rho} > \bar{\bar{\rho}}^{pre}(I, \rho_\ell, \sigma^2)$ such that $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0} \geq \mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]|_{\eta_t=0}$.

To satisfy condition (iii), we require the total welfare over all rounds is higher without pre-trade transparency than with pre-trade transparency, i.e. $W^{pre} \equiv T\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0} \leq W \equiv \sum_{t=1}^{T} \mathbb{E}[U_{i,t}^{\mathcal{M}^*}|\mathcal{H}_t]$. Given the same market choice, the expected utility weakly increases with the length of price history, i.e. $\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_\tau] \geq \mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]$ for $\tau > t$. For sufficiently small ξ and sufficiently large κ, there exists $\underline{t}$, such that $\eta_{\underline{t}}$ is sufficiently large, $\mathbb{E}[U_{i,\underline{t}}^{CM}|\mathcal{H}_{\underline{t}}] > \mathbb{E}[U_{i,\underline{t}}^{DM,pre}|\mathcal{H}_{\underline{t}}]|_{\eta_t=0}$. Therefore, we can rewrite the difference between welfares without and with pre-trade transparency as

$$
\begin{aligned}
W - W^{pre} &= \sum_{t=1}^{T} \mathbb{E}[U_{i,t}^{\mathcal{M}^*}|\mathcal{H}_t] - T\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0} \\
&\geq (T-\underline{t}) \underbrace{\left(\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t] - \mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0}\right)}_{>0} + \underbrace{\sum_{t=1}^{\underline{t}} \mathbb{E}[U_{i,t}^{\mathcal{M}^*}|\mathcal{H}_t] - \underline{t}\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0}}_{>-\underline{t}\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0}}
\end{aligned}
$$

Easy to see $W - W^{pre}$ is increasing in T. Given that the second part is bounded below, $\lim_{T\to\infty} W - W^{pre} > 0$. This implies we can find sufficiently small ξ and sufficiently large κ and sufficiently large T, such that condition (iii) is satisfied. ■