Corporate Finance Through Loyalty Programs*

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Abstract

Loyalty programs (LPs) are widely prevalent and typically analyzed in economic research for their role in boosting income. This paper uncovers a novel role of LPs as financing instruments. The rewards issued to and redeemed by consumers cause shifts in firms' present and future cash flows, effectively creating a form of borrowing from consumers. We document three stylized facts about LPs in the airline and hotel industries: 1) LPs serve as significant financing sources, with co-branded credit card programs contributing a large portion; 2) rewards are issued through broad consumption but are redeemed predominantly for consumption related to the issuing firm; 3) LPs generate countercyclical cash flows. We then build a dynamic model of LPs as financing instruments. The model features convenient rewards, which consumers can freely redeem. As a result, the funds raised through LPs emerge endogenously in equilibrium as a result of the interplay between reward issuance and redemption. The model suggests that 1) firms supplying high-value, low-frequency services can leverage LPs more effectively for financing; 2) firms should aim to decouple reward issuance from their business; 3) the cyclical nature of LPs bolsters the financial resilience of procyclical firms.

Keywords: loyalty programs, corporate finance, rewards, cobranded credit cards, financial resilience

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1 Introduction

Loyalty programs (LPs) are widely prevalent. According to 2017 COLLOQUY Loyalty Census, the total number of loyalty program memberships in the U.S. reached 3.8 billion. The distribution of these memberships spans various industries with 42% in retail, 29% in travel and hospitality, and 17% in financial services. Certain sectors have even embraced loyalty programs as a standard business practice. Indeed, almost all major airlines and hotel chains, along with 55 of the top 100 U.S. retailers, offer some form of loyalty programs (Sun and Zhang, 2019). A common and crucial feature of LPs is their provision of rewards to consumers. These rewards, usually presented as points, complimentary nights, or discounts on subsequent purchases, represent promises for future services. They are generally issued to consumers based on their purchases with the brand and its partners.

Economists have offered several theories to rationalize the emergence of LPs. The most straightforward explanation posits that LPs may provide psychological benefits or engender price misperceptions. They might also enable price discrimination based on quantity or purchasing history. In a market with multiple competing brands, LPs can heighten consumers' switching costs, thereby weakening competition. More recent research suggests that LPs are utilized to capitalize on agency conflicts between employers and employees, or for customer tracking and database marketing. Essentially, all these theories revolve around the central theme of boosting income.

In this paper, we intend to uncover a novel role of LPs as financing instruments. The idea is simple. When a firm issues rewards to consumers, it usually receives advance payments from consumers or some other benefits. In either scenario, the issuance of rewards boosts the firm's present cash flows. These rewards are recorded as the firm's liabilities, as it has yet to provide the promised services. Later, consumers will redeem these rewards for the firm's services at discounted prices or free of charge, reducing the firm's future cash flows. These shifts in the firm's present and future cash flows constitute de factor borrowings from consumers.

Although LPs have the potential to be used for financing in theory, do they truly serve as substantial funding sources for business in the real world? If so, how do they function in practice and what are their effects on firms? To address these questions, we document three stylized facts about LPs in the airline and hotel industries. We focus primarily on the two industries because their LPs are central to business operations and are among the most influential across all industries. Moreover, during the COVID-19 pandemic, major U.S. airlines leveraged their LPs as collateral, providing valuable observations into the inner workings of LPs.

First, LPs serve as significant financing sources, with cobranded credit card (CCC) programs contributing a large portion. Before the Covid-19 pandemic, the total LP liabilities of the two industries reached 40 billion dollars, accounting for 30% of the total debt. Whether intentionally or not, LPs have become de facto significant financing sources. A noteworthy finding is that airlines and hotels issue a large fraction of rewards through their CCC programs, exceeding even their own business. In order to encourage consumers to sign up and use CCCs, bank partners purchase rewards from airlines and hotels and offer them to consumers. For instance, in 2019, approximately 70% of the cash flows from Delta's LP, SkyMiles, were derived from its bank partner, American Express. More broadly, we find a significantly positive relationship between a firm's LP liabilities and the size of its CCC program within the two industries.

Second, rewards are issued through broad consumption, but are redeemed predominantly for consumption related to the issuing firm. American households extensively utilize the CCCs issued by airlines for a broad spectrum of consumption, much of which bears no direct relation to the issuing firm. Moreover, airlines deliberately enable consumers to earn miles on purchases with their partners, including those unrelated to their own business. This issuance practice is at odds with classical theories on LPs, which suggest that rewards should be issued based on a consumer's history of purchases related to the issuing firm. When it comes to the redemption of these rewards, the majority are used for air travel, thereby ensuring that the rewards are ultimately directed back towards the issuing firm.

Third, LPs generate countercyclical cash flows. For procyclical firms, reward redemption is closely tied to their business, thereby exhibiting a procyclical pattern. However, reward issuance is linked to broad consumption and thus less procyclical compared with redemption. This pattern was prominently evident during the Covid-19 pandemic. As a result, LP liabilities grew rapidly during that period. This countercyclical feature of LPs naturally enables procyclical firms to borrow more from consumers in bad times, helping them alleviate liquidity shortage and avoid costly bankruptcy.

Based on these stylized facts, we build a model of LPs as financing instruments. The model is an infinite-horizon continuous-time game with consumers, a single firm, and banks. All consumers are identical, with an exogenous demand for a service that only the firm provides. This setting naturally eliminates classical motives for LPs like price discrimination and competition. The firm has the option to finance its operations through debt or an LP. By adopting a LP, the firm essentially issues rewards to consumers and receives some form of advance payments, which are referred to as LP liabilities. For consumers, the rewards are convenient as there are no restrictions on their redemption—a consumer can purchase a service anytime he wishes, using any combination of cash and rewards he can afford. Due to the convenience of rewards, consumers discount their value at a lower interest rate than they would for an investment like lending. Consequently, as a substitute for debt financing, LP financing reduces the firm's interest costs. However, rewards are not perfectly convenient as cash. They are less liquid because they can only be redeemed for the service. Hence, we assume that consumers weakly prefer to hold cash rather than to hold rewards.

On the other hand, getting involved in a LP has associated costs for consumers. Enrolling in the LP and monitoring his status in the program requires time and attention from the consumer, which we refer to as the participation cost. Redeeming rewards is also more complicated than using cash, incurring a cost each time a consumer redeems. To compensate consumers for these costs, the firm must offer rewards at a discount, which drives the financing cost of LPs.

We consider two types of LPs that are the most prominent in practice: consumption-based LPs (CLPs) and purchase-based LPs (PLPs). Adopting a CLP , the firm cooperates with a partner bank in issuing a CCC. Since banks act competitively, it is essentially the firm that issues rewards to consumers for their spending through the CCC and receives the fees that the partner bank charges consumers. Adopting a PLP, the firm issues rewards to consumers based on their purchases directly with the firm. LPs are static in the sense that once a LP is adopted, its design does not change over time. For the baseline setup, the game is stationary, and we focus on the equilibrium path in the steady state, in which each consumer behaves optimally and the aggregate behavior of all consumers remains constant.

For various reasons, a consumer may not redeem rewards immediately after earning them, leading to a substantial accumulation of rewards. From the firm's perspective, the positive reward balance essentially constitutes a form of borrowing from consumers. Notably, the reward balance is not directly determined by the firm but rather emerges endogenously in equilibrium as a result of the interplay between reward issuance and redemption. Due to the convenience of rewards, consumers control the redemption process. Specifically, in equilibrium, when a consumer needs a service, he will purchase it using rewards alone if his reward balance is sufficient; otherwise, he will use cash alone. Despite having the ability to do so, a consumer never opts for partial redemption for a service, as this strategy minimizes redemption costs. Additionally, a consumer does not desire to stockpile rewards beyond what are sufficient for a service, as cash is generally preferred over rewards.

The first key result of our theory concerns how the way the firm issues rewards affects the funds it raises endogenously. Since a consumer never redeems rewards before he has accumulated sufficient ones for a service, a consumer's average reward balance must be at least half the value of a service. This mechanism applies to both types of LPs. However, a consumer may also overaccumulate—hold rewards more than sufficient for a service. Under a CLP, overaccumulation arises due to the asynchronous nature of reward issuance and redemption: a consumer consistently earns rewards through the CCC, even during periods when he does not need a service. However, such asynchrony is absent under a PLP, as both issuance and redemption are linked to the purchases of services and mutually exclusive. Instead, overaccumulation arises due to the lumpiness of reward earning. We analytically show that the asynchrony under a CLP is more powerful in generating overaccumulation than the lumpiness under a PLP. We also numerically analyze the case where both types are adopted and find that the reward balance increases as the CLP gains more importance relative to the PLP. A broader implication of the comparison between CLPs and PLPs is that for a firm to leverage LPs more effectively for financing, it should issue rewards in ways that are loosely related or even unrelated to the firm's business. This implication is consistent with the second stylized fact that rewards are also issued through consumption unrelated to firms' business.

The second key result of our theory is that firms supplying high-value, low-frequency services can leverage LPs more effectively for financing. This finding rests on three observations. First, the reward balance resulting from consumers' unwillingness to redeem partially amounts to half the value of a service. Second, under a CLP, the reward balance resulting from overaccumulation is primarily dependent on the frequency of service demand. Less frequent demand leads to a higher probability that a consumer does not need a service and thus continually overaccumulates for an extended period. Third, the lumpiness of reward earning is more pronounced for firms supplying high-value services, as a consumer earns more rewards each time he makes purchases using cash. This model prediction aligns well with empirical observations across various industries. In particular, it elucidates why airlines, hotels, and department stores are the most proactive in developing LPs and expanding the issuance of rewards.

Lastly, we examine the cyclical nature of LPs and their impact on a firm's financial resilience. We consider a scenario where the economy unexpectedly enters an adverse state. In this state, consumer demand for the service drastically declines compared with the normal state, resulting in negative cash flows for the firm. Consequently, the firm depletes its liquidity buffers to stay afloat. If the adverse state persists for an extended period, the firm will exhaust its liquidity buffers and face bankruptcy. We investigate how the firm's bankruptcy risk varies with its choice of LP. We find that both types of LPs renders the firm's cash flows less procyclical, thereby enhancing the firm's financial resilience. This cyclical nature of LP financing is especially appealing to firms whose business is highly procyclical such as airlines, hotels, upscale department stores, and durable goods retailers. In particular, airlines feature heavy assets, high financial expenses, and low profit margins. The combination of a highly procyclical business and financial vulnerability makes airlines particularly inclined to utilize LP financing.

A common mechanism that renders cash flows less procyclical under both types of LPs is the procyclicality of LPs' financing cost. LP financing is costly as consumers use rewards, issued at a discount, to purchase services instead of cash. As such, a LP effectively becomes a revenue-sharing arrangement where a fixed fraction of revenue is shared with consumers in the form of reward discounts. Consequently, when revenue decreases, the shared revenue follows suit. In addition to the procyclicality of LPs' financing cost, a CLP also helps the firm raise more funds from consumers in the adverse state. The key is that when reward issuance is linked to broad consumption, it is less procyclical than reward redemption for procyclical firms. This mechanism is clearly revealed by the third stylized fact. A general implication of this mechanism is that to strengthen the desirable cyclical nature of a LP, the firm should weaken the linkage between reward issuance and the firm's business. This implication aligns with the second stylized fact that rewards are also issued through consumption unrelated to firms' business.

The rest of this paper is organized as follows. In the rest of the introduction, we review the related literature. Section 2 documents three stylized facts about LPs in the airline and hotel industries. Section 3 outlines the baseline setup of the model. Section 4 derives the equilibrium path in the steady state of the baseline setup. Section 5 derives the implications about LPs as financing instruments. Section 6 examines the cyclical nature of LPs and its impact on the firm's financial resilience. All proofs that not provided in the main text are relegated to the Appendix.

Related literature

Our paper proposes a novel financing motive of LPs, thereby contributing to the extensive literature that seeks to rationalize LPs. Behavioral theories posit that LPs can yield psychological benefits or generate pricing misconceptions. For instance, Lim et al. (2021) discovers that certain consumers attribute more value to LP points than to actual money. In the literature of industrial organization, a traditional viewpoint is that LPs implement price discrimination based on purchase history or quantity. Cremer (1984) demonstrates how a monopolistic seller could employ coupons valid for subsequent purchases to differentiate between first-time and repeat buyers. Sun and Zhang (2019) postulate that the use of limited reward expiration terms could be driven by the intent to discriminate between frequent and infrequent customers. In a multi-brand environment, LPs can increase consumer switching costs, thereby weakening competition. The switching cost theory is initially proposed by Banerjee and Summers (1987), Klemperer (1987), and Klemperer (1995), and subsequently tested by Hartmann and Viard (2008), Orhun et al. (2022), and Rossi and Chintagunt (2023). Fong and Liu (2011) contends that LPs enable tacit collusion within a dynamic overlapping-generations model. Basso et al. (2009) argue that LPs, such as airlines' frequent-flier programs, can exploit agency conflicts between employers and employees. These frequent-flier programs essentially incentivize employees to book services like flights at inflated prices. LPs also provide firms with valuable information about their customers that may be useful for them and their partners in future marketing activities. Basically, the existing literature predominantly interprets LPs as income-boosting instruments. To the best of our knowledge, our paper is the first to empirically document the importance of LPs as financing instruments and to theoretically scrutinize their nature.

The recent literature has seen a surge in studies exploring firms' optimal decisions in the context of LPs. Chen and Pearcy (2010) notice that certain industries tend to encourage brand switching, while others reward consumer loyalty. They offer a comprehensive analysis of these two pricing policies in a dynamic setting, providing insights into the choice between them. Chun and Ovchinnikov (2019) consider a contemporaneous shift where firms across various industries transition their LPs from quantity-based to spending-based designs. They investigate consumers' strategic responses to this shift and the welfare implications thereof. Chun et al. (2020) examine how firms should adjust the value of LP rewards from a revenue management perspective. They discover that the deferred revenue associated with a LP can serve as a cushion or hedging mechanism against uncertainty in operating performance. Chung et al. (2022) explore how the redemption of LP rewards influences firms' daily pricing and inventory decisions. In contrast to these studies, our paper investigates the design of LPs from a financing perspective. This approach allows us to consider LPs not merely as marketing instruments, but as strategic financial instruments that can significantly impact a firm's financial health and resilience.

In finance, our paper is related to several strands of literature studying crowdfunding, tokens, and trade credit. Strausz (2017) considers the benefit of crowdfunding as a way of acquiring information about the eventual payoff of the project if demand is uncertain (see also Astebro et al. 2017; Chemla and Tinn 2020; Ellman and Hurkens 2019). Lee and Parlour (2022) find that while financial intermediaries fail to finance all efficient projects, crowdfunding can improve efficiency. Unique to crowdfunding is the ability of consumers to commit to pay for the consumption benefit ignored by financial intermediaries. Brown and Davies (2020) and Kumar et al. (2020) show that crowdfunding may lead to distortion in financing efficiency and product market output decisions respectively. Cong and Xiao (2024) show how in a dynamic setting, the all-or-nothing feature of crowdfunding can mitigate information cascades and provide information aggregation.

Firms can raise funds by issuing tokens, which is usually referred to as ICOs. Catalini and Gans (2018) show that the ICO mechanism allows entrepreneurs to generate buyer competition for the token, which in turn reveals consumer value. Chod and Lyandres (2021) consider ICOs as a funding method that allows risk averse entrepreneurs to transfer risk to well diversified investors without giving up control rights. Cong et al. (2021) argue that tokens are hybrids of money and investable assets, and align users' investment motive with usage motive. Garratt and van Oordt (2022) show how cryptocurrencies raised through ICOs can align the interests of entrepreneurs and investors better than traditional funding schemes. Rogoff and You (2023) address whether in theory, massive online retailers can issue digital tokens that potentially compete with bank debit accounts. Their central finding is that platforms can potentially earn higher revenues by making tokens non-tradable unless they can generate a sufficiently high outside-platform convenience yield.

Trade credit refers to that a firm, which is usually a large retailer, obtains de facto financing from its suppliers. A typical explanation for trade credit is that due to information or enforcement advantage, suppliers are better able to mitigate the financial frictions faced by external investors such as asymmetric information and lack of commitment (Biais and Gollier, 1997; Burkart and Ellingsen, 2004).

2 Three Facts about Loyalty Programs in the Airline and Hotel Industries

To illustrate the importance of loyalty programs in firms' financing and their features as financing instruments, we conduct a series of empirical analysis on the airline industry and the hotel industry, which are famous for adopting loyalty programs extensively. In this section, we start with an introduction of the accounting of loyalty program liabilities and our data collection. We then present three stylized facts regarding loyalty programs.

2.1 Data

Public firms with large loyalty programs disclose loyalty program liabilities in their annual reports, which represent what firms owe to loyalty program members. Different firms may report them using different terms such as loyalty program deferred revenue, contract liability, and gift card balance. Historically, there are two methods to calculate loyalty program liabilities. The first approach, known as the incremental cost method, involves the recognition of a liability corresponding to the marginal costs incurred in providing services to eligible customers. On the other hand, the second approach, referred to as the deferred revenue method, entails the recognition of a liability for advance payments received in anticipation of future service delivery. Under the deferred revenue method, loyalty program liabilities represent the amount of cash flows that the firm will forgo to fulfill its promises related to the advance payments and are close to the opportunity cost in economics. Since 2018, IFRS 15 and ASC 606 made it obligatory for companies to adopt the deferred revenue method. In our analysis, loyalty program liabilities refer to the numbers estimated using the deferred revenue method.

Estimating loyalty program liabilities using the deferred revenue method is not simple because it involves the estimation of consumers' redemption behavior. For example, airlines' loyalty program liabilities mainly take the form of miles. To determine the opportunity cost induced by promises related to miles, airlines estimate the equivalent selling price of miles and the amount of miles that will be redeemed. United Airlines describe its method as follows:

> "The Company's estimated selling price of miles is based on an equivalent ticket value, which incorporates the expected redemption of miles, as the best estimate of selling

> price for these miles. The equivalent ticket value is based on the prior 12 months' weighted average equivalent ticket value of similar fares as those used to settle award redemptions while taking into consideration such factors as redemption pattern, cabin class, loyalty status and geographic region."
>
> "The Company's breakage model is based on the assumption that the likelihood that an account will redeem its miles can be estimated based on a consideration of the account's historical behavior. The Company uses a logit regression model to estimate the probability that an account will redeem its current miles balance. The Company reviews its breakage estimates annually based upon the latest available information. "

According to the 2017 COLLOQUY loyalty census, loyalty programs are most prevalent in the retail sector, the travel and hospitality sector, and the financial services sector. They account for 42%, 29%, and 17% of all loyalty memberships in the United States, respectively. We choose to focus on the airline industry and the hotel industry because loyalty programs are considered to be an important part of the business for the two industries. The adoption of loyalty programs is ubiquitous, and the reporting of them is complete. Also, the features of their loyalty programs that we observe in practice are likely results of careful design.

In the empirical part of the paper, we document three stylized facts regarding the importance, the mechanics and the nature of loyalty programs as financing instruments. We collect information on the stock and the flow of loyalty program liabilities, debt, and revenue from 10-K reports. Some firms directly report loyalty program liabilities on the balance sheet, while others report them in the notes to financial statements. Table 3 shows how we collect information from 10-K reports and construct the relevant variables. Regarding debt, we include not only long-term debt and its current maturities, but also financial leases and operating leases.[1] In our within-industry analysis, we use revenue to control for the size of firms' business. To obtain a consistent measure within industries, we use total revenue for airlines and total revenue net of reimbursement costs for hotels.[2] Since firms all adopt the deferred revenue method in 2018, we focus on the sample period spanning from 2018 to 2022. During the Covid-19 pandemic, both the airline industry and the hotel industry experienced a severe downturn, and as a response, their capital structures changed massively. To provide a picture of loyalty programs in normal times, we focus on the year 2019 when conducting

[1] Disclosure of leases is required by IFRS 16. For more details, please see https://www.pwc.com/gx/en/services/audit-assurance/assets/ifrs-16-new-leases.pdf.

[2] Hotels do not own all the properties they manage. For those purely managed properties, hotels typically cover some operating expenses such as employees' payroll and later receive full reimbursement from property owners. As a result, two reimbursement terms appear in hotels' revenue and expense, respectively, and they are almost equal. Hence, the reimbursement practice affects only hotels' revenue but not their profits and cash flows. Note that the size of the reimbursement depends on the fraction of purely managed properties and the agreement between hotels and property owners, which are heterogeneous across hotels. We think that total revenue net of reimbursement costs is a better measure of hotels' true economic revenue.

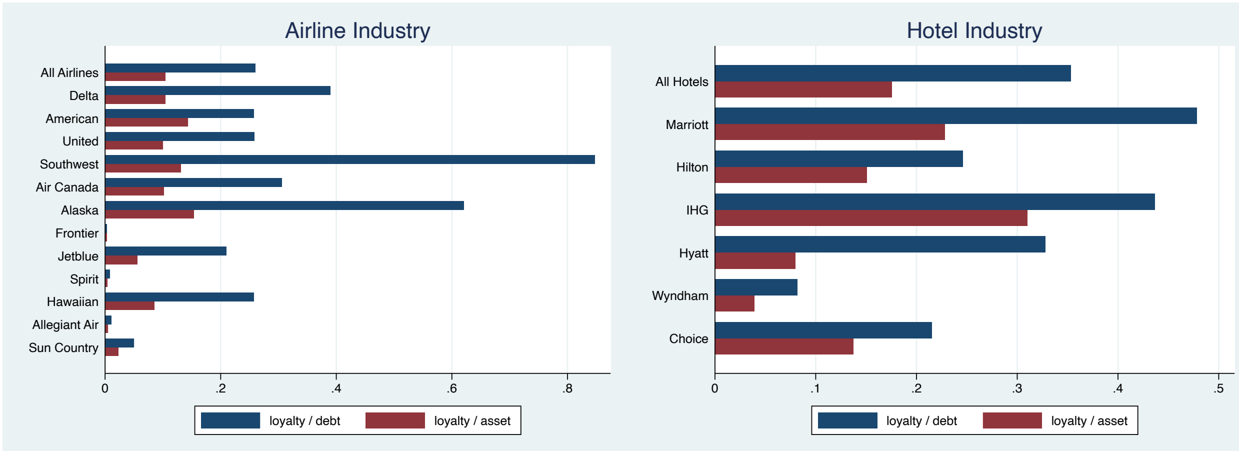

Figure 1: Loyalty program liabilities relative to total debt and total assets

cross-sectional analysis.[3] Unless otherwise specified, the scaled version of a variable refers to the ratio of the variable to revenue in 2019.

2.2 Fact 1: LPs serve as significant financing sources, with CCC programs contributing a large portion

In 2019, the LP liabilities of the airline industry amounted to $30 billion, while those of the hotel industry amounted to $10 billion. To put the numbers in perspective, we calculate the ratios of LP liabilities to total debt and total assets, which are shown in Figure 1. On average, LP liabilities constitute approximately 30% of total debt[4] and around 15% of total assets in the two industries. Whether intentionally or not, LPs are de facto important sources of financing for the two industries.

Given the large size of LP liabilities, one might wonder how firms accumulate them. During the COVID-19 pandemic, United, Delta, and American Airlines borrow against their LPs and disclosed information on the inner workings of LPs. Figure 2 illustrates Delta's LP, SkyMiles, based on Delta's investor presentation. Every time a customer flies with Delta and chooses to earn Delta miles, Delta purchases miles from SkyMiles and grants to him. This part accounts for 32% of SkyMiles' cash flows. The other 68% of the cash flows are mostly due to sales to Amex, Delta's partner bank. Amex issues a collection of cobranded credit cards with Delta and grants the miles to cardholders when they sign up for or spend with the cards.

Delta provided a more detailed breakdown over time. As shown in the lower half of Figure 2, sales to Amex have been the largest source of SkyMiles' cash flows and increasingly important in recent years, followed by sales to Delta, while sales to other non-air partners and sales directly to

[3]Due to IFRS 16, information on leases has been universally available since 2019.

[4]Total debt includes debt, operating leases, and finance leases.

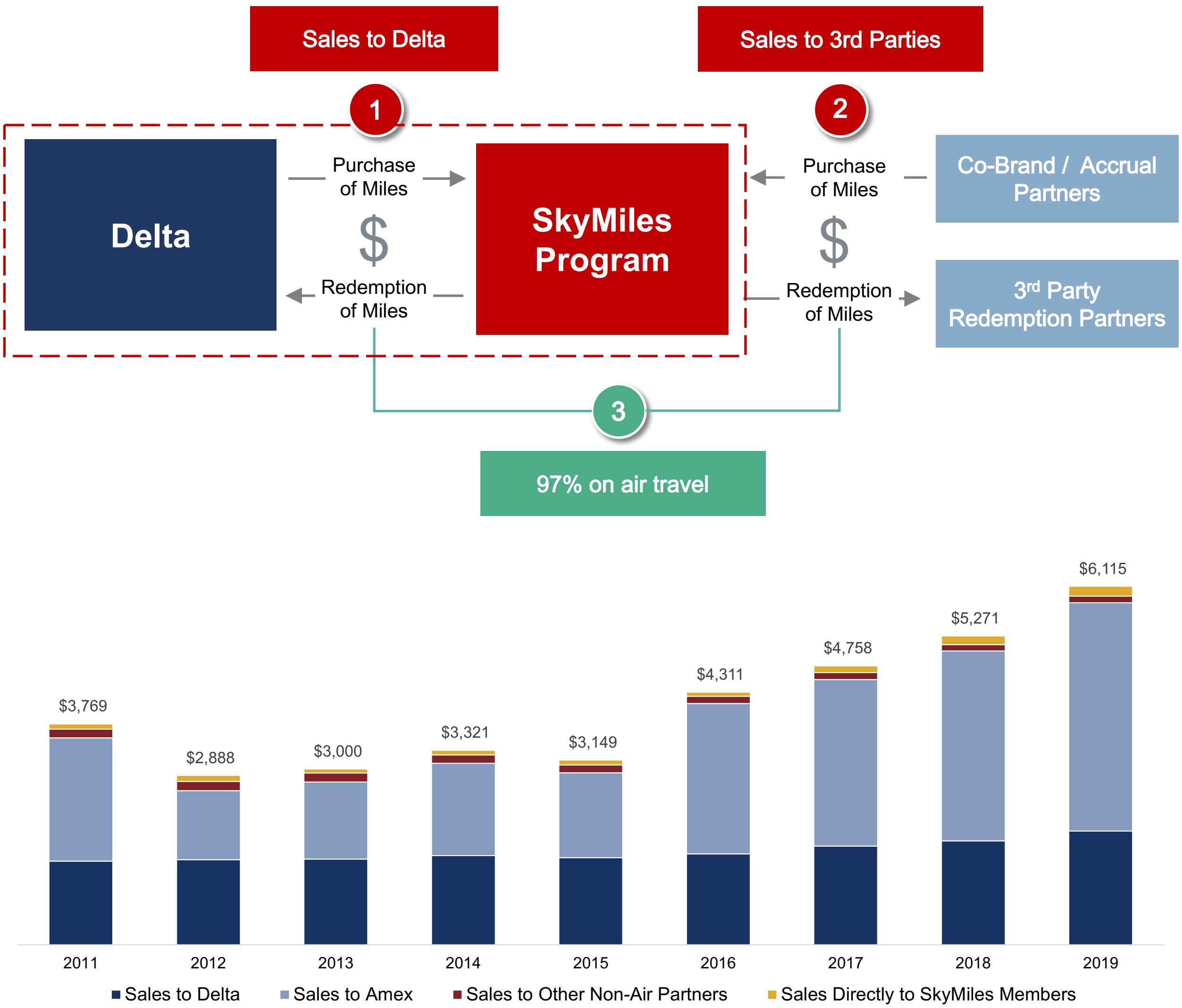

Figure 2: Delta's loyalty program

consumers were tiny. United and American Airlines are in similar situations. In 2019, the fractions of the loyalty program's cash flows due to sales to third parties were 71% for United and 65% for American.

In the case of the three U.S. airlines, it is clear that CCC programs account for a large fraction of LP liabilities. To obtain more systematic evidence of the role of CCC programs, we examine the relationship between LP liabilities and the sizes of CCC programs. Unfortunately, we do not observe the number of cardholders or card spend, so we use the variety of a firm's CCCs, or more specifically, the number of the types of a firm's CCCs, as a proxy for the sizes. The rationale is that when a firm's CCC program is larger and has more cardholders, the firm and its partner bank will develop a richer product line to attract and serve customers. As shown in Figure 3, we can see a clear positive relationship between the scaled loyalty program liabilities and the variety

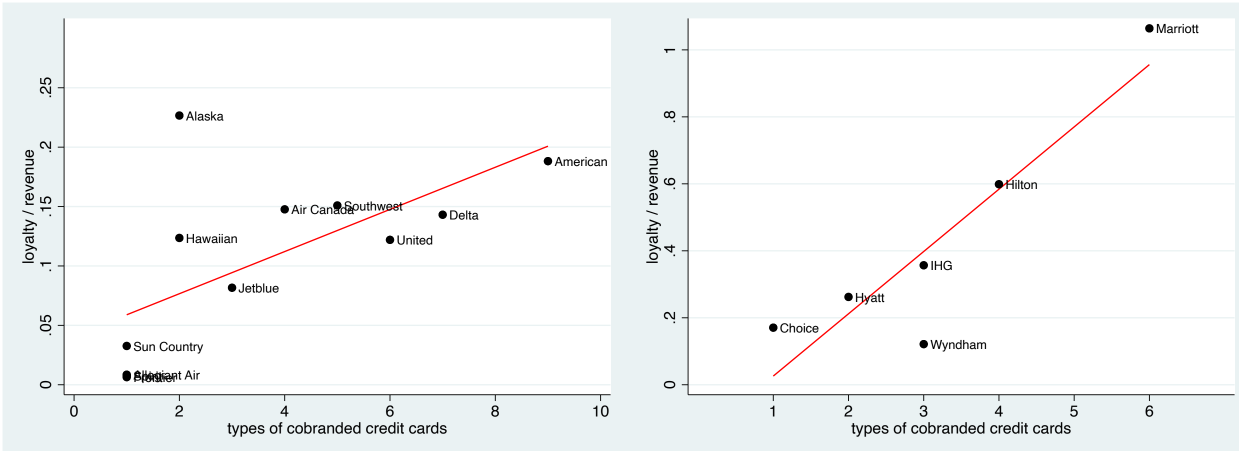

Figure 3: Loyalty program liabilities and varieties of CCCs

for both industries. The first two columns of Table 4 confirm that this positive relationship is statistically significant at 5% level, although there are only 12 observations in the airline industry and 6 observations in the hotel industry.

2.3 Fact 2: rewards are issued through broad consumption but are redeemed predominantly for consumption related to the firm

A CCC typically offers three tiers of rewards to cardholders. They receive the highest rewards for purchases made directly with the issuing firm, lower but still attractive rewards for purchases in certain categories, and the lowest rewards for other purchases. For example, Delta SkyMiles Gold Amex Card offers

- 2 miles per dollar for purchases made directly with Delta,
- 2 miles per dollar for purchases at restaurants and U.S. supermarkets,
- 1 miles per dollar for other purchases.

Given the reward structures of CCCs, we naturally expect that cardholders will use them for purchases with the issuing firm. In fact, consumers also use them extensively for purchases with other firms. Figure 4 shows the annual transaction volume of Delta's CCCs. The numbers are disclosed by Amex in its 10-K reports because Delta is its most important partner. It was 101 billion dollars and accounted for 12% of Amex US transaction volume in 2019; it later increased to 155 billion dollars and accounted for 14% of Amex US transaction volume. Meanwhile, Delta's annual revenue was close to 50 billion dollars. That means, at least 50% of the spend with Delta's CCCs was not related to Delta's business. The number is perhaps considerably higher since a large fraction

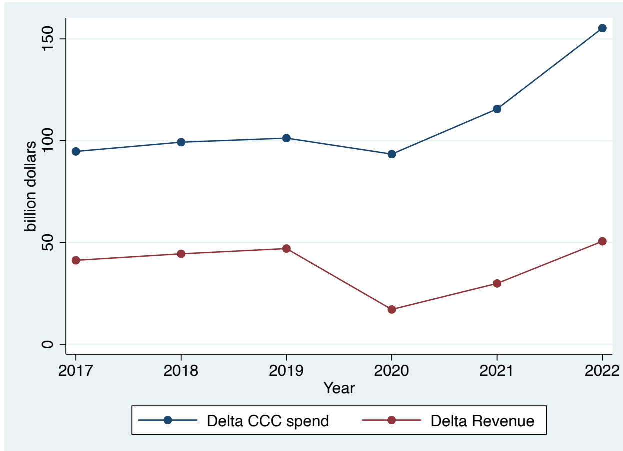

Figure 4: Delta's CCCs

of Delta's revenue is not paid for with its CCCs. This pattern is not unique to Delta. In 2019, the spend with American's CCCs was 109 billion dollars and accounted for 12% of Mastercard U.S. volume, while American's annual revenue was 45.8 billion dollars. Given the importance of CCC programs to LPs, these firms actually issue a large fraction of rewards through broad consumption unrelated to their business.

One may think that such issuance strategy is simply a by-product of CCCs and not intended by firms. A more direct indication of firms' intention is that they allow consumers to earn rewards on purchases with their partners. For example, consumers can earn Delta miles even without Delta CCCs, when they book hotel stays or car rental on Delta's website, shop at over 1000 retailers on SkyMiles Shopping, and dine out at over 10000 restaurants listed on SkyMiles Dining. Although issuance through hotel stays and car rental can potentially be ascribed to the synergy between different travel products, shopping and dining are not connected to Delta' business in particular ways. In fact, U.S. airlines clearly describe this issuance strategy as a key strength of their loyalty programs. For example, in its investors presentation, Delta states,[5]

> "SkyMiles has an extensive network of longstanding partner relationships, allowing members to earn miles on car rental, hotel, retail, dining and other partners."

In economics, the strategy of issuing rewards through unrelated consumption is at odds with classical theories on LPs. The theories of price discrimination and switching cost would suggest that rewards should be issued based on a consumer's history of purchases related to the firm. The agency theory would suggest that rewards should be issued through business consumption instead of personal consumption. Later, we show that this issuance strategy can be well reconciled with the finance theory of loyalty programs.

[5]United and American also made similar statements.

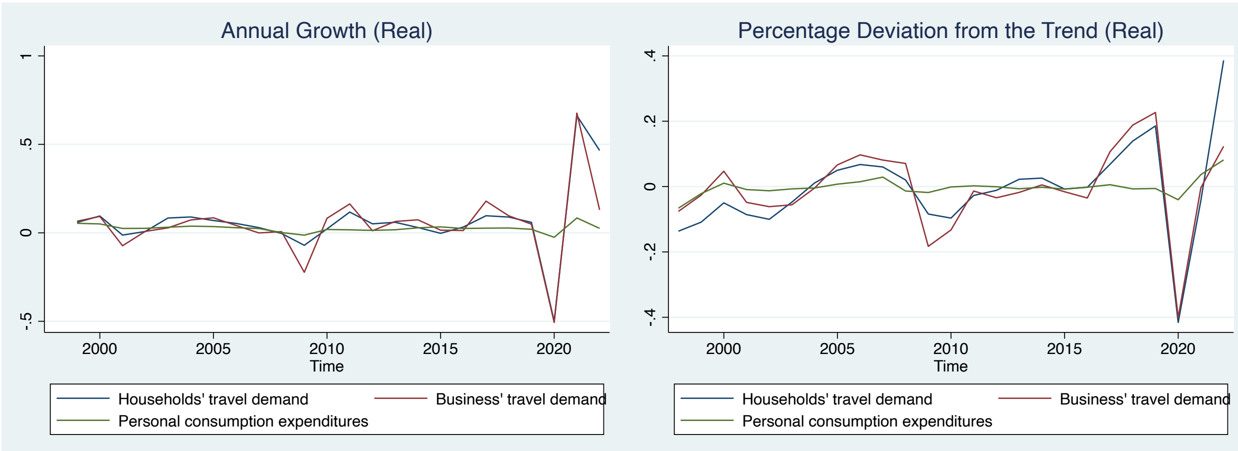

Figure 5: Cyclicality of travel demand and aggregate consumption

As for redemption of miles, around 95% of miles are redeemed for air travel for the three airlines. United further disclosed that it provided 80% of the air travel that its miles are redeemed for. The concentration of redemption on air travel is not surprising, as the value of miles is set to be the highest when they are redeemed in this way. For the same reason, we speculate that most of hotel points are redeemed for hotel stays.[6]

2.4 Fact 3: loyalty programs generate counter-cyclical cash flows

Travel consumption is highly volatile and procyclical, more than most consumption categories. Figure 5 compares the U.S. travel demand and the aggregate consumption. Real travel demand is obtained from Tourism Satellite Accounts Data of Bureau of Economic Analysis, and real personal consumption expenditures are obtained from St. Louis Fed. The left panel plots the annual growth rate, and the right panel plots the percentage deviation from the trend estimated using local polynomial regressions. Clearly, both resident households' travel demand (blue) and business' (red) go hand in hand with the aggregate consumption (green), but fluctuate on larger scales; business' demand is even more volatile than resident households'.

On the one hand, most rewards are redeemed for travel services, meaning that reward redemption tends to move in tandem with travel consumption and is highly procyclical. On the other hand, reward issuance through CCC programs and partners tends to move in tandem with broad consumption and is less procyclical. Taken together, for highly procyclical firms such as airlines and hotels, if a large fraction of rewards are issued through CCC programs and partners, the overall reward issuance will be less procyclical than the overall reward redemption. United reported that during 2008-2009 recession, its revenue declined 19% while its loyalty program revenue only

[6]In many cases, hotel rewards are directly free nights instead of hotel points that may be redeemed for other things.

declined 2%. Due to Covid-19, Delta's revenue dropped by 63.6% in 2020, but the transaction volume of its CCCs dropped only by 7.8%, as shown in Figure 4. The different cyclical nature of reward issuance and reward redemption implies that LPs generate counter-cyclical cash flows: in the bad states of their business, firms can receive more cash flows through issuance than those paid out through redemption. Recall that unredeemed rewards represent promises for future services. The essence of the counter-cyclical cash flows in finance and accounting is the counter-cyclical financing of firms through LPs.

To see more direct evidence of the cyclical nature of LP financing, we investigate how Covid-19 affects its flows and balances. Figure 6 presents the time series of the issuance and redemption of airlines' miles scaled by the revenue in the same year from 2018 to 2022. The airlines are ordered in descending order of scaled LP liabilities in 2019, where Alaska Airlines has the largest scaled LP liabilities. For most of the 12 airlines depicted, the scaled redemption remains stable, implying that redemption of miles closely follows the overall airline business. Meanwhile, the scaled issuance experienced a pronounced spike in 2020, for the airlines with large scaled LP liabilities, e.g. the first seven airlines from Alaska to United. The logic here is that a firm issuing rewards in more diverse ways will have larger LP liabilities relative to its business, and its reward issuance is less correlated with its business.

Figure 7 plots the time series of LP liabilities. To control for the cross-sectional variations in business and demonstrate the change in LP liabilities over time, we scale them by revenue in 2019. LP liabilities experienced a significant increase in 2020 for the seven airlines with large scaled LP liabilities, while this trend was not observed for other airlines. Similarly, Marriott and Hilton, which have the largest scaled LP liabilities among hotels, also saw a dramatic increase in their LP liabilities in 2020. We run OLS regressions to test this hypothesis in a more formal manner. We construct an indicator of large loyalty programs that equals 1 for the seven airlines from Alaska to United, Marriott, and Hilton, and 0 otherwise. We regress the change in scaled LP liabilities from 2019 to 2020 on the indicator. We also add the same change from 2018 to 2019 to control for potential time trends. As shown in Columns (3) to (6) of Table 4, the coefficient of the indicator is positive at 5% significance level for both industries. Note that this analysis is essentially a joint test of two statements: 1) firms issuing rewards in more diverse ways have larger LP liabilities compared with its business, and 2) their LP liabilities are more counter-cyclical.

One may wonder how the industries view these observations. Do they know, and do they care? Delta Airlines described one strength of its LP to investors as follows:

> "SkyMiles offers significant diversity of cash flows with a long-term track record of stable and growing performance through cycles."

United and American have almost the same description. Clearly, big U.S. airlines realize that more

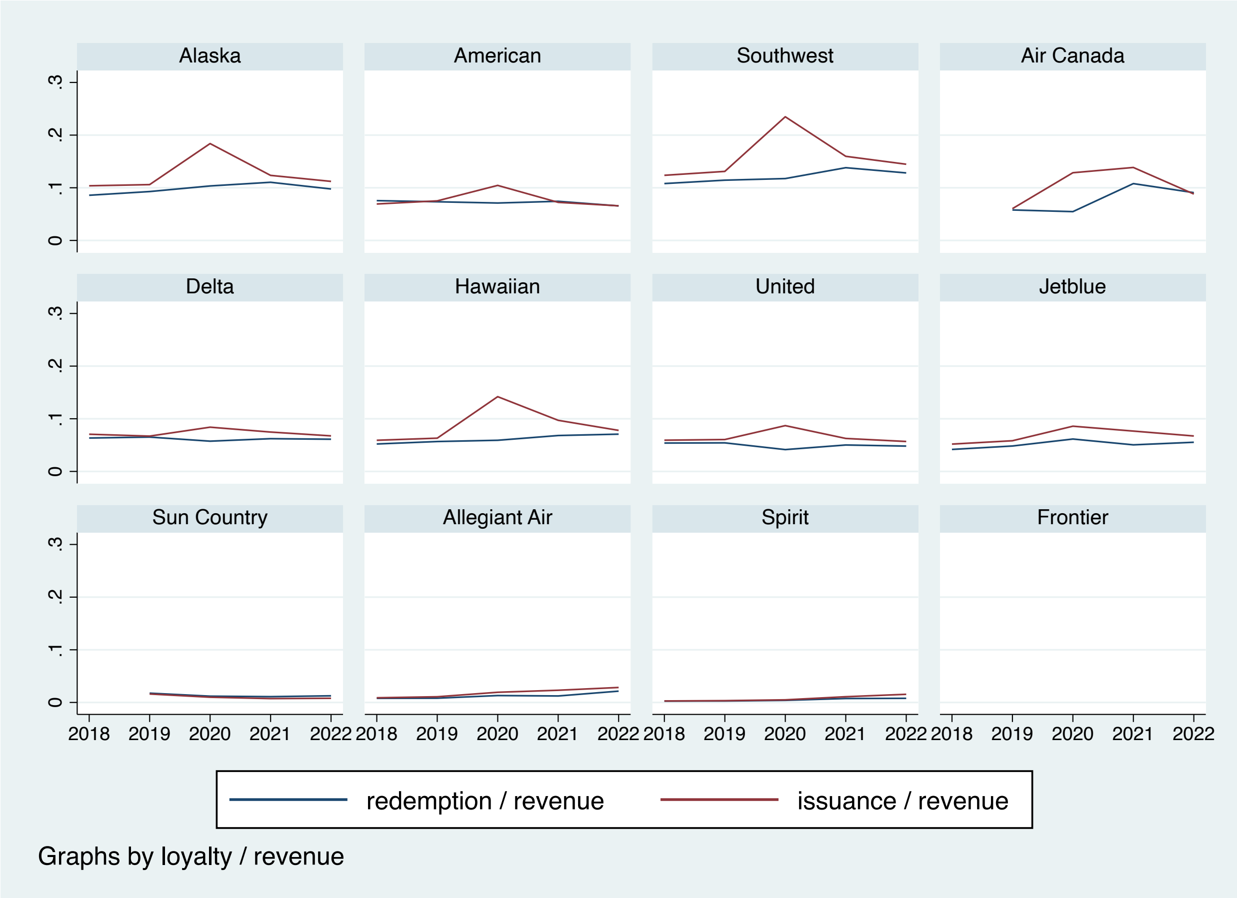

Figure 6: The time series of issuance and redemption

diverse and less procyclical issuance of miles can help them better survive business cycle fluctuation. They attempt to achieve it by expanding their partnership networks and allow consumers to earn miles on unrelated consumption.

3 The Baseline Setup

Motivated by empirical observations about airlines and hotels, we consider two types of loyalty programs: consumption-based LPs (CLPs), under which consumers earn rewards from make payment with cobranded credit cards, and purchase-based LPs (PLPs), under which consumers earn rewards from purchases made directly with the firm.

3.1 Players

We consider an infinite-horizon continuous-time game. The two main players in the game are consumers and a firm, while banks play minor roles. Players are all risk neutral. The return of

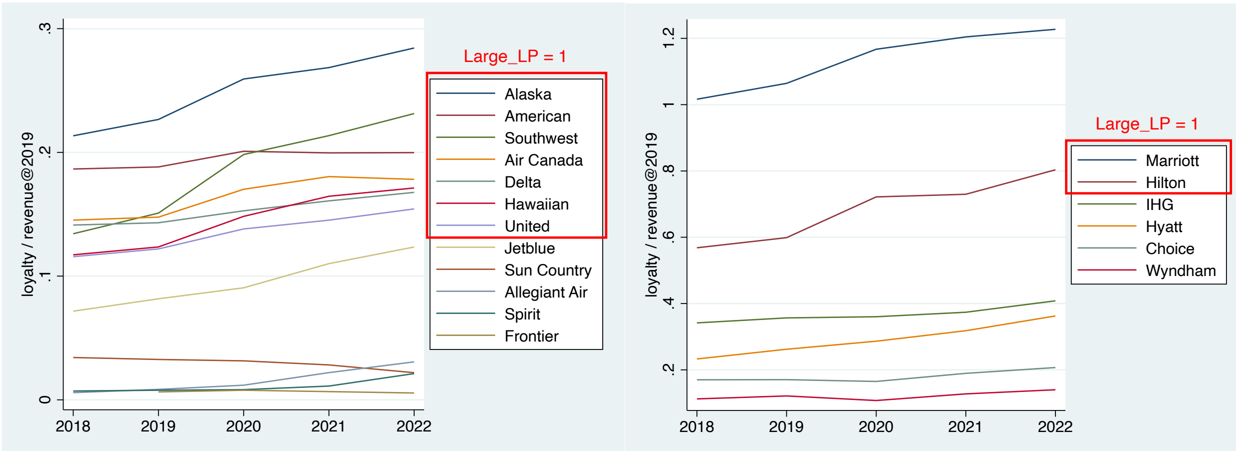

Figure 7: The time series of scaled loyalty program liabilities

investment in the game is uniformly r_0, and there is no tax.

Consumers are infinitesimal, and their total mass is normalized to 1. Each consumer needs a service, e.g., a flight or a hotel night, at the rate of q_0. The arrival of the need is independent across consumers and over time. A service is worth p_0 to a consumer when he needs it and 0 when he does not need. Taken together, the demand for service is p_0q_0. In addition to the service, each consumer also has exogenous consumption of other goods. Each consumer's total consumption, including the service and other goods, is completely smoothed out, so his consumption flow is constant, which is C_0 per unit of time.[7] Note that consumers are all homogeneous, so the firm does not benefit from price discrimination.

The firm monopolizes the supply of the service that consumers need, so the model abstracts from competition. To extract all the surplus, it sets the price of a service to p_0. To provide services, the firm needs to raise funds to buy and maintain certain assets and hire workers. Here, we do not specify the details of the firm's operation.[8] Instead, we assume that the firm's cash flows are positive in the normal state for all cases we consider, and the firm pays out the cash flows as dividend, so its balance sheet does not change. The firm's operating expense is then constant over time, which is OE_0 per unit of time. As a benchmark, we first consider the case where the firm adopts no LP and consumers can make purchases only with cash. A consumer will buy one whenever needed, and the firm receives p_0.

Proposition 1. *If no loyalty program is adopted, consumers buy services with cash whenever they*

[7] In practice, consumer credit products such as credit card loans and buy-now-pay-later are widely used, especially in the U.S.. These products relax consumers' liquidity constraints and help smooth out their consumption expenditure. Here, for simplicity, we make an extreme assumption that such smooth-out is complete.

[8] In Section 6, we introduce shocks to the game and consider the possibility of bankruptcy. There we specify more details.

need, and the firm's cash flows are

$$CF_0 \triangleq p_0 q_0 - OE_0. \tag{1}$$

Banks play passive roles in the game. Each bank can issue a bank credit card (BCC) to consumers and earn interchange fees paid by merchants. Interchange fees are ϕ of the consumption.[9] For the baseline setup, we assume that banks do not charge any fee to consumers. Banks act competitively. To attract consumers' usage of its BCC, a bank offers each consumer cashback as much as the fees it earns from him. One of the banks may also issue a cobranded credit card (CCC) together with the firm.

3.2 Loyalty programs

The firm can choose to adopt a LP and issue rewards. As observed in practice, we assume that rewards are convenient to use. First, although a service is indivisible, a consumer can purchase it with any combination of cash and rewards that are worth one service. Second, unlike debt whose payout follows a fixed schedule, consumers can redeem rewards whenever they want. Consumers enjoy substantial convenience when holding rewards, which makes it more similar to holding demand deposits than doing investment. To capture the benefits of convenience, we assume that consumers treat rewards like a checking account term and do not discount its future redemption value. We do not consider the possibility that the firm issues nonconvenient rewards because consumers will require more compensation for holding rewards, making the LP unattractive.

However, rewards are still not perfectly convenient as cash. On the one hand, rewards are less liquid than cash because rewards cannot be redeemed for consumption other than the service. Hence, we assume that consumers weakly prefer to hold cash rather than to hold rewards. On the other hand, getting involved in a LP incurs some use costs to consumers. Joining the LP and keeping track of his status in the LP take a consumer's time and attention. This *participation cost* is equivalent to a flow cost of η per unit of time. Unless a consumer decides not to buy the service at all, the participation cost is always incurred. Redeeming rewards involves a more troublesome procedure than using cash, so each redemption incurs a *redemption cost* ε, which is smaller than p_0. Since the firm needs to compensate consumers for the use costs, rewards must be issued at a discount in equilibrium. In the model, these use costs, instead of the discount rate or financial frictions, drive the financing cost of LPs.

We assume one reward is always worth $1/p_0$ of one service. Hence, a consumer can purchase one service with x rewards and $p_0 - x$ cash for any $x \in [0, p_0]$. The exact value of one reward

[9] Interchanges fees are roughly 2% to 3% of the consumption in the U.S. and lower in other countries. The exact fraction varies with consumption categories and merchants.

does not matter because the firm determines the issuance of rewards. The nontrivial part of the assumption is that this value is constant. In practice, firms like airlines and hotels do devalue their rewards occasionally.[10]

3.3 CLPs and PLPs

Adopting a CLP, the firm cooperates with a bank in issuing a cobranded credit card (CCC). For the CCC, the bank purchases rewards from the firm and offer them to consumers instead of cashback. Due to competition among banks for the cooperation, the firm receives all the fees earned by the bank from this CCC and decides how many rewards offered to consumers. The CCC is determined by the scope of the reward category γ and the reward rate a_c. The firm classifies certain consumption under the reward category. For each dollar of consumption in the reward category paid for with the CCC, a consumer receives a_c rewards, and 0 rewards for other consumption. The essence here is that the CCC's reward structure will induce consumers to pay for the reward category with the CCC and other consumption with BCCs. For each dollar of consumption in the reward category, consumers receive a_c rewards instead of ϕ cash. Hence, the price of a reward is

$$\theta_c \triangleq \phi / a_c.$$

The reward category accounts for a fraction γ of a consumer's total consumption.

We focus on the cases with $\gamma C_0 a_c < p_0 q_0$. This condition implies that even if a consumer pays for the whole reward category with the CCC, the rewards he receives are not enough to redeem for all services he needs in expectation. This condition generally holds in practice. On the one hand, the issuance rate of rewards of a CCC is typically small because it is related to the rate of interchange fees and the consumption paid for with credit cards. Interchange fees are roughly 2% to 3% of the consumption. A large fraction of consumption such as rents for housing is usually required to be paid for with cash in order to avoid interchange fees. On the other hand, consumers in the model are basically frequent users of the service and thus have high expected demand.

Adopting a PLP, the firm offers a consumer an option to buy rewards when he makes purchases. For each dollar spent on services, a consumer can choose to buy π rewards at the unit price of θ_p. That is, if a consumer buys one service with x rewards and $p_0 - x$ cash, he can pay another $(p_0 - x)\pi\theta_p$ cash and receive $(p_0 - x)\pi$ rewards. If a consumer exercises this option, he essentially spends x rewards and $(p_0 - x)(1 + \pi\theta_p)$ cash to receive one service and $(p_0 - x)\pi$ rewards. We focus on the cases with $\pi \leq 1$ because in practice, consumers usually need to accumulate rewards

[10] In practice, firms benefit from devaluation ex post as it reduces the actual LP liabilities. However, devaluation hurts firms' reputation for caring about customers and especially enrages loyal customers who use services frequently and accumulate sizable balances.

through multiple purchases to get one service for free.

In practice, it is more common to see that the firm requires consumers to buy rewards by selling a bundle of services and rewards, rather than offering an option. Specifically, a consumer can earn π' rewards for each dollar spent on services, and because of the rewards, the firm sets the price of a service to p' that is higher than p_0. Under this rule, a consumer essentially spends x rewards and $(1-x/p_0)\,p'$ cash to receive one service and $(1-x/p_0)\,p'\pi'$ rewards. Notice that the firm can replicate this issuance by offering an option with $\theta_p = (1-p_0/p')\,/\pi'$ and $\pi = \pi' p'/p_0$. Therefore, bundling and offering an option are equivalent in our setting. We choose the latter to draw a better parallel between the two types of LPs.

When we do not distinguish between the two types of LPs, we use θ to represent the unit price of rewards. We say a consumer fully uses the LP if he pays for the whole reward category with the CCC under a CLP or always exercises the option to buy rewards under a PLP.

3.4 The steady state

For the baseline setup, we consider that the firm can choose to adopt either a CLP or a PLP, which allows for analytical solutions of the model.[11] Also, we focus on a static LP—once chosen, its design does not change over time. Note that a consumer may not use rewards immediately after he earns them. That could be because he has no need for a service or because he find it more desirable to leave the rewards for future redemption. As a result, a consumer may hold a substantial amount of rewards, and his behavior may depend on his reward holdings. Since consumers are homogeneous, the state of the game can be summarized by the distribution of all consumers' reward holdings. Therefore, we define the steady state under a static LP as follows.

Definition 1. In a steady state under a static LP,

- each consumer's decision maximizes his expected average payoff flow;
- the distribution of consumers' reward holdings stays unchanged over time.

For the baseline setup, we focus on the equilibrium path in steady states. In a steady state, the aggregate behavior of all consumers remains constant, and the firm's cash flows are constant. Our analysis centers around how a LP affects the constant cash flows.

4 Model Solution

In this section, we derive the steady state under a static CLP or PLP.

[11] In Section 5.1, we numerically demonstrate the case when both types are adopted.

4.1 Consumers' behavior and the firm's cash flows

We first characterize consumers' behavior, assuming that they find it desirable to use the service. Since there are infinite horizons and a consumer does not discount his future payoff, a consumer cares about his average payoff flow. Then a consumer's strategy can be generally characterized as receiving rewards at the rate of z and redeems x rewards for a service at the rate of y on average, where $0 < x \leq p_0$ and $xy \leq z$ due to the budget constraint of rewards. For a service he buys with only cash, he pays p_0 cash. For a service he buys with a positive amount of rewards, he incurs a redemption cost ε and pays $p_0 - x$ cash in addition to x rewards on average. The cost of receiving rewards at the rate of z is $z\theta$, and the participation cost is η. Taken together, a consumer's average payoff flow is

$$(q_0 - y)(p_0 - p_0) + y[p_0 - \varepsilon - (p_0 - x)] - z\theta - \eta = y(x - \varepsilon) - z\theta - \eta. \tag{2}$$

Each redemption incurs a redemption cost ε. Given that a consumer uses a fixed amount of rewards, he would like to use more rewards in each redemption and redeem less frequently to save redemption cost. For one service, he can redeem at most p_0 rewards, so he would like to redeem p_0 rewards whenever he decides to redeem. On the other hand, a consumer weakly prefers to hold cash than hold rewards, so with at least p_0 rewards, he must redeem p_0 rewards when he needs a service. The two observations lead to the following lemma.

Lemma 1. *If a consumer never stops earning rewards, he redeems p_0 rewards for a service when he needs a service and have at least p_0 rewards, and does not redeem rewards otherwise.*

Based on Lemma 1, we derive the issuance rate of a LP if consumers fully uses the LP.

Lemma 2. *If a consumer fully uses the LP, he receives rewards at a rate of A on average.*

- *Under a CLP, $A = A_c \triangleq \gamma C_0 a_c$.*
- *Under a PLP, $A = A_p \triangleq \frac{p_0 q_0}{1/\pi + 1}$.*

It is straightforward to see $A = A_c$ under a CLP. As for a PLP, Lemma 1 implies that if a consumer fully uses the LP, almost all rewards will be redeemed because a consumer never holds more than $p_0 + p_0\pi$ rewards. Since a consumer receives a service and $p_0\pi$ rewards every time he pays cash, rewards will be redeemed for a fraction $\frac{1}{1/\pi+1}$ of services, which implies $A = A_p$. We regard A_c (A_p) as the issuance rate of a CLP (PLP).

Combining Lemmas 1 and 2, Proposition 2 characterizes consumers' behavior.

Proposition 2. *Suppose that the firm adopts a LP with the unit price of rewards θ and the issuance rate A. If $\theta \leq 1 - \frac{\varepsilon}{p_0} - \frac{\eta}{A}$, each consumer uses the service:*

- *he fully uses the LP;*
- *when needing a service, he redeems p_0 rewards if he has at least p_0 rewards and pays p_0 cash otherwise.*

If $\theta > 1 - \frac{\varepsilon}{p_0} - \frac{\eta}{A}$, a consumer does not use the service.

Under consumers' optimal redemption strategy, each redemption always involves p_0 rewards. The average benefit of a reward after the redemption cost is $1 - \theta - \frac{\varepsilon}{p_0}$. Because of the flow cost, only when $1 - \theta - \frac{\varepsilon}{p_0} > 0$, a consumer may use the service. And conditionally on using the service, he will fully uses the LP and receives rewards at the rate of A. Plugging $x = p_0$, $y = A/p_0$ and $z = A$ into the righthand side of equation 2, we obtain that a consumer's average payoff flow from using the service is

$$\frac{A}{p_0} \cdot (p_0 - \varepsilon) - A\theta - \eta.$$

Therefore, a consumer uses the service if and only if this average payoff flow is nonnegative, or

$$\theta \le 1 - \frac{\varepsilon}{p_0} - \frac{\eta}{A}. \tag{CC}$$

This condition is referred to as the consumer constraint.

Next, we calculate the firm's cash flows in the steady state if each consumer uses the service. Consumers as a whole redeem p_0 rewards for a service at the rate of A/p_0, and buys a service with purely cash at the rate of $q_0 - A/p_0$. The firm receives cash flows $A\theta$ from issuing rewards and $(q_0 - A/p_0)\,p_0$ from selling services to consumers. Since the distribution of consumers' reward holdings stays unchanged, consumers as a whole hold a constant reward balance. Denote it by B. Essentially, the firm pre-sells this amount of rewards to consumers and receives an amount $B\theta$ of cash, which is the firm's financing through the LP. With the $B\theta$ cash, the firm can reduce debt financing or increase investment by $B\theta$, which generates cash flows $r_0 B\theta$. Taken together, the firm's cash flows are

$$\begin{aligned} CF &\triangleq A\theta + \left(q_0 - \frac{A}{p_0}\right) p_0 + r_0 B\theta - OE_0 \\ &= \underbrace{r_0 B\theta}_{\text{interest from reward balance}} - \underbrace{A\,(1-\theta)}_{\text{discount of rewards}} + CF_0. \end{aligned} \tag{3}$$

Although the funds raised through the LP do not incur the typical interest cost, they are also costly because the firm needs to issue rewards at a discount to compensate consumers for use costs. The financing cost is effectively

$$\frac{A\,(1-\theta)}{B\theta} = \frac{A}{B} \cdot \frac{1-\theta}{\theta} \tag{4}$$

per unit of funds. Whether the LP is desirable as a financing instrument hinges on the relative size between its financing cost and the return of investment r_0. Conceptually, the financing cost of a LP depends on two factors. The first is the discount at which the firm issues rewards, $1-\theta$. The second is the balance-to-flow ratio, B/A. A LP is more desirable as a financing instrument if it can result in a higher reward balance given a certain issuance rate.

4.2 Reward balance under a CLP

Next, we derive the reward balance under a CLP. The key is to characterize the steady-state distribution of consumers' reward holdings. Let $\Gamma(N)$ be the mass of consumers holding rewards fewer than N, and $\gamma(N)$ be the density of consumers holding N rewards. Lemma 3 characterizes $\gamma(\cdot)$ in the steady state under a CLP.

Lemma 3. *In the steady state under a CLP, the density of consumers' reward holdings must satisfy the equation system*

$$\gamma'(N) = \begin{cases} \gamma(N+p_0)\frac{q_0}{A}, & \text{if } N \in [0, p_0), \\ -\gamma(N)\frac{q_0}{A} + \gamma(N+p_0)\frac{q_0}{A}, & \text{if } N \in [p_0, +\infty), \end{cases} \tag{5}$$

and three boundary conditions

$$\lim_{x \uparrow N} \gamma(x) = \lim_{x \downarrow N} \gamma(x), \forall N \tag{6}$$

$$\gamma(N) = 0, \text{ if } N < 0 \tag{7}$$

and

$$\int_0^{+\infty} \gamma(x)\, dx = 1. \tag{8}$$

The intuition of Lemma 3 is as follows. For a small τ, let $\Delta\Gamma(N+A\tau) \triangleq \Gamma(N+A\tau) - \Gamma(N)$ be the mass of consumers holding rewards between N units and $N+A\tau$ units at t. These consumers consist of three groups as follows.

- The first group is the consumers who do not redeem rewards for any service during the past τ units of time. Since they are always accumulating rewards at the rate of A, they must hold rewards between $N-A\tau$ units and N units at $t-\tau$.
- The second group is the consumers who redeem rewards for one service during the past τ units of time. They must hold rewards between $N+p_0-A\tau$ units and $N+p_0$ units at $t-\tau$.
- The third group is the consumers who redeem rewards for more than one service during the past τ units of time and end up holding rewards between N units and $N+A\tau$ units at t.

For $N \in [0, p_0 - A\tau)$, consumers holding rewards between $N - A\tau$ units and N units at $t - \tau$ do not have sufficient rewards to redeem for one service by t, so all of them belong to the first group. On the other hand, consumers holding rewards between $N + p_0 - A\tau$ units and $N + p_0$ units at $t - \tau$ have needs for services with probability $1 - e^{-q_0\tau}$. Since $N + p_0 + A\tau < 2p_0$, these consumers have sufficient rewards to redeem for only one service, so all the fraction $1 - e^{-q_0\tau}$ of them belong to the second group. The mass of the third group is of a higher order than τ. Hence, we obtain the following equation:

$$\Delta\Gamma(N + A\tau) = \Delta\Gamma(N) + \Delta\Gamma(N + p_0)\left(1 - e^{-q_0\tau}\right) + o(\tau).$$

For $N \in [p_0 + A\tau, +\infty)$, consumers holding rewards between $N - A\tau$ units and N units at $t - \tau$ have sufficient rewards to redeem for one service, so only those of them having no needs for services, which make up the fraction $e^{-q_0\tau}$, belong to the first group. On the other hand, consumers holding rewards between $N + p_0 - A\tau$ units and $N + p_0$ units at $t - \tau$ have sufficient rewards to redeem for more than one service, so only those of them having needs for exactly one service by t, which make up the fraction $q_0\tau e^{-q_0\tau}$, belong to the second group. The mass of the third group is of a higher order than τ. Hence, we obtain the following equation:

$$\Delta\Gamma(N + A\tau) = \Delta\Gamma(N)e^{-q_0\tau} + \Delta\Gamma(N + p_0)q_0\tau e^{-q_0\tau} + o(\tau).$$

Taking τ to 0, we obtain equation (5). The boundary conditions (6), (7) and (8) stem from that the distribution is continuous everywhere, no consumer holds a negative number of rewards, and the total mass of consumers is 1, respectively.

Solving the equation system in Lemma 3, we explicitly characterize the steady-state distribution of consumers' reward holdings $\gamma(\cdot)$ and pin down the reward balance through

$$B = \int_0^\infty N\gamma(N)\,dN.$$

Proposition 3. *The steady-state distribution of consumers' reward holdings is*

$$\gamma(N) = \begin{cases} 0, & \text{if } N \in (-\infty, 0), \\ \frac{1}{p_0} \cdot \left(1 - e^{-N/\lambda}\right), & \text{if } N \in [0, p_0), \\ \frac{1}{p_0\left(\lambda\frac{q_0}{A} - 1\right)} e^{-N/\lambda}, & \text{if } N \in [p_0, +\infty), \end{cases} \tag{9}$$

where λ is the unique positive root of

$$\left(1 - e^{-p_0/\lambda}\right) \cdot \lambda = \frac{A}{q_0}. \tag{10}$$

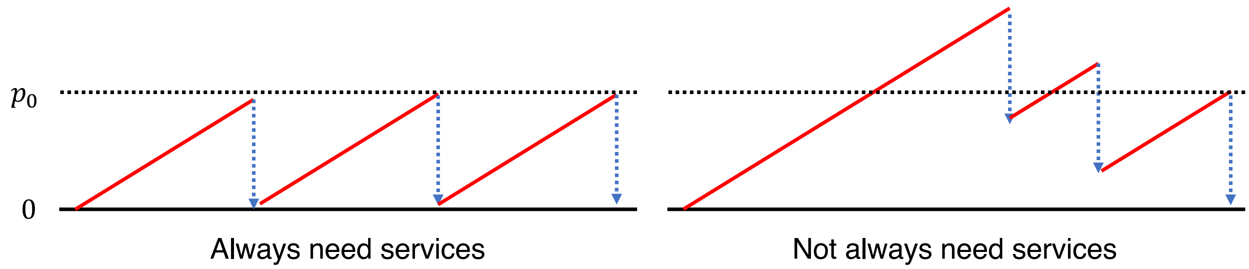

Figure 8: The dynamics of reward balance under CLPs

Further, the reward balance is

$$B = \frac{p_0}{2} + \lambda, \tag{11}$$

The reward balance consists of two parts. The first part is simply a half of p_0. It stems from that consumers are not willing to redeem partially. Consider the case that consumers always need services. Their reward holdings follow a cycle, as shown in the left panel of Figure 8. Starting from 0, a consumer's reward holdings increase at a constant rate up to p_0, then consumers redeem rewards, and they drop back to 0. In this case, the average reward holdings are a half of p_0. The second part stems from that consumers do not always need services, so they may continue to accumulate rewards even after holding p_0 rewards. Since the issuance rate is smaller than the expected demand, i.e.,

$$A = \gamma C_0 a_c < p_0 q_0,$$

a consumer never worries that some rewards are left unused, so he always fully uses the CCC and accumulates rewards at the rate of A. That means, the issuance of rewards to a consumer is constant and *independent* of his purchase and redemption of services. Therefore, a consumer may accumulate substantially more than p_0 rewards if he does not need services for a while. The right panel of Figure 8 shows a possible path of rewards balance. The second part is referred to as overaccumulation. Put intuitively, the accumulation of reward balance relies on both consumers' unwillingness to redeem partially and their temporary lack of needs for services

4.3 Reward balance under a PLP

Suppose that the firm adopts a PLP. A consumer's reward holdings change only when he buys a service. We can then represent a consumer's reward holdings by a sequence $\{h_i\}_{i\in\mathbb{N}}$, where h_i is the amount between his ith purchase and his $(i+1)$th purchase. By Lemma 1, if he holds fewer than p_0 rewards, he will pay p_0 cash and earn $p_0\pi$ rewards; if he holds at least p_0 rewards, he will redeem p_0 rewards. That means,

$$h_i = \begin{cases} 0, & \text{if } i = 0, \\ h_{i-1} + p_0\pi, & \text{if } h_{i-1} < p_0, \\ h_{i-1} - p_0, & \text{if } h_{i-1} \geq p_0. \end{cases}$$

Since the need for a service arrives independently over time, a consumer stays at each point of the sequence for the same expected length of time. This implies that the reward balance in the steady state equals the average of the sequence. Bases on this formulation, Proposition 4 characterizes the steady-state distribution of consumers' reward holdings and the reward balance.

Proposition 4.
- *If $1/\pi$ is a rational number, let m and n be the positive coprime integers such that $\mu/n = (1+\pi)/\pi$. Consumers are uniformly distributed over the set $\left\{\frac{i}{\mu}(p_0 + p_0\pi)\right\}_{i=0}^{\mu-1}$ in terms of reward holdings.*
- *If $1/\pi$ is an irrational number, consumers are uniformly distributed over the interval $[0, p_0 + p_0\pi)$ in terms of reward holdings.*
- *Let μ be $+\infty$, if $1/\pi$ is an irrational number. The reward balance is*

$$B = \frac{\mu - 1}{2\mu}(1+\pi)\,p_0.$$

Here, we present the proof of Proposition 4 and its economic intuition. We start with the case that $1/\pi$ is an integer. As demonstrated in the left panel of Figure 9, a consumer's reward holdings follow a cycle:

- Starting with zero rewards, a consumer will pay for the next $1/\pi$ purchases with cash, so his reward holdings consistently increase as he make purchases until he accumulates p_0 rewards.
- He will then redeem all his rewards for the next purchase, and his reward holdings go back to 0.

In the steady state, all consumers are uniformly distributed over $\{0, p_0\pi, 2p_0\pi, \ldots, p_0 - p_0\pi, p_0\}$ in terms of reward holdings, and the reward balance is $p_0/2$. This steady-state distribution is driven by two forces. First, a consumer is not willing to redeem partially. Second, his reward holdings never exceed p_0. Since $1/\pi$ is an integer, his reward holdings will exactly hit p_0 when they reach or exceed p_0, and he will then redeem rewards for the next purchase instead of earning more.

When $1/\pi$ is a rational number but not a integer, the case becomes more complicated. Under consumers' optimal strategy, reward earning is lumpy whenever it occurs, which is $p_0\pi$. Due to the lumpiness, consumer's reward holdings may exceed p_0. For example, as demonstrated in the right panel of Figure 9, if $\pi = 2/3$, a consumer's reward holdings are $4p_0/3$ when they first reach or exceed p_0; moreover, when the consumer redeems p_0 rewards for the next purchase, his reward holdings decrease to a positive amount instead of 0. To characterize the sequence of reward holdings in a general way, we resort to simple number theory. Notice that a redemption can be considered as that a consumer still earns $p_0\pi$ rewards but meanwhile gives up $p_0 + p_0\pi$ rewards. Therefore, we can write the sequence in modular arithmetic as follows:

$$h_i = i \cdot p_0\pi \pmod{(p_0 + p_0\pi)}.$$

Since $1/\pi$ is a rational number, there exists a unique pair of positive coprime integers μ and n such that $\mu/n = (1+\pi)/\pi$. We then obtain

$$\frac{h_i}{(p_0 + p_0\pi)/\mu} = i \cdot n \pmod{\mu}.$$

Since μ and n are coprime, $\frac{h_i}{(p_0+p_0\pi)/\mu}$ follows a cycle, and in one cycle, $\frac{h_i}{(p_0+p_0\pi)/\mu}$ hits each element in $\{0, 1, 2, \ldots, \mu-2, \mu-1\}$ once. That means, consumers are uniformly distributed over the set $\left\{\frac{i}{\mu}(p_0 + p_0\pi)\right\}_{i=0}^{\mu-1}$ in terms of reward holdings. Further, the reward balance is

$$B = \frac{1}{\mu}(p_0 + p_0\pi)\sum_{i=0}^{\mu-1} i \cdot \frac{1}{\mu} = \frac{\mu-1}{2\mu}(1+\pi)\,p_0.$$

Note that $1/\pi$ is a rational number but not a integer, so the integer n is strictly greater than 1. That means, $\mu > \frac{1+\pi}{\pi}$, and

$$\frac{\mu-1}{2\mu}(1+\pi)\,p_0 > \frac{\frac{1+\pi}{\pi}-1}{2\frac{1+\pi}{\pi}}(1+\pi)\,p_0 = \frac{p_0}{2}.$$

The part in addition to $p_0/2$ is ascribed to the lumpiness of reward earning discussed above.

When $1/\pi$ is an irrational number, the economic forces are basically the same. Conceptually, we can construct an infinite sequence of rational numbers that converges to $1/\pi$. During this process, μ will go to infinity, and the set $\left\{\frac{i}{\mu}(p_0 + p_0\pi)\right\}_{i=0}^{\mu-1}$ will get closer to the interval $[0, p_0 + p_0\pi)$. Formally, by Weyl's equidistribution theorem, if $\frac{\pi}{1+\pi}$ is an irrational number and

$$\frac{h_i}{p_0 + p_0\pi} = i \cdot \frac{\pi}{1+\pi} \pmod{1},$$

h_i is uniformly distributed over the interval $[0, p_0 + p_0\pi)$. Hence, the reward balance is $(p_0 + p_0\pi)/2$.

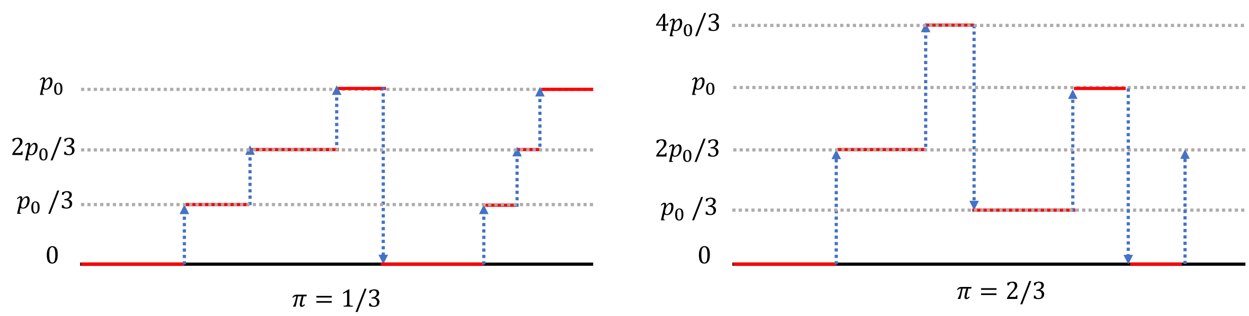

Figure 9: The dynamics of reward balance under CLPs

5 Implications of LPs as Financing Instruments

In this section, we derive the implications about LPs as financing instruments. As discussed in Section 4.1, the desirability of a LP depends on the discount at which the firm issues rewards, $1-\theta$, and the balance-to-flow ratio, B/A. The analysis centers around the two objects.

5.1 Comparison between CLPs and PLPs

Proposition 5. *Given the issuance rate, a CLP results in a greater reward balance than a PLP.*

In general, the reward balance consists of the half of p_0, due to consumers' unwillingness to redeem partially, and overaccumulation. Under a CLP, overaccumulation arises due to the asynchrony between the issuance and the redemption of rewards—a consumer consistently earns rewards, even during the period that he does not need a service. However, such asynchrony is missing under a PLP, because issuance and redemption are both linked to purchases of services. Opportunities to earn rewards and redeem rewards are synchronous and exclusive. This is why overaccumulation is zero when $1/\pi$ is an integer. A consumer with p_0 rewards will redeem rewards instead of earning more. When $1/\pi$ is not an integer, overaccumulation arises due to the lumpiness of reward earning and could reach $p_0\pi/2$ at maximum. Proposition 5 suggests that the asynchrony under a CLP is more powerful in generating overaccumulation than the lumpiness under a PLP.

Numerical analysis suggests that such superiority of CLPs over PLPs also holds when the firm adopts both types simultaneously. Suppose that the issuance rates under the CLP and the PLP are A_c and A_p respectively, where $A_p = \frac{p_0 q_0}{1/\pi+1}$. Similar to Lemma 3, the steady-state distribution of

A_p	200	199	150	120	100	80	50	0
A_c	0	1	50	80	100	120	150	200
B	150	180	184	186.6	188	191	194	200

Table 1: The reward balance when both types are adopted

This table presents the reward balance predicted by the model when both types are adopted. A_c and A_p are the issuance rates under the CLP and the PLP respectively. For all calculation, $p_0 = 300$, $q_0 = 4$, and the total issuance rate is set to 200.

consumers' reward holdings satisfies the equation system

$$\gamma'(N) = \begin{cases} -\gamma(N)\frac{q_0}{A_c} + \gamma(N+p_0)\frac{q_0}{A_c}, & \text{if } N \in [0, p_0\pi), \\ -\gamma(N)\frac{q_0}{A_c} + \gamma(N+p_0)\frac{q_0}{A_c} + \gamma(N-p_0\pi)\frac{q_0}{A_c}, & \text{if } N \in [p_0\pi, p_0+p_0\pi), \\ -\gamma(N)\frac{q_0}{A_c} + \gamma(N+p_0)\frac{q_0}{A_c}, & \text{if } N \in [p_0+p_0\pi, +\infty), \end{cases}$$

and three boundary conditions in Lemma 3. We numerically calculate the steady-state distribution and the reward balance. We set p_0 to 300, q_0 to 4, and the total issuance rate to 200. Table 1 presents the reward balance for different combinations of A_c and A_p. Clearly, given the total issuance rate, as more rewards are issued through the CLP and fewer through the PLP, the reward balance increases.

Combining Proposition 5 and the numerical analysis, we obtain a general implication regarding reward issuance. Redemption occurs only when a consumer purchases a service, so it is naturally tied to the firm's business. A weaker linkage between reward issuance and the firm's business will render issuance less synchronous with redemption. Therefore, to more effectively raise money through the LP (i.e., a high balance-to-flow ratio), the firm should issue rewards in ways loosely related or even unrelated to the firm's business. This implication is consistent with the second stylized fact documented in Section 2 that in practice, LPs issue rewards through consumption unrelated to firms' business.

5.2 Comparison across firms

In this subsection, we intends to assess how the desirability of LPs as financing instruments depends on the firm's characteristics. In the model, the firm's characteristics are reflected by the value of a service p_0 and the frequency of the need for a service q_0. To control for the size of the firm's business, we fix the demand for the service p_0q_0 and vary p_0 (q_0). Intuitively, we are comparing firms providing high-value, low-frequency services with those providing low-value, high-frequency services.

Proposition 6. *Given the demand for the service and the issuance rate, the reward balance is greater under both CLPs and PLPs if p_0 is higher and q_0 is lower.*

Under a CLP, the overaccumulation λ is determined by equation (10). It is decreasing in p_0, as each redemption reduces the balance by p_0. It is decreasing in q_0, as less frequent demand means that the likelihood that a consumer does not need a service for a long time is greater. When p_0q_0 is held fixed, a higher p_0 is associated with a lower q_0, and the impact of q_0 turns out to be dominant. To see this, let $\hat{\lambda} \triangleq \lambda / p_0$. Then $\overline{\lambda}$ satisfies

$$\left(1 - e^{-1/\hat{\lambda}}\right) \cdot \hat{\lambda} = \frac{A}{p_0 q_0},$$

and is thus fixed given p_0q_0 and A. The reward balance is

$$B = \left(\frac{1}{2} + \hat{\lambda}\right) p_0,$$

which is greater if p_0 is higher and q_0 is lower.

Under a PLP, the overaccumulation derives from the lumpiness of reward earning, which is $p_0\pi$. It is not hard to imagine that the higher $p_0\pi$ is, the stronger the overaccumulation is. To see it concretely, by $A = \frac{p_0 q_0}{1/\pi + 1}$, π is fixed given p_0q_0 and A. So is μ, as it is completely determined by π. The reward balance is

$$B = \frac{\mu - 1}{2\mu} (1 + \pi) p_0$$

which is greater if p_0 is higher and q_0 is lower.

In fact, Proposition 6 is well consistent with empirical observations across different industries. Table 2 shows the balances and flows of LPs for firms in different industries. Marriott and Delta provides upscale travel services and have the highest balance-to-flow ratios. Macy's, Target, GAP and ULTA Beauty retail middle-end goods and have lower but still sizable balance-to-flow ratios. TJX is an off-price retailer, and Starbucks mainly sells coffee and bakery, which features low-value and high-frequency. No wonder they have very low balance-to-flow ratios.

Proposition 7. *Given the demand for the service and the issuance rate, if η is fixed and ε is concave in p_0, the consumer constraint is more relaxed if p_0 is higher and q_0 is lower.*

The firm would like the discount of rewards to be lower, but it is subject the consumer constraint (CC), which implies that the discount cannot be lower than

$$\frac{\varepsilon}{p_0} + \frac{\eta}{A}.$$

To compare the discount of rewards across firms, we need to take a stance on how the participation

Firms in 2019	B (deferred revenue*)	A (deferral of revenue*)	B/A (balance-to-flow ratios)
Marriott	5718	2468	2.32
Delta	6728	3156	2.13
Macy's	839	554	1.51
Target	935	729	1.28
GAP	226	187	1.21
ULTA Beauty	230	206.7	1.11
TJX	501	1690	0.30
Starbucks	1114	10984	0.10

*in million dollars

Table 2: The balances and flows of LPs

cost and the redemption cost vary with the firm's characteristics. Unfortunately, we do not have good observations on these costs, and they certainly depend on the details of the service. A conjecture that is reasonable in general would be that given the demand for the service, the participation cost is fixed, and the redemption cost is concave in p_0. The concavity can result from common cost structures in economics. For example, the redemption cost consists of a fixed part and a variable part that is proportional to the value of a service. Under this conjecture, the consumer constraint is more relaxed if p_0 is higher and q_0 is lower.

Propositions 6 and 7 suggest that firms supplying high-value, low-frequency services can leverage LPs more effectively for financing. Firms with this feature can borrow more from consumers without compromising the convenience of rewards or incurring more use costs. In practice, airlines, hotels, department stores provide high-value, low-frequency services. Consistent with the model prediction, they are the most active in developing LPs and expanding the issuance of rewards.

5.3 The design of LPs

In the final part of this section, we shed light on the design of LPs as financing instruments. In practice, the design of LPs may involve other motives such as price discrimination and competition. Instead of rationalizing the LPs observed in practice as optimal financing instruments, here we just want to highlight potential tradeoffs related to the finance motive. For convenience and without much loss of generality, we focus on the case that π is an irrational number under a PLP.

According to equation (3), the firm's cash flows are

$$r_0 B\theta - A(1-\theta) + CF_0.$$

For both types of LPs, B is not affected by θ. Hence, given A, the firm's cash flows are increasing in θ. The firm would like to choose θ such that the consumer constraint binds:

$$\theta = 1 - \frac{\varepsilon}{p_0} - \frac{\eta}{A}.$$

With the binding consumer constraint, the firm's cash flows are a function of A as follows

$$\begin{aligned} CF(A) \triangleq\ & r_0 \underbrace{\left(1 - \frac{\varepsilon}{p_0} - \frac{\eta}{A}\right)}_{\text{the unit price of rewards}} \times \underbrace{B}_{\text{the reward balance}} \\ & - \underbrace{\left(\frac{A}{p_0}\varepsilon + \eta\right)}_{\text{the compensation to consumers for inconvenience}} + CF_0. \end{aligned} \tag{12}$$

Proposition 8. *With respect to the issuance rate A,*

- *the unit price of rewards $1 - \frac{\varepsilon}{p_0} - \frac{\eta}{A}$ is increasing and concave,*
- *the reward balance $\frac{p_0}{2} + \lambda$ and $\frac{p_0}{2}(1+\pi)$ are increasing and convex, and*
- *the compensation to consumers for the use costs $\frac{A}{p_0}\varepsilon + \eta$ is increasing and linear.*

As stated in Proposition 8, the issuance rate A affects the firm's cash flows through three channels. First, a higher A allows the firm to set a higher unit price of rewards. The discount of rewards partially derives from the compensation to consumers for the participation cost, which is independent of the amount of redemption. If more rewards are issued and used, the amount of compensation that each unit of reward needs to contain is smaller, so the unit price is higher. Second, a higher A translates into more redemptions of services and thus requires a higher compensation to consumers for the redemption cost. Third, a higher A results in a higher reward balance.

The strengths of the three effects vary in different regions of the issuance rate. Lemma 8 demonstrates the concavity of the three terms in equation (12) with respect to A. Since the unit price of rewards involves the inverse of A, it is extremely sensitive to A when A is close to 0, and its sensitivity diminishes rapidly as A increases. On the contrary, the reward balance becomes more and more sensitive to A as A increases. Since the redemption cost is constant for each redemption, the compensation to consumers for the use costs is linear in the number of redemptions and thus linear in A. By and large, when A is small, the first effect dominates; and when A is large, the second effect dominates. When A is in an intermediate range, the dominant effect is ambiguous.

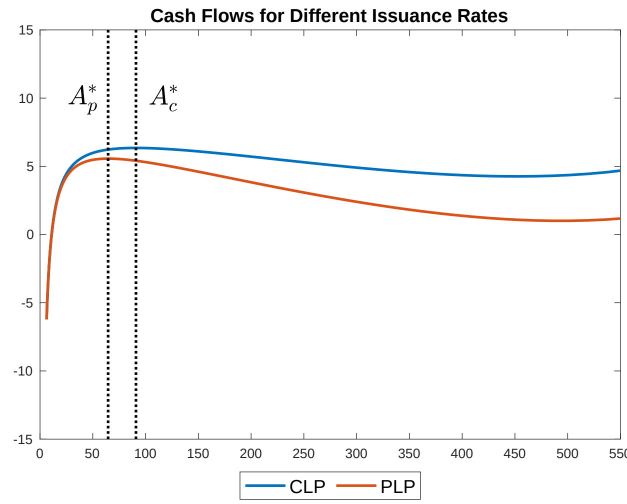

Figure 10: The firm's cash flows for different issuance rates
This figure presents the firm's cash flows for different issuance rates when $r_0 = 0.1$, $p_0 = 300$, $q_0 = 4$, $\eta = 6$, $\varepsilon = 10.5$.

6 The Cyclical Nature of LPs

In this section, we examine the cyclical nature of LPs and its impact on the firm's financial resilience. We refer to the state in the baseline setup as the normal state. The economy may enter an adverse state in which consumers' demand for the service drops dramatically. The firm's cash flows cannot cover its expenses, so it has to expend liquidity buffers to maintain the business. If the liquidity buffers are depleted before the consumers' demand returns to the normal level, the firm will go bankrupt and shut down the business. Since bankruptcy is value-destroying, the firm's financial resilience in the adverse state is valuable. In practice, the firm may raise funds from outside investors to obtain additional liquidity. But such emergency financing is far from perfect, especially in adverse states of the economy, due to various financial frictions. This leaves the room for LPs to make improvement. We will illustrate that LPs can improve the firm's financial resilience.

6.1 The adverse state

To consider the possibility of bankruptcy, we need more details about the firm's balance sheet and operation in the normal state. To provide services, the firm needs to hold an illiquid asset and operate it at a flow cost of O_0. In practice, the flow cost contains depreciation of equipment, employees' compensation, and any other operating expense. Purchasing and deploying the illiquid asset incurs a one-time cost I_0 to the firm. Since the variable cost of providing one service must be smaller than consumers' reservation value p_0, it does not affect the firm's decision in the model.

Hence, we assume it is zero. The firm also holds liquidity buffers that are worth L_0 and earns interest at the rate of r_0. Liquidity buffers are used for daily operation as well as to help withstand temporary operating loss in adverse states. The firm raises funds through debt and potentially through LPs. We assume zero equity for simplicity. Debt requires a coupon rate of r_0.[12] I_0, O_0, L_0, r_0, m_0 are exogenously given. Since the financial cost of the liquidity buffers equal their interest income, the operating expense in the baseline setup equals the operating and financing cost of the illiquid asset:

$$OE_0 = O_0 + r_0 I_0.$$

The firm's cash flows are positive in the normal state. It pays out the cash flows as dividend and maintain the balance sheet.

Suppose that the economy is in the normal state, and the firm is in the steady state, whether it adopts a LP. At $t = 0$, the economy is hit by an unexpected shock, for example Covid-19, and enters an adverse state. A fraction σ of consumers are affected by the shock: they do not need the service, and their consumption in all categories decreases to δC_0, where $\delta > 0$. Other consumers remain unchanged. I refer to σ as the magnitude of the shock. The assumption about the adverse state implies that the firm's business is procyclical: the demand for the service decreases by a fraction of σ, while the aggregate consumption decreases by a fraction of $\sigma(1-\delta)$. Higher σ and δ imply higher procyclicality of the firm's business. The probability that the adverse state last longer than t units of time is $\Phi(t)$, where $\Phi(t)$ is strictly decreasing in t and converges to 0. Hence, the economy will return to the normal state at some points almost surely.

The shock σ is so large that the firm's cash flows are negative in all cases we consider. It enters the adverse state with the liquidity buffers L_0 and goes bankrupt if the buffers are depleted. To avoid costly bankruptcy, the firm pays no dividend and rollovers all debt. To manifest the impact of the LP and simplify the illustration, we assume that the firm can respond to the shock only in a limited way. It cannot change operating expense or the design of the LP, which we refer to as operational inflexibility. It can roll over existing debt but cannot issue new debt, which we refer to as financial inflexibility.

Consumers' behavior does not change and still follows the characterization in Proposition 2. This assumption is consistent the observation that during the Covid pandemic, although consumers did not fly much with airlines and airlines suffered tremendous losses, their aggregate spend on airlines' CCCs declined mildly. There are two potential reasons why consumers do not change their behavior systematically. First, consumers are not concerned about the firm's bankruptcy that much. Airline bankruptcies are common in history. Big bankrupt airlines usually emerged from Chapter 11 and kept their LPs intact so that LP members would like to continue flying with the

[12] We assume equal interest rates on liquidity buffers and debt purely for simplicity. All results remain if they are different.

new firm. Second, consumers regard the adverse state as temporary and expect their demand for flying to bounce back.

6.2 The impact of the LP on the bankruptcy probability

We will show that the LP makes the firm's cash flows less procyclical and thus makes the firm more financially resilient. The formal result is stated as follows.

Proposition 9. *If adopting an LP leads to weakly higher cash flows than adopting no LP in the normal state, it also leads to a strictly lower bankruptcy probability than adopting no LP in the adverse state.*

We first analyze the case where the firm adopts no LP. The firm borrows $I_0 + L_0$ debt, and the debt coupon is

$$r_0 (I_0 + L_0).$$

In the adverse state, the firm's cash flows from sales dwindle to

$$(1 - \sigma) p_0 q_0.$$

The firm's negative cash flows translate into decreases in the liquidity buffer. Denote the liquidity buffer at t by $L_0(t;\sigma)$. Then its dynamics are characterized by the following differential equation:

$$\frac{dL_0(t;\sigma)}{dt} = (1 - \sigma) p_0 q_0 - O_0 - r_0 (I_0 + L_0) + r_0 L_0 (t;\sigma),$$

where $r_0 L_0(t;\sigma)$ is the interest income on the liquidity buffer at t. With the boundary condition $L_0(0;\sigma) = L_0$, we obtain the liquidity buffer at t as follows:

$$L_0(t;\sigma) = L_0 e^{r_0 t} + \frac{e^{r_0 t} - 1}{r_0} \left[(1 - \sigma) p_0 q_0 - O_0 - r_0 (I_0 + L_0)\right].$$

We then analyze the case where the firm adopts a LP with the consumer constraint (CC) being satisfied. To incorporate all cases in one framework, we consider that the firm adopts a CLP and a PLP simultaneously. Their issuance rates are A_c and A_p respectively, the unit price of rewards is θ, and the combined reward balance is B. When either of A_c and A_p is zero, the LP is actually a pure CLP or a pure PLP.

The firm obtain $B\theta$ through the LP and borrows $I_0 + L_0 - B\theta$ debt, so the debt coupon is

$$r_0 (I_0 + L_0 - B\theta).$$

In the adverse state, the firm's cash flows from sales dwindle to

$$(1-\sigma)\left[p_0q_0-(A_c+A_p)(1-\theta)\right]+\sigma\delta A_c\theta.$$

Only the consumers not affected by the shock buy services and contribute the cash flows equal to the first term. The consumers affected by the shock still use the CCC and earn rewards at the rate of δA_c, which generates cash flows equal to the second term. Denote the liquidity buffer at t by $L(t;\sigma)$. Then its dynamics are characterized by the following differential equation:

$$\begin{aligned}\frac{dL(t;\sigma)}{dt}=&(1-\sigma)\left[p_0q_0-(A_c+A_p)(1-\theta)\right]+\sigma\delta A_c\theta\\&-O_0-r_0(I_0+L_0-B\theta)+r_0L(t;\sigma),\end{aligned}$$

We then obtain the liquidity buffer at t as follows:

$$L(t;\sigma)=L_0(t;\sigma)+\frac{e^{r_0t}-1}{r_0}\left[r_0B\theta-(1-\sigma)(A_c+A_p)(1-\theta)+\sigma\delta A_c\theta\right].$$

Note that the difference in the cash flows between adopting the LP and not in the normal state is

$$CF-CF_0=r_0B\theta-(A_c+A_p)(1-\theta).$$

We can write the liquidity buffer at t as

$$\begin{aligned}L(t;\sigma)=&\frac{e^{r_0t}-1}{r_0}\sigma(A_c+A_p)(1-\theta)+\frac{e^{r_0t}-1}{r_0}\sigma\delta A_c\theta\\&+\frac{e^{r_0t}-1}{r_0}(CF-CF_0)+L_0(t;\sigma).\end{aligned}\tag{13}$$

The first two terms in the right-hand side of equation (13) reflect the two mechanisms affecting the cyclical nature of LP financing. The first term reflects the procyclicality of LPs' financing cost, which applies to both types of LPs. LP financing is costly because consumers use rewards to buy services instead of cash, and rewards are issued at a discount. Given the unit price of rewards, the total discount on rewards that consumers actually enjoy is proportional to the redemption rate of rewards, which is lower in the adverse state due to the lower demand for services. An alternative way to view this is that a LP is de facto a revenue sharing arrangement—a fixed fraction of revenue is shared with consumers in the form of discounts on rewards. When revenue drops, so does the shared revenue.

The second term is unique to the CLP and stems from that the firm raises more funds from consumers in the adverse state. Under the assumption that consumers do not change their behavior, the issuance of rewards is always linked to consumers' aggregate consumption while the redemption is linked to their demand for services. Then for procyclical firms analyzed here, reward issuance is less procyclical than reward redemption, and the former becomes higher than the latter in the adverse state. Such divergence in the two rates results in the firm raising more funds from consumers, which is reflected by the reward balance increasing at the rate of $\sigma\delta A_c$. This mechanism is consistent with the third stylized fact documented in Section 2 that in practice, LPs generate counter-cyclical cash flows.

If the difference in the cash flows in the normal state is nonnegative, i.e., $CF - CF_0 \geq 0$, the liquidity buffer under the LP is strictly higher than that under no LP. This implies that adopting the LP leads to a strictly lower bankruptcy probability, so we obtain Proposition 9. This cyclical nature of LP financing is especially attractive to highly procyclical firms such as airlines, hotels, upscale department stores, and durable goods retailers. Airlines feature heavy assets, high financial expenses, and low profit margins. Highly procyclical business plus financial weakness makes airlines particularly eager to utilize LP financing.

Proposition 10. *Given the total issuance rate $A_c + A_p$, the higher the issuance rate under the CLP A_c is, the less sensitive the liquidity buffer is to the magnitude of the shock; i.e.,*

$$\frac{\partial^2 L(t;\sigma)}{\partial\sigma\partial A_c} > \frac{\partial^2 L(t;\sigma)}{\partial\sigma\partial A_p}$$

Proposition 10 suggests that given the total issuance rate, more issuance through the CLP strengthens the cyclical nature. The reason is that the second mechanism is unique to the CLP. The issuance under the CLP is not linked to purchases, while the issuance under the PLP is. As a result, the former does not decrease with the demand for the service, while the latter does. A more general implication is that to strengthen the cyclical nature of a LP, the firm should weaken the linkage between reward issuance and the firm's business. This implication is consistent with the second stylized fact documented in Section 2 that in practice, rewards are also issued through consumption unrelated to firms' business.

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Appendix

A Tables & Figures

Table 3: Data Collection

This table summarizes how we construct loyalty program liabilities and revenue for firms in the airline industry and the hotel industry.

(1) Brand	(2) Loyalty Program Liabilities	(3) Revenue
American Airlines	See notes to consolidated financial statements	Total Revenue on the Consolidated Statements of Operations
Delta Airlines	See the Loyalty program section	Total Revenue on the Consolidated Statements of Operations
United Airlines	On the consolidated balance sheets	Total Revenue on the Consolidated Statements of Operations
Southwest Airlines	See the revenue section	Total Revenue on the Consolidated Statements of Operations
Alaska Airlines	See the mileage plan liabilities section	Total Revenue on the Consolidated Statements of Operations
Jetblue Airlines	See the Contract Liabilities section	Total Revenue on the Consolidated Statements of Operations
Hawaiian Airlines	See notes to consolidated financial statements	Total Revenue on the Consolidated Statements of Operations
Spirit Airlines	See notes to consolidated financial statements	Total Revenue on the Consolidated Statements of Operations
Marriott	See Accounting for the Loyalty Program section	Total Revenue on the Consolidated Statements of Operations - reimbursement costs
Hilton	See Accounting for the Loyalty Program section	Total Revenue on the Consolidated Statements of Operations - reimbursement costs
IHG	See Breakage assumption used to estimate IHG One Rewards deferred revenue section	Total Revenue on the Consolidated Statements of Operations - reimbursement costs
Hyatt	See the Loyalty Program Future Redemption Obligation and Revenue Recognition section	Total Revenue on the Consolidated Statements of Operations - reimbursement costs
Wyndham	"The recorded liability related to the program" under the loyalty program section + "Deferred Loyalty program revenues" under the Deferred revenues section	Total Revenue on the Consolidated Statements of Operations - reimbursement costs
Choice	See Accounting for Choice Privileges Loyalty Program section, point liability + deferred revenue	Total Revenue on the Consolidated Statements of Operations

Table 4: The regression results about LP liabilities

	loyalty / revenue		Change in loyalty / revenue from 2019 to 2020			
	(1)	(2)	(3)	(4)	(5)	(6)
	Airlines	Hotels	Airlines	Airlines	Hotels	Hotels
Varieties of CCCs	0.018**	0.186**				
	(0.007)	(0.044)				
Large_LP			0.021***	0.014***	0.111***	0.096**
			(0.006)	(0.004)	(0.014)	(0.022)
Change in loyalty / revenue				1.727***		0.590
from 2018 to 2019				(0.315)		(0.669)
Constant	0.041	-0.160	0.003	-0.002	0.002	-0.006
	(0.029)	(0.155)	(0.005)	(0.003)	(0.008)	(0.012)
Observations	12	6	12	11	6	6
R-squared	0.411	0.819	0.546	0.896	0.942	0.954

B Proofs

Proof of Lemma 1

Suppose that a consumer does not follow the strategy in Lemma 1 and use a combination of cash and rewards with positive probability. That implies $y > 0$ and $x < p_0$. If $x \leq \varepsilon$, then $y(x-\varepsilon) - z\theta - \eta < 0$, so the consumer is better off if he does not join the LP and buy a service with only cash. If $x > \varepsilon$, consider an alternative strategy that he still receives rewards at the rate of z, and when needing a service, he buys it with only rewards if he has at least p_0 rewards and with only cash otherwise. Under this strategy, he redeems p_0 rewards for a service at the rate of z/p_0. Then his average payoff flow is

$$\frac{z}{p_0}(p_0 - \varepsilon) - z\theta - \eta.$$

By $\varepsilon < x < p_0$ and $xy \leq z$,

$$\frac{z}{p_0}(p_0 - \varepsilon) - z\theta - \eta > \frac{z}{x}(x - \varepsilon) - z\theta - \eta \geq y(x - \varepsilon) - z\theta - \eta.$$

Hence, the consumer is better off by using the alternative strategy under which he buys a service with either only cash or only rewards.

Proof of Lemma 3

Denote the probability that a consumer needs exactly one service over t units of time as $P_1(t)$ and the probability that a consumer needs at least one service over t units of time as $P_0(t)$. We first show that $P_1(t) = q_0 t e^{-q_0 t}$ and $P_0(t) = 1 - e^{-q_0 t}$.

$P_0(t)$ satisfies

$$\begin{aligned} P_0(t+dt) &= P_0(t) + (1 - P_0(t)) P_0(dt) \\ \Rightarrow -\frac{d\ln(1 - P_0(t))}{dt} &= \frac{P_0(dt)}{dt} = P_0'(0) = q_0. \end{aligned}$$

Combining with the boundary condition $P_0(0) = 0$, we obtain $P_0(t) = 1 - e^{-q_0 t}$.

$P_1(t)$ satisfies

$$\begin{aligned}
P_1(t+dt) &= P_1(t)(1-P_0(dt)) + (1-P_0(t))P_1(dt) \\
\Rightarrow P_1(t+dt) - P_1(t) &= -P_1(t)q_0dt + e^{-q_0t}P_1(dt) \\
\Rightarrow \frac{dP_1(t)}{dt} &= -P_1(t)q_0 + e^{-q_0t}q_0 \\
\Rightarrow \frac{de^{q_0t}P_1(t)}{dt} &= q_0
\end{aligned}$$

Combining with the boundary condition $P_1(0) = 0$, we obtain $P_1(t) = q_0te^{-q_0t}$.

Next, we show that the mass of the third group is of a higher order than τ. The probability that a consumer needs more than one service over t units of time is

$$\begin{aligned}
P_0(t) - P_1(t) &= 1 - e^{-q_0t} - q_0te^{-q_0t} \\
&= 1 - (1+q_0t)\left[1 - q_0t + \sum_{k=2}^{+\infty}\frac{1}{k!}(-q_0t)^k\right] \\
&= (q_0t)^2 - (1+q_0t)\sum_{k=2}^{+\infty}\frac{1}{k!}(-q_0t)^k.
\end{aligned}$$

The mass of the third group is smaller than $P_0(t) - P_1(t)$.

Proof of Proposition 3

The equation system in Lemma 3 is an advance differential equation system. We conjecture

$$\gamma(N) = e^{-N/\lambda}\gamma_0 \tag{14}$$

for $N \in (p_0, +\infty)$, where $\lambda > 0$. Then according to equation (5), we have

$$\begin{aligned}
-\frac{1}{\lambda}e^{-N/\lambda}\gamma_0 + e^{-N/\lambda}\gamma_0\frac{q_0}{A} - e^{-N/\lambda}e^{-p_0/\lambda}\gamma_0\frac{q_0}{A} &= 0 \\
\Leftrightarrow \left(1 - e^{-p_0/\lambda}\right)\cdot\lambda &= \frac{A}{q_0}.
\end{aligned}$$

Plugging the conjectured distribution (14) into equation (5), we obtain that for $N \in [0, p_0)$,

$$\begin{aligned}
\gamma(N) = \int_0^N \gamma(x+p_0)\frac{A}{q_0}dx + \gamma(0) &= \int_0^N e^{-x/\lambda}e^{-p_0/\lambda}\cdot\gamma_0\frac{q_0}{A}dx \\
&= \lambda e^{-p_0/\lambda}\cdot\gamma_0\frac{q_0}{A}\cdot\left(1 - e^{-N/\lambda}\right).
\end{aligned}$$

Note that the boundary condition (6) naturally holds:

$$\lim_{N \uparrow p_0} \gamma(N) = \lambda e^{-p_0/\lambda} \cdot \gamma_0 \frac{q_0}{A} \cdot \left(1 - e^{-p_0/\lambda}\right) = e^{-p_0/\lambda} \cdot \gamma_0 = \gamma(p_0).$$

The boundary conditions (7) and (8) require

$$\int_0^{+\infty} \gamma(N)\, dN = 1.$$

Since

$$\begin{aligned}
\int_0^{+\infty} \gamma(N)\, dN =& \lambda e^{-p_0/\lambda} \cdot \gamma_0 \frac{q_0}{A} \cdot \int_0^{p_0} \left(1 - e^{-N/\lambda}\right) dN + \gamma_0 \int_{p_0}^{+\infty} e^{-N/\lambda} dN \\
=& \lambda e^{-p_0/\lambda} \cdot \gamma_0 \frac{q_0}{A} \cdot \left(p_0 + \lambda e^{-p_0/\lambda} - \lambda\right) + \gamma_0 \lambda e^{-p_0/\lambda} \\
=& \lambda e^{-p_0/\lambda} \cdot \gamma_0 \frac{q_0}{A} \cdot \left(p_0 - \frac{A}{q_0}\right) + \gamma_0 \lambda e^{-p_0/\lambda} \\
=& \gamma_0 \lambda e^{-p_0/\lambda} \frac{p_0 q_0}{A} = \gamma_0 p_0 \left(\lambda \frac{q_0}{A} - 1\right)
\end{aligned}$$

So,

$$\gamma_0 = \frac{1}{p_0 \left(\lambda \frac{q_0}{A} - 1\right)}.$$

And for $N \in [0, p_0)$,

$$\begin{aligned}
\gamma(N) &= \lambda e^{-p_0/\lambda} \cdot \gamma_0 \frac{q_0}{A} \cdot \left(1 - e^{-N/\lambda}\right) \\
&= \left(\lambda \frac{q_0}{A} - 1\right) \cdot \frac{1}{p_0 \left(\lambda \frac{q_0}{A} - 1\right)} \left(1 - e^{-N/\lambda}\right) \\
&= \frac{1}{p_0} \left(1 - e^{-N/\lambda}\right).
\end{aligned}$$

Next, we show that the equation

$$\left(1 - e^{-p_0/\lambda}\right) \cdot \lambda = \frac{A}{q_0}$$

has a unique positive root λ. First, since

$$\lim_{\lambda \to 0} \left(1 - e^{-p_0/\lambda}\right) \cdot \lambda = 0 < \frac{A}{q_0}$$

and

$$\lim_{\lambda\to+\infty}\left(1-e^{-p_0/\lambda}\right)\cdot\lambda=\lim_{\lambda\to+\infty}\frac{1-e^{-p_0/\lambda}}{p_0/\lambda}\cdot p_0=\lim_{x\to 0}e^{-x}\cdot p_0=p_0>\frac{A}{q_0},$$

the equation must have positive roots. Second, since

$$\frac{d\left[1-e^{-x}-e^{-x}x\right]}{dx}=e^{-x}x>0,$$

we have for $\lambda>0$,

$$\frac{d\left[\left(1-e^{-p_0/\lambda}\right)\cdot\lambda\right]}{d\lambda}=1-e^{-p_0/\lambda}-e^{-p_0/\lambda}\frac{p_0}{\lambda}>1-e^{-0}-e^{-0}\cdot 0=0.$$

This implies that $\left(1-e^{-p_0/\lambda}\right)\cdot\lambda$ is strictly increasing in λ for $\lambda>0$. Therefore, the equation has a unique positive root.

Finally, we derive the reward balance.

$$\begin{aligned}
B&=\int_0^{+\infty}\gamma(N)NdN\\
&=\frac{1}{p_0}\cdot\int_0^{p_0}\left(1-e^{-N/\lambda}\right)NdN+\frac{1}{p_0\left(\lambda\frac{q_0}{A}-1\right)}\cdot\int_{p_0}^{+\infty}e^{-N/\lambda}NdN\\
&=\frac{1}{p_0}\cdot\frac{1}{2}p_0^2-\frac{1}{p_0}\cdot\int_0^{p_0}e^{-N/\lambda}NdN+\frac{1}{p_0\left(\lambda\frac{q_0}{A}-1\right)}\cdot\int_{p_0}^{+\infty}e^{-N/\lambda}NdN\\
&=\frac{p_0}{2}-\frac{1}{p_0}\frac{\lambda\frac{q_0}{A}}{\lambda\frac{q_0}{A}-1}\cdot\int_0^{p_0}e^{-N/\lambda}NdN+\frac{1}{p_0\left(\lambda\frac{q_0}{A}-1\right)}\cdot\int_0^{+\infty}e^{-N/\lambda}NdN\\
&=\frac{p_0}{2}-\frac{1}{p_0}\frac{\lambda\frac{q_0}{A}}{\lambda\frac{q_0}{A}-1}\cdot\lambda^2\left[e^{-p_0/\lambda}\left(-\frac{p_0}{\lambda}-1\right)+1\right]+\frac{1}{p_0\left(\lambda\frac{q_0}{A}-1\right)}\cdot\lambda^2\\
&=\frac{p_0}{2}+\frac{1}{p_0}\frac{1}{\lambda\frac{q_0}{A}-1}\cdot\lambda^2\left[e^{-p_0/\lambda}\left(\frac{p_0}{\lambda}+1\right)\lambda\frac{q_0}{A}-\lambda\frac{q_0}{A}+1\right].
\end{aligned}$$

By $\left(1-e^{-p_0/\lambda}\right)\cdot\lambda=\frac{A}{q_0}$, we have

$$\begin{aligned}
B&=\frac{p_0}{2}+\frac{1}{p_0}\frac{1}{\lambda\frac{q_0}{A}-1}\cdot\lambda^2\left[\left(\lambda\frac{q_0}{A}-1\right)\left(\frac{p_0}{\lambda}+1\right)-\lambda\frac{q_0}{A}+1\right]\\
&=\frac{p_0}{2}+\frac{1}{p_0}\frac{1}{\lambda\frac{q_0}{A}-1}\cdot\lambda^2\left(\lambda\frac{q_0}{A}-1\right)\frac{p_0}{\lambda}\\
&=\frac{p_0}{2}+\lambda.
\end{aligned}$$

Proof of Proposition 5

Under a PLP with π, the issuance rate is

$$A = \frac{p_0 q_0}{1/\pi + 1},$$

and the reward balance is no greater than

$$\frac{p_0}{2} + \frac{p_0 \pi}{2}.$$

On the other hand, for the same issuance rate, the reward balance under a CLP is

$$\frac{p_0}{2} + \lambda,$$

where

$$\left(1 - e^{-p_0/\lambda}\right) \cdot \lambda = \frac{A}{q_0}.$$

It is easy to see

$$\lambda > \frac{A}{q_0} = \frac{p_0 \pi}{1+\pi} \geq \frac{p_0}{2}\pi.$$

The last inequality follows $\pi \leq 1$.

Proof of Proposition 8

The concavity of the unit price of rewards and the linearity of the compensation to consumers for the use costs are obvious. For the reward balance under a CLP,

$$\frac{d\lambda}{dA} = \frac{1}{q_0} \frac{1}{1 - e^{-p_0/\lambda} - p_0/\lambda \cdot e^{-p_0/\lambda}} > 0,$$

$$\frac{d^2\lambda}{dA^2} = \frac{1}{q_0} \frac{p_0/\lambda \cdot e^{-p_0/\lambda}\left(p_0/\lambda^2\right)}{\left(1 - e^{-p_0/\lambda} - p_0/\lambda \cdot e^{-p_0/\lambda}\right)^2} \frac{d\lambda}{dA} > 0.$$

For the reward balance under a PLP,

$$\frac{d\pi}{dA} = \frac{p_0 q_0}{\left(p_0 q_0 - A\right)^2} > 0,$$

$$\frac{d^2\pi}{dA^2} = 2\frac{p_0 q_0}{\left(p_0 q_0 - A\right)^3} > 0.$$

Proof of Proposition 10

The liquidity buffer at t is decreasing in the magnitude of the shock σ:

$$\frac{\partial L(t;\sigma)}{\partial \sigma} = \frac{e^{r_0 t}-1}{r_0}(A_c + A_p)(1-\theta) + \frac{e^{r_0 t}-1}{r_0}\delta A_c \theta - \frac{e^{r_0 t}-1}{r_0} p_0 q_0 < 0.$$

Since

$$\frac{\partial^2 L(t;\sigma)}{\partial \sigma \partial A_c} = \frac{e^{r_0 t}-1}{r_0}(1-\theta) + \frac{e^{r_0 t}-1}{r_0}\delta\theta > \frac{\partial^2 L(t;\sigma)}{\partial \sigma \partial A_p} = \frac{e^{r_0 t}-1}{r_0}(1-\theta),$$

an increase in A_c weakens the sensitivity of the liquidity buffer to the magnitude of the shock more than an increase in A_p. The difference $\frac{e^{r_0 t}-1}{r_0}\delta\theta$ stems from the second mechanism.