Erratum

Product Market Competition, Insider Trading and Stock Market Efficiency

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Abstract

There is an error in my 2010 paper “Product Market Competition, Insider Trading and Stock Market Efficiency”. This note shows how to correct it by adjusting the hypotheses of the model, specifically, by assuming that investors learn about the stock’s mean payoff rather than about its realization. All the predictions of the model remain valid, except for the second part of Proposition 5 regarding the distribution of stock returns.

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There is an error in my paper “Product Market Competition, Insider Trading and Stock Market Efficiency” (Journal of Finance, 65(1) (2010)). The issue is easily remedied by adjusting the assumption on the signal structure. All propositions continue to hold, except for the second part of Proposition 5 regarding the indirect information effect of market power. The issue and remedy are identical to those in my paper “Wealth, Information Acquisition and Portfolio Choice” (Review of Financial Studies, 17(3) (2004)), which employs the same small risk approximation, and was later corrected (“Erratum: Wealth, Information Acquisition and Portfolio Choice”, Review of Financial Studies, 17(3) (2011)).\footnote{At the time I discovered the issue in my 2004 article, I thought it was confined to investors’ information choice (which it affected through the period-0 expected utility). I thank Sergei Glebkin for pointed out that the the issue affects also the period-1 portfolio choice.}

1 The problem

The paper resorts to an approximation to solve for the rational expectations equilibrium under CRRA utility. However, the proposed solution does not satisfy the approximation’s assumption. Specifically, the change in wealth under the equilibrium price in Proposition 1 is not of order $z$, implying that higher order terms cannot be neglected in the Taylor expansion of the expected utility. Indeed, consider the portfolio share in a generic stock given in Equation (5):

$$f_l = \frac{h_s}{\gamma(1-\omega)} \varepsilon_l - \theta + \frac{(1 - \omega)K_0^{1-\omega}}{w}.$$ 

In this equation, $\varepsilon_l$ and $\theta$ have variances of order $1/z$, specifically $1/(h_s z)$ and $\sigma^2_\theta/z$. This implies that the variance of $f_l$ is also of order $1/z$; i.e., the portfolio share grows larger as $z$ shrinks. The portfolio return, $r_{1z} = \ln \left[ R^f + \sum_{m=1}^{M} f^m_l (R^m - R^f) \right]$, contains terms of the type $f_l (R - R^f)$ (here, I dropped the superscript $m$ to simplify the notation), which in turn contain terms of the type $f_l \alpha z$, $f_l \beta z$, $f_l \theta z$, $f_l \varepsilon_l z$ and $r^f z$. The variance of
such terms is of order \((1/z) \times z = 1 = z^0\). As a result, the portfolio return, and hence the change in wealth, are of order 0 and do not converge to zero as \(z\) converges to zero.

2 A resolution

I offer a solution to the problem outlined above. We assume that the unconditional mean of the technology (aka productivity) shock is random and that investors learn about this mean rather than about the shock’s realization. Specifically, we assume that \(\ln(A) = az\) is normally distributed with mean \(bz\) and precision \(h_a/z\) (variance \(z/h_a\)), where \(b\) is normally distributed with mean 0 and precision \(h_b\) (variance \(1/h_b\)). Agents’ signals \(s_t\) are about \(b\) rather than about \(a\): \(s_t = b + \varepsilon_t\) where \(\varepsilon_t\) is independent of \(a, b, \theta\) and across agents and firms, and is normally distributed with mean zero and precision \(h_s\) (variance \(1/h_s\)). Because the signal is about \(b\), not \(a\), its variance is not scaled by \(z\). The aggregate demand for the stock emanating from noise traders as a fraction of investors’ wealth, denoted \(\theta\), (i.e. they demand \(\theta w/P\) shares where \(w\) is investors’ initial wealth) is normally distributed with mean 0 and precision \(h_\theta\) (variance \(1/h_\theta\)) and is independent from all other random variables and across stocks. Everything else, including the equilibrium concept, are as in the published article.

The equilibrium stock price resembles closely that given in Proposition 1 of the published article, but for two main differences: \(a\) and \(p_a\) are replaced with, respectively, \(b\) and \(p_b\), and \(h/h_a\) appears in several places.

**Proposition 1.** *(Equilibrium stock price)* There exists a log-linear rational expectations equilibrium characterized as follows.

Shares trade at a price \(P = (1 - \omega)K_0^{1-\omega} \exp(pz)\) where \(p = p_0(\omega) + p_b(\omega)b + p_\theta(\omega)\theta\),

\[
p_0(\omega) \equiv \frac{(1 - \omega)^2}{h_a} \left( \frac{1}{2} - \frac{\gamma(1 - \omega)K_0^{1-\omega}}{w} \right) - r^f, \tag{1}
\]

\[
p_b(\omega) \equiv (1 - \omega) \left( 1 - \frac{h_b}{h(\omega)} \right) \geq 0, \quad p_\theta(\omega) \equiv (1 - \omega) \frac{\gamma h}{h_a h_s} p_b(\omega), \tag{2}
\]
\[ h_p(\omega) \equiv \frac{h_a^2 h_s^2}{\gamma^2 (1 - \omega)^2 h^2 \sigma^2_\theta} \quad \text{and} \quad h(\omega) \equiv h_b + h_p(\omega) + h_s. \quad (3) \]

Investor \( l \) allocates a fraction \( f_l \) of her wealth to each stock such that

\[ f_l = \frac{h_s h_a}{\gamma (1 - \omega) h} \varepsilon_l - \theta + \frac{(1 - \omega) K_0^{1-\omega}}{w}. \quad (4) \]

All proofs are presented in the Appendix. The proof of Proposition 1 is similar to that in the paper. We conjecture that the equilibrium price is given by Equations 1 to 3, solve for optimal portfolios, determine the price that clears the stock market and confirm that the conjecture is valid. The main difference is that the variance of the technology shock, conditional on information (i.e., the equilibrium price and the private signal), is a function only of the prior precision of the shock, \( h_a \), at the order \( z \); it does not depend on the informativeness of the price. Indeed,

\[
V(\ln A | \mathcal{F}_i) = V(az | \mathcal{F}_i) = E_i(V(az | b, \mathcal{F}_i) | \mathcal{F}_i) + V(E_i(az | b, \mathcal{F}_i) | \mathcal{F}_i)
= E_i(V(az | b) | \mathcal{F}_i) + V(E(az | b) | \mathcal{F}_i)
= E\left(\frac{z}{h_a} \right) | \mathcal{F}_i) + V(bz | \mathcal{F}_i)
= \frac{z}{h_a} + o(z),
\]

because \( V(bz | \mathcal{F}_i) = z^2/h = o(z) \) is of order \( z^2 \), and thus negligible. This is the reason the term \( \frac{h}{h_a} \) enters the equations.

The issue with the approximation encountered in the published article does not arise here because the variance of portfolio shares in Equation 4 remains finite as \( z \) shrinks towards zero (the variances of \( \varepsilon_l \) and \( \theta \) are independent of \( z \)). As a result, portfolio returns, which involve the product of portfolio shares (of order 0) with stock returns (of order \( z \)), and hence the change in wealth converge to zero as \( z \) converges to zero.

Next, we list the propositions that characterize the effect of market power, \( \omega \), on various aspects of the firms. They all continue to hold, except for the second part of
Proposition 5 regarding the distribution of stock returns. We start with trading volume.

**Proposition 2. (Trading volume)** Trading volume is larger for firms with more market power.

The price provides a signal for the mean technology shock, \( b \), with error \( (1 - \omega)h_{\omega}\theta \). Its precision—price informativeness—equals \( h_p = \frac{h^2 h_a^2}{\gamma^2 (1 - \omega)^2 h^2 \sigma^2} \) and increases in market power \( \omega \), as the following proposition establishes.

**Proposition 3. (Informational efficiency)** Stock prices are more informative for firms with more market power.

We turn to the dispersion of investors’ forecasts of productivity, \( \text{Var} [E(a | \mathcal{F}_t) | b, \theta] \), of profit, \( \text{Var} [E(\pi | \mathcal{F}_t) | b, \theta] \) and of return, \( \text{Var} [E(r | \mathcal{F}_t) | b, \theta] \).

**Proposition 4. (Dispersion of investors’ forecasts)** Investors’ productivity, profit and return forecasts are less dispersed for firms with more market power.

The following proposition considers the impact of market power on the distribution of stock returns.

**Proposition 5. (Stock returns)** Firms enjoying more market power have less volatile returns, unconditionally and conditional on public information, lower expected returns and higher Sharpe ratios. They also have less volatile profits.

The expectation and variance of returns, the Sharpe ratio and the variance of profit are all reduced by market power, in line with risk; that’s the direct effect of market power. But contrary to what I wrote in the published paper, market power has no indirect effect operating through information. More precisely, this effect is negligible. This is the only result that fails to hold in the current setup. The next proposition establishes that the stock’s liquidity, represented by the coefficient \( p_\theta \), increases in market power.

**Proposition 6. (Liquidity)** Firms enjoying more market power have stock prices that are less sensitive to noise shocks.
We consider next the effect of market power on the efficiency of the capital allocation. We assume that the firm starts with $K_0$ units of capital and one share outstanding and issues $\alpha$ new shares (an arbitrary positive number), thus raising $K = \alpha P$ units of capital.

**Proposition 7.** *(Equilibrium when the firm raises capital)* Assume that the firm issues $\alpha$ new shares. There exists a log-linear rational expectations equilibrium characterized as follows.

Shares trade at a price $P = \overline{P} \exp(pz)$ such that $p = p_0(\omega, \delta) + p_b(\omega, \delta) b + p_\theta(\omega, \delta) \theta$,

$$
\overline{P}(\omega) = \left( \frac{K_0}{1 - \delta} \right)^{(1-\omega)} \left( (1 - \omega) - \delta \left( \frac{K_0}{1 - \delta} \right)^\omega \right),
$$

(5)

$$
p_0(\omega, \delta) \equiv \frac{1}{1 - \delta + \delta \omega} \left\{ \frac{(1 - \omega)^2}{h_a} \left( \frac{1}{2} - \frac{\gamma (1 - \omega) K_0^{(1-\omega)}}{w(1 - \delta)^{(1-\omega)}} \right) - rf \right\},
$$

(6)

$$
p_b(\omega, \delta) \equiv \frac{1 - \omega}{1 - \delta + \delta \omega} \left( 1 - \frac{h_b}{h(\omega)} \right) \geq 0, \quad p_\theta(\omega, \delta) \equiv (1 - \omega) \frac{\gamma h}{h_a h_s} p_b(\omega).
$$

(7)

$\delta(\omega) \equiv \alpha \overline{P}/(K_0 + \alpha \overline{P})$ is the dilution factor and $h(\omega)$ is defined in Proposition 1.
The firm raises $K = \alpha \overline{P} \exp(kz)$ units of capital where $k = p$.

The next proposition describes how the accuracy of information affects the economy’s allocative efficiency, holding fixed the degree of market power. Allocative efficiency is measured by the elasticity of investments to technology shocks, $\partial(\ln K)/\partial(\ln A)$, which equals $p_b(\omega, \delta)$ in equilibrium.

**Proposition 8.** *(Allocative efficiency & information)* Capital is more efficiently allocated when information is more accurate.

The proposition establishes that the elasticity of investments to technology shocks increases with the level of information, $h$, holding fixed the degree of market power, $\omega$. Next, we vary the degree of market power. To isolate the informational effect of market power on allocative efficiency, we analyse, as in the article, the elasticity of investments
with respect to technology shocks, relative to the benchmark with perfect information. This benchmark refers to the case in which the precision of private signals about \( b \) is infinite. This doesn’t imply that there is no risk as in the published article; rather, it signifies that the technology shock’s learnable component (\( b \)), is completely certain, while there remains an element of unlearnable risk (the deviations of \( a \) from its mean \( b \)). In the perfect-information limit (\( h = \infty \)), the elasticity reaches \( p_b^P = (1 - \omega)/(1 - \delta + \delta \omega) \). Accordingly, the indirect effect of market power on allocative efficiency is measured \( p_b/h_b^P \), which equals \( 1 - h_b/h \). As the level of market power increases, both the informativeness of prices and the total precision, \( h \), also rise, resulting in improved allocative efficiency, as the next proposition establishes.

**Proposition 9.** *(Allocative efficiency & market power)* Capital is more efficiently allocated when firms enjoy more market power.

We end with the impact of market power when past profits are informative about the current technology shock. Specifically, we assume that 1) the mean of shocks, \( b \), displays persistence, i.e., \( b_0 = \rho b + \eta \) where \( \rho \) is a positive parameter, and 2) the past profit is a noisy signal for the mean of shocks, \( \pi_0 = (1 - \omega)b_0 + \nu \) where the coefficient \( (1 - \omega) \) corresponds to the fact that the firms uses its market power to insulate its profit from shocks, thereby weakening the link from productivity to profits. In these equations, \( \eta \) and \( \nu \) are error terms independent of all random variables, and normally distributed with mean zero and precisions \( h_\eta \) and \( h_\nu \). Combining both equations implies that the past profit is equivalent to a signal, \( b + u \), about \( b \) where \( u = b + (\eta + \nu/(1 - \omega))/\rho \) has precision \( h_{\pi_0}(\omega) = \frac{\rho^2}{1/h_\eta + 1/((1 - \omega)h_\nu)} \). The following proposition describes the equilibrium.

**Proposition 10.** *(Learning from past profits)* There exists a log-linear rational expectations equilibrium in which shares trade at a price \( P = (1 - \omega)K_0^{1-\omega} \exp(pz) \) where

\[
p = p_0(\omega) + p_{\pi_0}(\omega)u + p_b(\omega)b + p_\theta(\omega)\theta, \quad u \equiv b + (\eta + \nu/(1 - \omega))/\rho,
\]  

(8)
\[ p_0(\omega) \equiv \frac{(1 - \omega)^2}{h_a} \left( \frac{1}{2} - \frac{\gamma(1 - \omega)K_0^{1-\omega}}{w} \right) - r^f, \quad p_b(\omega) \equiv (1 - \omega) \left( 1 - \frac{h_b}{h(\omega)} \right), \quad (9) \]

\[ p_\theta(\omega) \equiv \frac{(1 - \omega)\gamma h}{h_a(h_{\pi_0}(\omega) + h_s)} p_b(\omega), \quad p_{\pi_0}(\omega) \equiv \frac{(1 - \omega)h_{\pi_0}(\omega)}{h(\omega)}, \quad (10) \]

\[ h_p(\omega) \equiv \frac{h_a^2(h_{\pi_0}(\omega) + h_s)^2}{\gamma^2(1 - \omega)^2h(\omega)^2\sigma_\theta^2}, \quad h_{\pi_0}(\omega) = \frac{\rho^2}{1/h_\eta + 1/((1 - \omega)^2h_\nu)}, \quad (11) \]

and \[ h(\omega) \equiv h_b + h_{\pi_0}(\omega) + h_p(\omega) + h_s. \quad (12) \]

In this equilibrium, market power plays a dual role. On the one hand, stronger market power implies a more informative stock price (as in Proposition 3). On the other hand, it means a less informative past profit (\( h_{\pi_0} \) is lower).
References


Appendix - Proofs

Proposition 1 (Stock prices)

The proof of proposition 1 builds on Peress (2010) and Peress (2011). We guess that equilibrium prices are given by equations 1 to 3 and solve for an investor’s optimal portfolio by driving $z$ toward zero. The first step is to relate stock returns to technology shocks.

- Stock returns

For a given stock of capital $K_0$, intermediate goods prices are determined by the market clearing condition, $AK_0 = ((1 - \omega)/Q)^{1/\omega}$. The resulting monopoly profits equal $\Pi = YQ = (1 - \omega)(AK_0)^{1-\omega}$.

Since there is one share outstanding, the gross stock return is $R = \Pi/P$. Writing $P = P_0 \exp(pz) + o(z)$ where $o(z)$ captures terms of order larger than $z$ implies that $R = (1 - \omega)K_0^{1-\omega}/P_0 \exp[((1 - \omega)a^m - p)z] + o(z)$. When $z = 0$ (no risk), $R = (1 - \omega)K_0^{1-\omega}$.

Thus, the log return on stock is $rz = \ln(R) = (1 - \omega)az - pz$. The next step is to estimate the mean and variance of stock returns using the equilibrium prices and private signals $s_l$.

- Signal extraction

We guess that prices are approximately normally distributed and given in Equation 1, i.e. $pz = p_0z + p_b \xi z + o(z)$ where $\xi \equiv b + \mu \theta$, $\mu$ is a constant to be determined. The conditional mean and variance of $az$ for agent $l$ are given by:

$$V(az | \mathcal{F}_l) = E(V(az | b, \mathcal{F}_l) | \mathcal{F}_l) + V(E(az | b) | \mathcal{F}_l)$$

$$= E(V(az | b) | \mathcal{F}_l) + V(E(az | b) | \mathcal{F}_l)$$

$$= E\left( \frac{z}{h_a} | \mathcal{F}_l \right) + V(bz | \mathcal{F}_l)$$

$$= \frac{z}{h_a} + o(z).$$

since $V(bz | \mathcal{F}_l) = z^2/h = o(z)$ is of order $z^2$.

$$E(az | \mathcal{F}_l) = E(bz | \mathcal{F}_l) = a_{\xi} \xi z + a_s s_l z + o(z),$$

where $h \equiv h_b + \frac{1}{\mu^2 \sigma^2} + h_s$, $a_{\xi} h \equiv \frac{1}{\mu^2 \sigma^2} = h_p$ and $a_s h \equiv h_s$.

We note that the variance of returns is constant at the order $z$ since $V(bz | \mathcal{F}_l) = z^2/h = o(z)$ is of order $z^2$. 

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The conditional variance $\text{Var}(az \mid F_t)$ depends only on the prior precision of the technology shock $h_a$. The conditional expectation $E(az \mid F_t)$ is a weighted average of priors, public and private signals where the weight on the private signal (the public signal) is increasing in $h_a$ (in $1/(\mu^2\sigma^2_\theta)$). The conditional expectation and variance of stock excess returns follow:

$$E(rz \mid F_t) = E((1 - \omega)az \mid F_t) - pz = (1 - \omega)(a_\xi z + a_s s_t z) - pz + o(z).$$

$$\text{Var}(rz \mid F_t) = \text{Var}((1 - \omega)az \mid F_t) = (1 - \omega)^2 z^2 h_a + o(z).$$

- Individual portfolio choice

We turn to the portfolio choice. Agent $l$ is endowed with wealth $w$ and forms her portfolio to maximize $E \left[ \left( c_l^{1-\gamma} - 1 \right) / (1 - \gamma) \mid F_t \right]$ subject to $c_l = w \exp(r_lz)$ where $r_lz = \ln \left[ R_f + \sum_{m=1}^{M} f_m^m (R_m - R_f) \right]$ is investor $l$’s log portfolio return. $r_lz$ is approximately normal when $z$ is small (e.g., Campbell and Viceira (2002)). Therefore,

$$E \left[ \left( c_l^{1-\gamma} - 1 \right) / (1 - \gamma) \mid F_t \right] = E \left[ \{ w^{1-\gamma} \exp((1 - \gamma)r_lz) - 1 \} / (1 - \gamma) \mid F_t \right] = \left\{ w^{1-\gamma} \exp \left[ (1 - \gamma)E(r_lz \mid F_t) + (1 - \gamma)^2 \text{Var}(r_lz \mid F_t)/2 \right] - 1 \right\} / (1 - \gamma),$$

where $E(r_lz \mid F_t) = \sum_{m=1}^{M} \{ f_m^m (E(r_mz \mid F_t) - r_f z) + f_m^m (1 - f_m^m) \text{Var}(r_mz \mid F_t)/2 \} + o(z)$ and $\text{Var}(r_lz \mid F_t) = \sum_{m=1}^{M} f_m^m 2 \text{Var}(r_mz \mid F_t) + o(z)$. Maximizing $E \left[ \left( c_l^{1-\gamma} - 1 \right) / (1 - \gamma) \mid F_t \right]$ with respect to $f_m^m$ leads to the fraction of wealth allocated to stock $m$ (at the order 0 in $z$):

$$f_m^m \equiv f_l = \frac{E(rz \mid F_t) - r_f z + \text{Var}(rz \mid F_t)/2}{\gamma \text{Var}(rz \mid F_t)} + o(1). \quad (13)$$

Substituting the above expressions for $E(rz \mid F_t)$ and $\text{Var}(rz \mid F_t)$ yields:

$$f_l = \frac{(1 - \omega)(a_\xi z + a_s s_t z) - pz - r_f z}{(1 - \omega)^2 z h_a} + \frac{1}{2\gamma} + o(1) \quad (14)$$

$$\Leftrightarrow f_l = \frac{(1 - \omega)(a_\xi z + a_s s_t z) - p - r_f}{\gamma (1 - \omega)^2 h_a} + \frac{1}{2\gamma} + o(1) \quad (15)$$

$$\Leftrightarrow f_l = \frac{h_a}{(1 - \omega)\gamma} \left\{ a_\xi z + a_s s_t - \frac{1}{(1 - \omega)}(p + r_f) \right\} + \frac{1}{2\gamma} + o(1) \quad (16)$$
The final step involves aggregating stock demands and clearing the market. We multiply equation 4 by investors’ wealth $w$ and sum over all investors to obtain investors’ aggregate demand for stock $m$ (at the order 0 in $z$):

$$
\int_{0}^{1} f_{1w} dl = \frac{h_{w}}{(1-\omega)^{\gamma}} \left\{ \frac{1}{h} \left( \frac{1}{\mu^{2} \sigma_{\theta}^{2}} \xi + h_{s1} \right) - \frac{1}{(1-\omega)} \left( p + r^{f} \right) \right\} + \frac{w}{2\gamma} + o(1) \tag{18}
$$

$$
\Leftrightarrow \int_{0}^{1} f_{1w} dl = \frac{h_{w}}{(1-\omega)^{\gamma}} \left\{ h_{s} b + \int_{0}^{1} h_{s} \xi dl + \frac{1}{\mu^{2} \sigma_{\theta}^{2}} \xi - \frac{h}{(1-\omega)} \left( p + r^{f} \right) \right\} + o(1), \tag{19}
$$

since $\int_{0}^{1} h_{s} \xi dl = \int_{0}^{1} h_{s} (b + \xi) dl = \int_{0}^{1} h_{s} b dl + \int_{0}^{1} h_{s} \xi dl$ and $\int_{0}^{1} h_{s} b dl = h_{s} b$. Applying the law of large numbers to the sequence $\{h_{s} \xi\}$ of independent random variables with the same mean 0 leads to $\int_{0}^{1} h_{s} \xi dl = 0$ (see He and Wang (1995) for more details). Finally, the market clearing condition for the stock is $(\int_{0}^{1} f_{1} dl + \theta)w / P = 1$. The left-hand side is the total demand for the stock which consists of investors’ and noise traders’ demands. The right-hand side is the supply of shares. Plugging in the expression for investors demand and dropping terms of order $z$ and above yields $\mu = \frac{\gamma(1-\omega)h}{h_{w} h_{v}}$. The equilibrium price given in Proposition 1 follows. It is linear in $b$ and $\theta$ as guessed. Finally, rearranging equation 17 leads to equation 4.

**Proposition 2 (Trading volume)**

An investor’s trades are worth $w |f_{1} - f_{1,0}|$. Hence, the aggregate value of informational trades equals $\int_{0}^{1} \frac{w}{2} |f_{1} - f_{1,0}| dl$ where the factor $1/2$ avoids double counting trades. Since $f_{1} - f_{1,0}$ is approximately normally distributed, the expected value of aggregate trades, conditional on the initial distribution of shares $f_{1,0}$, is given by $V_{1} = \mathbb{E} \left( \int_{0}^{1} \frac{w}{2} |f_{1} - f_{1,0}| dl | \{ f_{1,0} \} \right) + o(1) = \frac{w}{2} \sqrt{\frac{2}{\pi}} \text{Var}(f_{1} - f_{1,0} | f_{1,0}) + o(1) = \frac{w}{2} \sqrt{\frac{2}{\pi}} \text{Var}(f_{1}) + o(1)$ (see, e.g. He and Wang (1995)). Replacing $f_{1}$ with its expression in equation 4 yields $V_{1} = \frac{w}{2} \sqrt{\frac{2}{\pi}} \sqrt{\frac{h_{s} \xi^{2}}{(1-\omega) \mu^{2} \sigma_{\theta}^{2}}} + o(1)$. Noise traders generate an average trading volume of $E(\frac{1}{2} w |\theta|) = \frac{w}{2} \sqrt{\frac{2}{\pi}} \sqrt{\frac{h_{s} \xi^{2}}{(1-\omega) \mu^{2} \sigma_{\theta}^{2}}} + o(1)$. Adding information- and noise- motivated trades leads to a (dollar) total trading volume $V = \frac{w}{2} \sqrt{\frac{2}{\pi}} \left( \frac{h_{s} \xi^{2}}{(1-\omega) \mu^{2} \sigma_{\theta}^{2}} + \sigma_{\theta}^{2} + \sqrt{\sigma_{\theta}^{2}} \right) + o(1)$. To assess the impact of market power on trading volume, we need to compute $\frac{\partial (1-\omega)h(\omega)}{\partial \omega}$. We start with $\frac{\partial h}{\partial \omega}$. Substituting the expression for
$h_p(\omega)$ into the expression for $h(\omega)$ (both given in Equations 1) yields an implicit equation in $h$:

$$h^3 - (h_b + h_s)h^2 - \frac{h^2 h^2_a}{\gamma^2 \sigma_\theta^2} \frac{1}{(1 - \omega)^2} = 0.$$  

Differentiating this equation with respect to $\omega$ leads to:

$$\frac{\partial h(\omega)}{\partial \omega} = \frac{2h^2 h^2_a}{\gamma^2 \sigma_\theta^2(1 - \omega)^3 h(h + 2h_p)} > 0.$$

Therefore, $h$ increases in $\omega$. Next, we differentiate $(1 - \omega)h(\omega)$ with respect to $\omega$:

$$\frac{\partial(1 - \omega)h(\omega)}{\partial \omega} = -h + (1 - \omega) \frac{\partial h}{\partial \omega}$$

$$= -h + \frac{2h^2 h^2_a}{\gamma^2 \sigma_\theta^2(1 - \omega)^3 h(h + 2h_p)}$$

$$= \frac{1}{h(h + 2h_p)} \left( -h^2(h + 2h_p) + \frac{2h^2 h^2_a}{\gamma^2 \sigma_\theta^2(1 - \omega)^2} \right)$$

$$= \frac{1}{h(h + 2h_p)} \left( -h^3 - 2h_p h^2 + \frac{2h^2 h^2_a}{\gamma^2 \sigma_\theta^2(1 - \omega)^2} \right)$$

$$= -\frac{h^2}{h + 2h_p} < 0,$$

since $h_p h^2 = \frac{h^2 h^2_a}{\gamma^2 \sigma_\theta^2(1 - \omega)^2}$. This establishes that $h_p = h^2 h^2_a/(\gamma^2 \sigma_\theta^2(1 - \omega)^2 h^2)$ increases in $\omega$. It follows that $(1 - \omega)h(\omega)$ decreases in $\omega$. As a result, $V$ increase in $\omega$.

Turnover is obtained by dividing (dollar) total trading volume by the stock’s market capitalization, $(1 - \omega)K_0^{1-\omega} + o(1)$ (recall that the firms has one share outstanding). The effect of market power on the stock’s market capitalization, and hence on turnover, depends on $K_0$. If $K_0 \geq 1$,then capitalization decreases in $\omega$ and hence turnover increases in $\omega$. If $K_0 < 1$,then capitalization increases in $\omega$ over $[0, 1 + 1/ln(K_0)]$ and decreases in $\omega$ over $[1 + 1/ln(K_0), 1]$. As a result, if $K_0 < 1$ and $\omega \geq 1 + 1/ln(K_0)$,then turnover increases in $\omega$; if $K_0 < 1$ and $\omega < 1 + 1/ln(K_0)$, then $\omega$ has an ambiguous effect on turnover.

**Proposition 3 (Stock price informativeness)**

See the proof of Proposition 2 (Trading volume) which establishes that the informativeness of prices, defined as $h_p = h^2 h^2_a/(\gamma^2 \sigma_\theta^2(1 - \omega)^2 h^2)$, increases in $\omega$. 

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Proposition 4 (Dispersion of investors’ forecasts)

We showed in the proof of Proposition 1 that investors’ productivity forecasts equal $E(a | F_t) = E(b | F_t) = a_\xi + a_s s_t + o(1)$. The dispersion of these forecasts across investors, for a given realization of the shocks $b$ and $\theta$, is measured by $D \equiv Var[E(b | F_t) | b, \theta] = Var(a_\xi + a_s s_t | b, \theta) = Var(a_s \xi_t) = a_s^2 Var(\xi_t) = (h_s/h)^2/h_s = h_s/h^2$. To assess the impact of market power on $D$, it suffices to note that $h$ is increasing in $\omega$ (see the proof of Proposition 2 on trading volume) and therefore that $D$ is decreasing in $\omega$. Similarly, investors profit and return forecasts equal $E(\pi | F_t) = E((1-\omega)a | F_t)$ and $E(r | F_t) = E((1-\omega)a | F_t) - p$ so their dispersions, conditional on the shocks $b$ and $\theta$, equal $(1-\omega)^2 D$. Since $D$ is decreasing in $\omega$, so is $(1-\omega)^2 D$. Thus, investors make less dispersed forecasts about the productivity, profit and return of more monopolistic firms.

Proposition 5 (Distribution of stock returns)

- **Expected stock returns**

The stock’s expected excess return equals $E(R) - R^f = E[E(rz | F_t) - r^f z + Var(rz | F_t)/2] = E(rz) - r^f z + Var(rz | F_t)/2 = (-p_0 - r^f + (1-\omega)^2/h_a/2)z = \gamma(1-\omega)^3 K_0^{1-\omega} z/(h_a w)$ because all shocks have zero unconditional expectations, $p_0 + r^f = (1-\omega)^2 \left(\frac{1}{2} - \frac{\gamma(1-\omega)K_0^{1-\omega}}{w}\right)$ and $Var(rz | F_t) = Var((1-\omega)a z | F_t) = (1-\omega)^2 \frac{z}{h_a} + o(z)$ from the proof of Proposition 1. The numerator, $(1-\omega)^3 K_0^{1-\omega}$, reflects the direct effect of $\omega$ on the expected excess return: as $\omega$ increases, risk, and hence expected returns, fall.

- **Sharpe ratios**

The average Sharpe ratio equals $SR = \frac{E(R) - R^f}{\sqrt{Var(rz | F_t)}} = \frac{\gamma(1-\omega)^3 K_0^{1-\omega} z/(h_a w)}{\sqrt{(1-\omega)^2 \frac{z}{h_a} + o(z)}} = \gamma(1-\omega)^2 K_0^{1-\omega} \sqrt{z}/(h_a w)$. The direct effect of $\omega$ (the term $(1-\omega)^2 K_0^{1-\omega}$ in the numerator) decreases the Sharpe ratio.

- **Stock return volatility**

The variance of stock returns is $Var(rz) = Var[E(rz | F_t)] = E[Var(rz | F_t) | F_t]$ where $E[Var(rz | F_t)] = E[Var((1-\omega)a z | F_t)] = (1-\omega)^2 \frac{z}{h_a} + o(z)$ from the proof of Proposition 1. Therefore, return volatility is decreasing in $\omega$.

- **Profit volatility**

We compute the volatility of log profits to factor out the order-0 term, conditional on stock prices: $Var(ln \Pi | P) = Var((1-\omega)a z | P) = (1-\omega)^2 \frac{z}{h_a} + o(z)$. A strengthening market power reduces the volatility of profits through the direct effect of market power.
Contrary to what I wrote in the published paper, \( \omega \) has no indirect effect operating through information on any of these variables (they depend, not on \( h \), but on \( h_a \), which is unrelated to \( \omega \)). The reason is that, in the variance of stock returns, \( \text{Var}(rz) = \text{Var}[E(rz \mid F_t)] + E[\text{Var}(rz \mid F_t)] \), the first term, \( \text{Var}[E(rz \mid F_t)] \), is of order \( \omega^2 \) and is negligible. As a result, there is no information effect on the variance of returns. It follows that there is no such effect on the expected return, the Sharpe ratio and profit volatility either.

**Proposition 6 (Liquidity)**

In the model, liquidity represents the sensitivity of stock prices to (uninformative) noise shocks and is measured by \( p_\theta = \partial(\ln P)/\partial(\theta z) = \gamma(1-\omega)p_b \frac{h}{h_a h_s} = \gamma(1-\omega)^2(1-h_b/h) \frac{h}{h_a h_s} = \gamma(1-\omega)^2(h-h_b) \frac{1}{h_a h_s} = \gamma(1-\omega)^2(h-h_b) \frac{1}{h_a h_s} \). To assess the impact of competition on liquidity, we differentiate \( p_\theta \) with respect to \( \omega : \partial p_\theta / \partial \omega = \frac{\gamma}{h_a h_s} \{ -2(1-\omega)(h-h_b) + (1-\omega)^2 \partial h / \partial \omega \} \). Plugging in the expression for \( \partial h / \partial \omega \) from the proof of Proposition 2 on trading volume yields:

\[
\frac{\partial p_\theta}{\partial \omega} = \frac{\gamma}{h_a h_s} \left\{ -2(1-\omega)(h-h_b) + (1-\omega)^2 \frac{2h_a^2 h_s^2}{2\gamma^2 \sigma_a^2 (1-\omega)^3 h(h+2h_p)} \right\}
\]

\[
= \frac{\gamma(1-\omega)}{h_a h_s} \left\{ -2(h-h_b) + \frac{2h_a^2 h_s^2}{\gamma^2 \sigma_a^2 (1-\omega)^3 h(h+2h_p)} \right\}
\]

\[
= \frac{\gamma(1-\omega)}{h_a h_s} \left\{ -2(h_p + h_s) + \frac{2h_a h_s^2}{h(h+2h_p)} \right\},
\]

using the relations \( h \equiv h_b + h_p + h_s \) and \( h_p = \frac{h_a h_s^2}{\gamma^2 (1-\omega)^2 h^2 \sigma_a^2} \). It follows that:

\[
\frac{\partial p_\theta}{\partial \omega} = \frac{2\gamma(1-\omega)}{h_a h_s} \left\{ -(h_p + h_s) + \frac{h_p h}{h+2h_p} \right\}
\]

\[
= \frac{2\gamma(1-\omega)}{h_a h_s} \left\{ -(h_p + h_s)(h+2h_p) + h_p h \right\}
\]

\[
= \frac{2\gamma(1-\omega)}{h_a h_s} \frac{-h_s(h+2h_p) + 2h_p^2}{h+2h_p} < 0.
\]

Hence \( \partial p_\theta / \partial \omega < 0 \) and stock prices of more monopolistic firms are less sensitive to noise shocks, i.e. more liquid.

**Proposition 7 (Stock prices when shares are issued)**

The proof follows that of Proposition 1 except that the stock of capital is now endogenous. The amount of new capital raised equals the value of the \( \alpha \) new shares, i.e. \( K = \alpha P \). Its expanded capital stock, \( K_0 + K \), allows a monopoly to generate a profit \( \Pi = YQ = (1-\omega)(A(K_0 + K))^{1-\omega} \). Since there are \( 1 + \alpha \) shares outstanding, the resulting gross stock return is \( R = \Pi/(P(1+\alpha)) = \)
\[(1 - \omega)[A(K_0 + \alpha P)]^{1-\omega}/(P(1 + \alpha))\).

We express stock prices as \(P = \overline{P}\exp(pz) + o(z)\) and expand returns around \(z = 0\). We obtain
\[
R = (1 - \omega)(K_0 + \alpha\overline{P})^{1-\omega}/(\overline{P}(1 + \alpha)) \exp\{(1 - \omega)az - (1 - \delta + \delta\omega)pz\}
\]
where \(\delta \equiv \alpha\overline{P}/(K_0 + \alpha\overline{P})\) is the dilution factor. When \(z = 0\) (no risk), \(R = 1\) so \(P\) is the (implicit) solution to \(P(1 + \alpha) = (1 - \omega)(K_0 + \alpha\overline{P})^{1-\omega}\). Therefore, the log stock return is
\[
\ln(R) = [(1 - \omega)a - (1 - \delta + \delta\omega)p]z.
\]

The subsequent steps are identical to those that comprise the proof of Proposition 1. We solve the signal extraction and portfolio problems of an investor who observes \(p\) and \(s_l\). We aggregate stock demands using the law of large numbers, add noise trades and equate the total demand to the total supply of shares, \(1 + \alpha\). The resulting stock price \(p\) is linear in \(a\) and \(\theta\) as guessed. Its expression is provided in Proposition 7.

Propositions 8 and 9 (Allocative efficiency)

Differentiating equation 7 defining \(p_b(\omega, \delta)\) with respect to \(h\) holding \(\omega\) fixed yields \(\partial p_b/\partial h = (1 - \omega)/(1 - \delta + \delta\omega)h_b/h^2 > 0\). Thus, \(p_b\) increases in \(h\): investments are more efficient when information is more accurate. The efficiency of investments relative to the perfect-information benchmark is given by \(p_b/p_b^P = 1 - h_b/h\). It increases in \(\omega\) since \(h\) increases with \(\omega\) (see the proof of Proposition 2) and \(p_b/p_b^P\) increases with \(h\), \(p_b/p_b^P\). Thus, capital is more efficiently allocated across more monopolistic firms.

Proposition 10 (Learning from past profits)

The proof is identical to that of Proposition 1, except that investors observe an additional public signal \(\pi_0\). We guess that the stock price is approximately given in equation 8, i.e. \(pz = p_0z + p_{\pi_0}\xi_{\pi_0}z + z\xi_\pi z + o(z)\) where \(\xi_\pi \equiv b + \mu\theta\) (\(\mu\) is a constant to be determined) and \(\xi_{\pi_0} \equiv b + u\). Thus, observing \(p\) and \(\pi_0\) is equivalent to observing \(\xi_\pi\) and \(\xi_{\pi_0}\). Based on her information set \(\mathcal{F}_l = \{s_l, \xi_\pi, \xi_{\pi_0} \text{ for all stocks}\}\), the conditional mean and variance of \(az\) are for agent \(l\):

\[
V(az \mid \mathcal{F}_l) = E(V(az \mid b, \mathcal{F}_l) \mid \mathcal{F}_l) + V(E(az \mid b, \mathcal{F}_l) \mid \mathcal{F}_l)
\]
\[
= E(V(az \mid b) \mid \mathcal{F}_l) + V(E(az \mid b) \mid \mathcal{F}_l)
\]
\[
= E\left(\frac{z}{h_\alpha} \mid \mathcal{F}_l\right) + V(bz \mid \mathcal{F}_l)
\]
\[
= \frac{z}{h_\alpha} + o(z).
\]
since \( V(bz | F_1) = z^2/h = o(z) \) is of order \( z^2 \).

\[
E(az | F_1) = E(bz | F_1) = a_p \xi_p z + a_{\pi_0} \xi_{\pi_0} z + a_s s_l z + o(z),
\]

where \( h \equiv h_b + h_p + h_{\pi_0} + h_s \), \( a_p h = h_p = \frac{1}{\mu^2 \sigma^2_\theta} \), \( a_{\pi_0} h = h_{\pi_0} \) and \( a_s h \equiv h_s \).

The conditional expectation and variance of stock excess returns follow:

\[
E(rz | F_1) = E((1 - \omega)az | F_1) - pz = (1 - \omega) (a_p \xi_p z + a_{\pi_0} \xi_{\pi_0} z + a_s s_l z) - pz + o(z).
\]

\[
Var(rz | F_1) = Var((1 - \omega)az | F_1) = (1 - \omega)^2 \frac{z}{h_a} + o(z).
\]

Investors’ stock demand are given by equation 13, which yields after substitution:

\[
f_l^m \equiv f_l = \frac{E(rz | F_1) - r^f z + Var(rz | F_1)/2}{\gamma Var(rz | F_1)} + o(1). \tag{20}
\]

Substituting the above expressions for \( E(rz | F_1) \) and \( Var(rz | F_1) \) yields:

\[
f_l = \frac{(1 - \omega) (a_p \xi_p z + a_{\pi_0} \xi_{\pi_0} z + a_s s_l z) - pz - r^f z}{(1 - \omega)^2 z/h_a} + \frac{1}{2\gamma} + o(1) \tag{21}\]

\[
\Leftrightarrow f_l = \frac{h_a}{(1 - \omega)\gamma} \left\{ \frac{1}{h} \left( h_p \xi_p + h_{\pi_0} \xi_{\pi_0} + h_s s_l \right) - \frac{1}{(1 - \omega)} (p + r^f) \right\} + \frac{1}{2\gamma} + o(1). \tag{22}\]

The final step involves aggregating stock demands and clearing the market. We multiply equation 4 by investors’ income \( w \) and sum over all investors to obtain investors’ aggregate demand for stock \( m \) (at the order 0 in \( z \)):

\[
\int_0^1 f_l w dl = \frac{wh_a}{(1 - \omega)\gamma} \left\{ \frac{1}{h} \left( \frac{1}{\mu^2 \sigma^2_\theta} \xi_p + h_{\pi_0} \xi_{\pi_0} + \int_0^1 h_s s_l dl \right) - \frac{1}{(1 - \omega)} (p + r^f) \right\} + \frac{1}{2\gamma} + o(1) \tag{23}\]

\[
\Leftrightarrow \int_0^1 f_l w dl = \frac{wh_a}{(1 - \omega)\gamma} h \left\{ \frac{1}{\mu^2 \sigma^2_\theta} (b + \mu\theta + h_{\pi_0} (b + u) + h_b + \int_0^1 h_s \varepsilon_l dl - \frac{h}{(1 - \omega)} (p + r^f) \right\} + \frac{1}{2\gamma} + o(1), \tag{24}\]

since \( \int_0^1 h_s s_l dl = \int_0^1 h_s (b + \varepsilon_l) dl = \int_0^1 h_s b dl + \int_0^1 h_s \varepsilon_l dl \) and \( \int_0^1 h_s b dl = h_s b \). Applying the law of large numbers to the sequence \( \{h_s \varepsilon_l\} \) of independent random variables with the same mean 0 leads to \( \int_0^1 h_s \varepsilon_l dl = 0 \) (see He and Wang (1995) for more details). As a result,
\[
\int_0^1 f_{t} \, dl = \frac{wh_a}{(1 - \omega)\gamma h} \left\{ \frac{1}{\mu^2 \sigma^2_\theta} (b + \mu \theta) + h_\pi_0 (b + u) + h_s b - \frac{h}{(1 - \omega)} (p + r^f) \right\} + \frac{1}{2\gamma} + o(1).
\]

Finally, the market clearing condition, \((\int_0^1 f_{t} \, dl + \theta)w/P = 1\), implies:

\[
\int_0^1 f_{t} \, dl = \frac{h_a}{(1 - \omega)\gamma h} \left\{ (h_s + \frac{1}{\mu^2 \sigma^2_\theta} + h_\pi_0) b + h_\pi_0 u + \frac{1}{\mu^2 \sigma^2_\theta} \mu \theta - \frac{h}{(1 - \omega)} (p + r^f) \right\} + \frac{1}{2\gamma} + o(1) = \bar{P}/w - \theta
\]

\[
\Leftrightarrow \int_0^1 f_{t} \, dl = \frac{h_a}{(1 - \omega)\gamma h} \left\{ (h_s + \frac{1}{\mu^2 \sigma^2_\theta} + h_\pi_0) b + h_\pi_0 u + \left( \frac{1}{\mu \sigma^2_\theta} + \frac{(1 - \omega)\gamma h}{h_a} \right) \theta - \frac{h}{(1 - \omega)} (p + r^f) \right\} = \bar{P}/w - \frac{1}{2\gamma}
\]

\[
\Leftrightarrow (h_s + \frac{1}{\mu^2 \sigma^2_\theta} + h_\pi_0) b + h_\pi_0 u + \left( \frac{1}{\mu \sigma^2_\theta} + \frac{(1 - \omega)\gamma h}{h_a} \right) \theta - \frac{h}{(1 - \omega)} (p + r^f) = \frac{1 - \omega}{h_a} \gamma h \left( \frac{\bar{P}/w}{h_a} - \frac{1}{2\gamma} \right)
\]

\[
\Leftrightarrow (h_s + \frac{1}{\mu^2 \sigma^2_\theta} + h_\pi_0) b + h_\pi_0 u + \left( \frac{1}{\mu \sigma^2_\theta} + \frac{(1 - \omega)\gamma h}{h_a} \right) \theta - \frac{h}{(1 - \omega)} (p + r^f) = \frac{1 - \omega}{h_a} \gamma h \left( \frac{\bar{P}/w}{h_a} - \frac{1}{2\gamma} \right)
\]

\[
\Leftrightarrow \frac{1 - \omega}{h_a} \left( h_s + \frac{1}{\mu^2 \sigma^2_\theta} + h_\pi_0 \right) b + \frac{1 - \omega}{h_a} h_\pi_0 u + \frac{1 - \omega}{h_a} \left( \frac{1}{\mu \sigma^2_\theta} + \frac{(1 - \omega)\gamma h}{h_a} \right) \theta - (p + r^f) = \frac{(1 - \omega)^2 \gamma}{h_a} \left( \frac{\bar{P}/w}{h_a} - \frac{1}{2\gamma} \right)
\]

\[
\Leftrightarrow p = -r^f - (1 - \omega)^2 \gamma \left( \frac{\bar{P}/w}{h_a} - \frac{1}{2\gamma} \right) + \frac{1 - \omega}{h_a} \left( h_s + \frac{1}{\mu^2 \sigma^2_\theta} + h_\pi_0 \right) b + \frac{1 - \omega}{h_a} h_\pi_0 u + \frac{1 - \omega}{h_a} \left( \frac{1}{\mu \sigma^2_\theta} + \frac{(1 - \omega)\gamma h}{h_a} \right) \theta.
\]

Comparing this equation to the conjectured price implies \(\mu = \frac{\gamma (1 - \omega)h}{h_a (h_s + h_\pi_0)}\). The equilibrium price is linear in \(\xi_p\) and \(\xi_\pi_0\) as guessed. Rearranging this equation leads to the price given in Proposition 10.