Technical note on the treatment of taxes in a static principal agent model of executive compensation *

Ingolf Dittmann[†] Ernst Maug[‡]

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1 New symbols

We use the same notation as in the paper. Superscript 'd' refers to observed contracts ("data"), without this superscript contracts are other contracts evaluated b the algorithm. We need 4 additional symbols for incorporating the tax effect into our model:

- The CEO's holding of unrestricted stock: n_S^u.

*This note is a technical document to accompany our paper "Lower salaries and no options: the optimal structure of executive pay." It was originally intended as an internal document for the process of developing and writing the paper. We make it now publically available to provide the interested reader with additional details about our approach in the paper. We cannot guarantee that this document is self-contained. It should therefore not be cited without the permission of the authors. For a discussion of the relationship between the U.S. tax system and executive compensation the reader is referred to Hall and Liebman (2000). We are grateful to David Yermack to lead our analysis here. All errors and omissions are our own responsibility.

[†]Erasmus University Rotterdam, P.O. Box 1738, 3000 DR Rotterdam, The Netherlands. Email: dittmann@few.eur.nl. Tel: +31 10 408 1283.

[‡]University of Mannheim, Chair for Corporate Finance, 68131 Mannheim, Germany. Email: maug@bwl.uni-mannheim.de.

- The CEO's holding of restricted stock: n_S^r. **Note:** restricted and unrestricted stock are calculated at the start of the contract period. We do not take into account stock and option grants given during the contract period. Both restricted and unrestricted stock are held until the end of the contract period. The only difference is that the value of restricted stock is taxed at the end of the contract period, whereas unrestricted stock is not taxed.

- The personal tax rate τ_p. From Hall & Liebman (2000), we use 31% for 1992, 39.6% for 1993 and 42% from 1994 onwards.

- The corporate tax rate τ_c. From Hall & Liebman (2000), we use 34% from 1988 to 1992 and 35% from 1993 onwards.

2 Additional Assumptions

- We assume that the "fixed" salary ϕ is bonus only. This acknowledges that bonus is the biggest part of ϕ for most CEOs. Hence, ϕ is not subject to the million-dollar rule and leads to a tax credit to the firm.

- We assume that the "million-dollar" rule is binding, i.e., that restricted stock leads to no additional tax credit to the firm.

- We do not include capital gains taxes as these are difficult to model. Also they might be evaded by never selling the shares.

- We assume that investing money at the risk-free rate is not taxed.

- The contract parameters remain ϕ, n_O, and n_S. If ϕ is negative, the CEO buys additional (unrestricted) shares from her wealth. So $n_S^u = n_s^{u,d} - \min\{\phi, 0\}/P_0$, where $n_S^{u,d}$ is the proportion of unrestricted stock in the observed contract. Then,

$$\begin{aligned} n_S^r &= \max\{n_S - n_S^u, 0\} \\ &= \max\left\{n_S - n_S^{u,d} + \min\{\phi, 0\}/P_0, 0\right\} \ . \end{aligned}$$

  Note that the max operator is necessary, because otherwise n_S^r could be negative if $n_S < n_S^{u,d}$.

3 The effect of taxes on the executive's income

The after-tax income of an executive that has wealth W_0 and observed unrestricted stock $n_S^{u,d}$ and who is given a contract with parameters ϕ, n_O, n_S will realize the following after-tax income at the end of the contract period (i.e., at time $t = T$):

- Value of wealth, fixed salary and bonus:

$$(W_0 + \min\{\phi, (1 - \tau_p)\phi\}) \exp(r_f T) \ .$$

- Value of stock held over the contract period after taxes on dividends. We define:

$$P_T = P_0 \exp\left\{\left(r_f - d - \frac{\sigma^2}{2}\right) T + u\sigma\sqrt{T}\right\} \ ,$$

  If all dividends are reinvested, then the end of period wealth from holding one share of common stock is $P_T \exp(dT)$, which is the performance index of this stock. However, the CEO can only reinvest the after-tax dividend, therefore the value of the company P_0 at time T *including reinvested dividends* is $P_T \exp\{(1 - \tau_P) dT\}$.

- Tax to be paid on restricted stock:

$$n_S^r P_T \tau_p = \max\left\{n_S - n_S^{u,d} + \min\{\phi, 0\}/P_0, 0\right\} P_T \tau_p$$

- Value of the options exercised at the end of the period after taxes on option gain:

$$n_O(1-\tau_p)\max\{P_T - K, 0\} \ .$$

Hence, the end-of-period wealth W_T of the executive is given by:

$$\begin{aligned} W_T = &(W_0 + \min\{\phi, (1-\tau_p)\phi\})\exp(r_f T) + n_S P_T \exp\{(1-\tau_P)\,dT\} \\ &- n_S^r P_T \tau_p + n_O(1-\tau_p)\max\{P_T - K, 0\} \end{aligned}$$

4 Tax credit for the company

In addition, the company realizes the following tax credit (expressed in expected dollars at time $t = 0$):

- Tax credit from bonus: $\max(\phi, 0)\tau_c$
- Tax credit from exercising the options at the end of the period:

$$n_O E\left[\max\{P_T - K, 0\}\tau_c \exp(-r_f T)\right] = n_O BS\ \tau_c$$

- Tax credit from vesting restricted options at the end of the period:

$$n_S^r E[P_T]\tau_c \exp(-r_f T) = n_S^r P_0 \tau_c \exp(-dT)$$

So altogether the company expects tax savings of

$$X = \max(\phi, 0)\tau_c + n_O BS\ \tau_c + n_S^r P_0 \tau_c \exp(-dT)$$

The objective function (that is to be minimized) then is:

$$\begin{aligned} \mathit{objective} &= \phi + n_S P_0 + n_O BS - X \\ &= \min\{\phi, (1-\tau_c)\phi\} + n_S P_0 + n_O BS(1-\tau_c)\ - n_S^r P_0 \tau_c \exp(-dT). \end{aligned}$$

Internal note: Comparison to previous versions: in Version 20040622, the tax effect of restricted stock $(-n_S^r P_0 \tau_c \exp(-dT))$ is not included. For the optimization in the first version of the paper, however, we allowed for the tax effect of restricted stock by $(-n_S^r P_0 \tau_c)$.

5 Advantages of options in this setting

Options have the following advantages over stock in this setting:

- Options allow risk-sharing between CEO and the state. This improves the CEO's utility but also reduces her incentives.

  Additional advantages of options that are not captured in this model:

- Restricted stock might not be tax deductible for firms. This is the case if (1) the value of stock granted plus fixed salary exceeds the one million dollar threshold and if (2) the restricted stock are not part of a shareholder approved compensation plan.

- Options avoid capital gains tax that is paid on the capital gains of stock. On the other hand, options are associated with higher salaries, which in turn would lead to higher income from the CEO's wealth investment that would also be taxed. We do not model the taxation of wealth accumulation.

- Options are not expensed in the income statement. In practice, this seems to be a major advantage of options. In our model, we cannot include this feature.

6 EU and UPPS with taxes

The EU and UPPS functions for the grid search are:

$$
\begin{aligned}
&EU(\phi, n_S, n_O)\\
&= \frac{1}{1-\gamma}\frac{1}{\sqrt{2\pi}} \times \Bigg(\int_{-\infty}^{MD2} \Bigg[TW + \underbrace{(n_S \exp\{(1-\tau_p)\, dT\} - n_S^r \tau_p)}_{=n_S^{e1}} \underbrace{PC \exp\{uCV\}}_{=P_T} \Bigg]^{1-\gamma} \\
&\qquad \times \exp\left\{-\frac{u^2}{2}\right\} du \Bigg) \\
&\qquad + \Bigg(\int_{MD2}^{+\infty} \Bigg[\underbrace{(n_S \exp\{(1-\tau_p)\, dT\} - n_S^r \tau_p + n_O(1-\tau_p))}_{=n_S^{e2}} \underbrace{PC \exp\{uCV\}}_{=P_T} \\
&\qquad + TW - n_O(1-\tau_p)K\Bigg]^{1-\gamma} \exp\left\{-\frac{u^2}{2}\right\} du \Bigg) \\
&= \frac{1}{1-\gamma}\frac{1}{\sqrt{2\pi}} \Bigg(\int_{-\infty}^{MD2} \left[TW + n_S^{e1} PC \exp\{uCV\}\right]^{1-\gamma} \exp\left\{-\frac{u^2}{2}\right\} du \\
&\qquad + \int_{MD2}^{+\infty} \left[TW + n_S^{e2} PC \exp\{uCV\} - n_O(1-\tau_p)K\right]^{1-\gamma} \exp\left\{-\frac{u^2}{2}\right\} du \Bigg)
\end{aligned}
$$

with

$$
\begin{aligned}
TW &= (\min\{(1-\tau_p)\phi, \phi\} + W_0) \exp\{r_f T\}\ ,\\
PC &= P_0 \exp\left\{\left(r_f - d - \frac{\sigma^2}{2}\right) T\right\}\ ;\\
CV &= \sigma\sqrt{T}\ ,\\
MD2 &= \frac{\ln\left(\frac{K}{P_0}\right) - (r_f - d)\, T}{\sigma\sqrt{T}} + \frac{\sigma\sqrt{T}}{2}\ ,\\
n_S^{e1} &= n_S \exp\{(1-\tau_p)\, dT\} - n_S^r \tau_p\ ,\\
n_S^{e2} &= n_S \exp\{(1-\tau_p)\, dT\} + n_O(1-\tau_p) - n_S^r \tau_p\quad .\\
n_S^r &= \max\left\{n_S - n_S^{u,d} + \min\{\phi, 0\}/P_0, 0\right\}
\end{aligned}
$$

For $\gamma = 1$, we obtain:

$$
\begin{aligned}
EU(\phi, n_S, n_O) = {} & \frac{1}{\sqrt{2\pi}} \times \left(\int_{-\infty}^{MD2} \log \left[TW + \underbrace{(n_S \exp\{(1-\tau_p)\, dT\} - n_S^r \tau_p)}_{=n_S^{e1}} \underbrace{PC \exp\{uCV\}}_{=P_T} \right] \right. \\
& \left. \times \exp\left\{ -\frac{u^2}{2} \right\} du \right) \\
& + \left(\int_{MD2}^{+\infty} \log \left[\underbrace{(n_S \exp\{(1-\tau_p)\, dT\} - n_S^r \tau_p + n_O(1-\tau_p))}_{=n_S^{e2}} \underbrace{PC \exp\{uCV\}}_{=P_T} \right. \right. \\
& \left. \left. + TW - n_O(1-\tau_p)K \right] \exp\left\{ -\frac{u^2}{2} \right\} du \right) \\
& = \frac{1}{\sqrt{2\pi}} \left(\int_{-\infty}^{MD2} \log \left[TW + n_S^{e1} PC \exp\{uCV\} \right] \exp\left\{ -\frac{u^2}{2} \right\} du \right. \\
& \left. + \int_{MD2}^{+\infty} \log \left[TW + n_S^{e2} PC \exp\{uCV\} - n_O(1-\tau_p)K \right] \exp\left\{ -\frac{u^2}{2} \right\} du \right) \quad .
\end{aligned}
$$

For UPPS, we get:

$$
\begin{aligned}
& UPPS(\phi, n_S, n_O) \\
= {} & LD \left(n_S^{e1} \int_{-\infty}^{MD2} \left[TW + n_S^{e1} PC \exp\{uCV\} \right]^{-\gamma} \exp\left\{ uCV - \frac{u^2}{2} \right\} du \right. \\
& \left. + n_S^{e2} \int_{MD2}^{+\infty} \left[TW + n_S^{e2} PC \exp\{uCV\} - n_O(1-\tau_p)K \right]^{-\gamma} \exp\left\{ uCV - \frac{u^2}{2} \right\} du \right),
\end{aligned}
$$

where:

$$
LD = \frac{1}{\sqrt{2\pi}} \exp\left\{ \left(-d - \frac{\sigma^2}{2} \right) T \right\} \quad .
$$

For the derivative of UPPS with respect to P_0, we obtain:

$$
\begin{aligned}
\frac{dUPPS}{dP_0} = & -\gamma \frac{\exp\{(r_f - 2d - \sigma^2)\, T\}}{\sqrt{2\pi}} \\
& \left(\int_{-\infty}^{MD2} \left(TW + n_S^{e1} PC \exp\{uCV\}\right)^{-(1+\gamma)} \left(n_S^{e1}\right)^2 \exp\left\{2uCV - \frac{u^2}{2}\right\} du \right. \\
& \left. + \int_{MD2}^{\infty} \left(TW + n_S^{e2} PC \exp\{uCV\} - n_O(1-\tau_p)K\right)^{-(1+\gamma)} \left(n_S^{e2}\right)^2 \exp\left\{2uCV - \frac{u^2}{2}\right\} du \right) \\
& + \frac{\left(TW + n_S^{e1} K\right)^{-\gamma} n_O(1-\tau_p) \exp\{-dT\} \exp\left\{-\frac{1}{2}\left(MD2 - CV\right)^2\right\}}{\sqrt{2\pi} CV \; P_0}
\end{aligned}
$$