

**Internet Appendix to**  
**“EX ANTE SKEWNESS AND EXPECTED STOCK RETURNS”\***

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**ABSTRACT**

This document provides supplementary material to the paper *Ex Ante Skewness and Expected Stock Returns*. The document provides tables pertaining to moments computed using option maturities closest to one and six months, as well as additional results for three- and 12-month option moments that use alternative specifications of the stochastic discount factor. Finally, we conduct a simulation exercise using a Heston model with plausible parameter values, to compare the performance of our skewness metric to those proposed by Xing, Zhang and Zhao (2010) in a setting where skewness is known.

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## **I. Results for Additional Option Maturities**

Tables IA-I-IA-VIII present results complementary to Tables II - X in the main body of the text. The latter cover the three- and 12-month maturities, while in this document we supplement these results with two additional maturities: the one- and six-month maturities. Details and discussion of the tables in the main paper can be found in its Section II. All tables reported here follow the same methodology.

Table IA-I presents descriptive statistics for risk-neutral moment-sorted portfolios using options closest to one and six months to maturity to calculate volatility, skewness, and kurtosis. Table IA-II presents multi-way sorts on volatility, skewness, and kurtosis. In the three-way sorts on volatility, skewness, and kurtosis, some portfolios do not have firms in the three-way intersection for some months. Specifically, the low skew, low volatility, low kurtosis portfolios for both maturities do not have observations for July through December 2003. As a result, we report means for this portfolio over the available months.

Table IA-III presents results of risk adjustments using the Fama and French (1993) three-factor model for these option maturity moments whereas Table IA-IV adds the Pástor and Stambaugh (2003) liquidity factor. Table IA-V provides descriptive statistics for co-moment-sorted portfolios, and in Table IA-VI we adjust for Fama and French (1993) three factor risk in these co-moment-sorted portfolios. Tables IA-VII and IA-VIII provide complements to the main paper's Tables V and VI, which report the idiosyncratic portion of the moments.

## **II. Robustness Checks**

As discussed in the main body of the text, we analyze the robustness of our main results to alternative screens on the options data used to calculate risk-neutral moments. In particular, we examine the sensitivity of the results to four criteria. The first is that we impose no volume requirement on the options included in our analysis. The second criterion imposes a higher threshold on the price of options excluded from the analysis, requiring that option prices be greater than \$1. The third requires a lower threshold on option prices, excluding any options with prices less than or equal to \$0.25. The

final robustness check requires a greater number of both put and call options out of the money (OTM) for inclusion in the analysis.

Results of the first robustness check are presented in Table IA-IX. As shown in the table, the results of our sorting procedure are largely unchanged. Across all four option maturity horizons (one month, three months, six months, and 12 months), the patterns in average returns mirror those shown in the main body of results. Firms with low risk-neutral volatility, skewness, and kurtosis earn high average returns relative to their high risk-neutral volatility, high skewness, and high kurtosis counterparts. These returns are robust to characteristic adjustment. The magnitude of average return spreads across terciles is somewhat smaller than for firms with volume screens imposed, particularly at the one month horizon. Nevertheless, the qualitative conclusions of the main text are robust to omitting volume screens.

In Table IA-X, we increase the minimum option price considered to be valid for the calculation and to be \$1. We present summary statistics for portfolios formed on risk-neutral volatility, skewness, and kurtosis, when options are closest to one, three, six, and 12 months to maturity. Again, as shown in the table, the broad conclusions of the main body of the text are preserved. High volatility, skew and, kurtosis firms have average returns that are below those of low volatility, skew and, kurtosis firms, respectively. These results are further corroborated in Table IA-XI, where we reduce the price screen and require that options have prices greater than \$0.25 to be included in the analysis.

In Table IA-XII, we require three OTM puts and three OTM calls in order for an option to be included in the calculation of volatility, skewness, and kurtosis. This screen contrasts to the main body of the text, in which we require only two OTM puts and two OTM calls. As shown in the table, results are again qualitatively unchanged. There are volatility and skewness discounts (high volatility and high skewness firms earn lower average returns than their low volatility and low skewness counterparts), and a kurtosis premium. The results suggest that the findings documented in the main body of the paper are robust to alternative criteria for determining a minimum price for options to be included or a minimum number of OTM contracts.

### **III. Alternative Specifications of the Stochastic Discount Factor**

In this section, we analyze the extent to which the relations between higher *total* moments and subsequent returns are due to investors seeking compensation for higher *co-moment* risk, rather than idiosyncratic moments. We perform a series of tests; in each succeeding test, we decrease the restrictions placed on the stochastic discount factor. Our main focus is to test whether the relation between higher moments and subsequent risk-adjusted returns persists.

#### A. Adjusting for Co-Moment Risk

We test whether the returns related to the total moments presented in the previous section can be traced to co-moments. Specifically, we regress the returns of total moment portfolios on the returns of co-moment portfolios. We estimate

$$r_{it}(\tau) = \alpha_i + \beta_{i,CV}r_{CV,t} + \beta_{i,CS}r_{CS,t}(\tau) + \beta_{i,CK}r_{CK,t}(\tau) + \varepsilon_{i,t}, \quad (1)$$

where  $r_{it}(\tau)$  is the High-Low moment portfolio constructed by taking the time  $t$  return of the  $\tau$ -maturity option highest tercile moment portfolio in excess of the lowest tercile moment portfolio,  $r_{CV,t}(\tau)$  is the return of the  $\tau$ -maturity option highest tercile covariance portfolio in excess of the lowest tercile return,  $r_{CS,t}(\tau)$  is the return of the  $\tau$ -maturity option highest tercile co-skewness portfolio in excess of the lowest tercile return, and  $r_{CK,t}(\tau)$  is the return of the  $\tau$ -maturity option highest tercile co-kurtosis portfolio in excess of the lowest tercile return. Details of the co-moment portfolio construction are discussed in Section III of the main paper. Results of these regressions are shown in Table IA-XIII.

As shown in Table IA-XIII, the index and co-moment portfolios explain much of the time-series variation in the returns on volatility-sorted portfolios. The  $R^2$ 's from the regressions exceed 70% for all four maturities, and the slope coefficients are all precisely estimated. The results suggest that the volatility returns load positively on the covariance and coskewness mimicking portfolios, but negatively on co-kurtosis. However, the portfolios retain substantial returns in excess of that explained by the co-moments. The intercepts are economically and statistically large, ranging from  $-66$  basis points to  $-90$  basis points. Thus, the table suggests that while co-moment adjustment can explain much of the

time series variation in the return on volatility-sorted portfolios, it fails to capture the average return associated with these portfolios.

Similar to the Fama and French (1993) three-factor regressions in Table IV of the main paper, the co-moment factors are much less successful in capturing time series variation in the returns on skewness, and kurtosis-sorted portfolios. The intercepts remain economically and statistically large. In the case of skewness, these intercepts range from -79 basis points for the one-month maturity returns to -104 basis points for the six-month maturity returns. Intercepts for the kurtosis-sorted portfolios range from 72 basis points for the one-month maturity returns to 113 basis points for the six-month maturity returns.

Overall, we note that while risk-neutral co-moments, constructed from a single-factor model, do have some association with returns, portfolios sorted on total moments bear premia that do not appear to be related to these co-moment returns. Of course, this may be due to the way in which we measure sources of co-moment risk. In the subsequent subsections, we analyze progressively less restrictive measures of co-moment risk to investigate whether these total moments are in fact attributable to co-movement with some source of aggregate risk.

### *B. Parametric Stochastic Discount Factors with Higher Moments*

In the previous subsection, we attempted to form portfolios that capture time series variation in co-moment risk to isolate sources of total moment risk from co-moment risk. In this subsection, we follow an approach that similarly assumes that risk premia arise due to exposure to a common discount factor. However, we relax the functional form of this relationship and the nature of the risk premia. Specifically, we start from the observation that, under the law of one price, there exists a stochastic discount factor (SDF),  $M_t(\tau)$  that satisfies the Euler equation

$$E_t [M_t(\tau) r_{i,t}(\tau)] = 0, \tag{2}$$

where  $r_{i,t}$  is an excess return for asset  $i$ . Under a correctly specified SDF, this relation will hold exactly, implying that the payoff to asset  $i$  is determined by the covariance of the payoff with the SDF. In

contrast, if this condition does not hold, the implication is that payoffs to the asset cannot be described by covariance with the SDF; in our context, where assets are moment-sorted portfolios, the failure of equation (2) suggests that idiosyncratic moments are associated with returns, even after controlling for co-moments with the SDF.

Of course, inferences about the importance of idiosyncratic moments are relative to a particular specification of the SDF. Failure of the Euler equation condition to hold may represent the importance of idiosyncratic risk or misspecification of the SDFs. In the next three subsections, we use several methods to estimate SDFs that allow for higher co-moments to influence required returns. These methods differ in the details of specific factor proxies, the number of higher co-moments allowed, and the construction of the SDF. However, the goal in each case is to estimate the relation between idiosyncratic moments and residual returns, after adjusting for risk.

We begin by considering a parametric SDF that incorporates information about higher moments of the SDF, and consequently adjusts for co-moment risk with the SDF. In particular, Harvey and Siddique (2000) and Dittmar (2002) examine polynomial SDFs that account for co-skewness, and co-kurtosis risk, respectively. These SDFs are nested in the polynomial specification

$$M_t(\tau) = d_0 + d_1 (R_t^*(\tau)) + d_2 (R_t^*(\tau))^2 + d_3 (R_t^*(\tau))^3 \quad (3)$$

where  $R_t^*(\tau)$  is the  $\tau$ -period return on a traded portfolio that captures the relevant risks in the SDF. We now discuss various approaches to this formulation of the SDF.

### *B.1 The S&P 500 Index*

Our first test uses the S&P 500 as the tangency portfolio in estimating  $M_t$  using equation (3). While numerous studies document violations of the CAPM, evidence in support of higher co-moment CAPMs is stronger. For example, Harvey and Siddique (2000) investigate an SDF that is quadratic in the return on the market portfolio, consistent with a three-moment CAPM. Dittmar (2002) investigates an SDF that is cubic in the return on the market, consistent with a four-moment CAPM. Both studies document

empirical evidence suggesting that higher-moment CAPMs improve upon the standard two-moment CAPM.<sup>1</sup>

The parameters in equation (3) are estimated via GMM using the sample moment restrictions

$$\hat{\alpha} = \frac{1}{T} \sum_{t=1}^T (R_{i,t}(\tau) M_t - 1_N) = 0, \quad (4)$$

where  $R_{i,t}(\tau)$  is a  $10 \times 1$  vector of gross returns comprising three portfolios sorted on  $\tau$ -maturity risk-neutral volatility, three portfolios sorted on  $\tau$ -maturity risk-neutral skewness, three portfolios sorted on  $\tau$ -maturity risk-neutral kurtosis, and a Treasury bill return. We include the risk-free return since Dahlquist and Söderlind (1999) show that failing to do so can result in an SDF that implies a downward-sloping capital market line. We examine three versions of the polynomial SDF,  $M_t$ . The first is linear ( $d_2 = d_3 = 0$ ), accounting for covariance with the tangency portfolio, the second is quadratic ( $d_3 = 0$ ), accounting for co-skewness, and the unrestricted version accounts for co-kurtosis.

In Table IA-XIV, Panels A through D, we report the parameter estimates,  $J$ -statistic of overidentifying restrictions, and point estimates of the excess returns (pricing errors) implied by the SDF for the High-Low moment portfolio returns. In addition, we present Newey-West standard errors or  $p$ -values for the  $J$ -statistic in parentheses. Panel A presents results for the moment-sorted portfolios based on one-month maturity options; Panels B to D present complementary results for options based on three, six, and 12 months to maturity. In all cases, we use data over the period April 1996 through December 2005 for 117 monthly observations. The results in Panels A and B suggest that at shorter maturities, the candidate models cannot be rejected at conventional significance levels. However, examination of the standard errors of the parameter estimates suggest that this failure to reject is more likely attributable to lack of power than fit of the model. With the exception of the intercept term, few of the parameter estimates are statistically different than zero at conventional levels.<sup>2</sup> Further, at the longer-horizon maturities shown in Panels B, C, and D, the specifications are formally rejected at the 10% significance level. One positive result is that the point estimates correspond with economic arguments about co-moment preference; negative signs on the coefficients  $d_1$  and  $d_3$  suggest aversion to covariance and co-kurtosis, whereas the positive sign on  $d_2$  suggests preference for co-skewness.

More importantly, the point estimates of the excess returns on High-Low volatility-, skewness-, and kurtosis-sorted portfolios are large in magnitude. Excess returns average -157, -72, and 86 basis points per month across specifications and maturities for the volatility-sorted, skewness-sorted, and kurtosis-sorted High-Low portfolios, respectively. The precision of the errors varies greatly, and tends to be greater with longer-maturity (six-month and 12-month option) moment-sorted portfolios.

In summary, the evidence suggests that the payoffs to higher moment-sorted portfolios cannot be traced to higher co-moments with respect to a value-weighted market proxy. While the statistical magnitude of the pricing errors is not consistent across all specifications, the economic magnitude of the pricing errors is large. Relative to the risks associated with returns on an S&P 500 tangency portfolio, the returns to the moment-sorted High-Low portfolios appear to be idiosyncratic.

### *B.2 Industry Tangency Portfolio*

Our second investigation of the systematic and idiosyncratic components of the payoffs to higher moment-sorted portfolios estimates the parameters of an SDF polynomial in the returns on the tangency portfolio constructed by a set of basis assets. Our choice to use this proxy is motivated by several considerations. First, we focus on a tangency portfolio as it correctly prices the assets included in its formation by construction. As discussed in Hansen and Jagannathan (1991), there is a one-to-one correspondence under the law of one price between this tangency portfolio and the minimum variance SDF that correctly prices assets. Second, as mentioned above, although the CAPM suggests that the value-weighted market is the tangency portfolio, a large body of empirical evidence suggests that this hypothesis is violated. King (1966) and Ahn, Conrad and Dittmar (2009) suggest that industry portfolios represent a reasonable basis for asset pricing, as sorting on industries tends to maximize within-portfolio covariation and minimize across-portfolio covariation. Consequently, we use a set of 14 industry portfolios to form our tangency portfolio. Descriptions of the industry indices and the tangency portfolio are presented in Table IA-XVII.

Table IA-XV, Panels A to D contains results from estimating the polynomial equation (3) using the industry tangency portfolio to estimate  $M_t$  via GMM. As shown in the table, the results are qualitatively



unchanged from those estimated using the S&P 500 index. There is a slightly larger tendency to reject the overidentifying restrictions of the model, as indicated by the relatively smaller  $p$ -values of the tests compared to those in Panels A to D. However, as in the previous table, any failure to reject seems likely to be due to lack of power, as suggested by the large standard errors of the point estimates of the parameters. We cannot reject the null hypothesis that the parameters are significantly different than zero at conventional levels for any of the specifications.

It should finally be noted that the point estimates of pricing errors in Panels E through H remain large. The average excess return on the High-Low volatility portfolio varies from -98 to -189 basis points per month depending on maturity, comparable to that estimated using the value-weighted market portfolio. Similar results for skewness portfolios indicate average excess returns varying between -21 and -63 basis points, whereas average excess returns for kurtosis-sorted portfolios range from 14 to 70 basis points. Several of these estimates are statistically different from zero at the 10% level. Thus, similar to our conclusion for the value-weighted market portfolio, we conclude that relative to the risks present in the industry tangency portfolio, the returns to moment-sorted extremum portfolios appear to be idiosyncratic.

### *C. Non-Parametric Stochastic Discount Factors with Higher Moments*

In the preceding sections, we estimate the parameters of polynomial SDFs using different proxies for the tangency portfolios, and examine whether these discount factors could explain the returns on moment-sorted portfolios. The evidence suggests that they cannot, indicating that the returns related to these moments appear to be idiosyncratic to the risks embodied in the returns employed in the SDFs. In this section, we pursue a more nonparametric approach for investigating the SDF using the relation between the risk-neutral and physical densities of a candidate asset.

The no-arbitrage condition in asset pricing suggests that the risk-neutral and physical probability measures are related by the equation

$$M_t(s, \tau)P_t(s) = \exp(r\tau)Q_t(s), \quad (5)$$

where  $M_t(s, \tau)$  is the  $\tau$ -period SDF at time  $t$ , contingent on state  $s$ ,  $P_t(s)$  is the physical probability of state  $s$  occurring at time  $t$ , and  $Q_t(s)$  is the risk-neutral probability of state  $s$  occurring at time  $t$ . Given an estimate of the physical and risk-neutral probabilities, this equation implies

$$M_t(s, \tau) = \exp(r\tau) Q_t(s) / P_t(s). \quad (6)$$

Researchers have used this relation in several ways. It is possible to use restrictions on  $M$ , combined with estimates of the risk-neutral distribution  $Q$ , to generate an estimate of the physical distribution  $P$ . For example, Bliss and Panagirtzoglou (2004) assume that investors have either power or exponential utility functions and estimate the risk-neutral distribution of the FTSE100 and S&P500 using options data in order to generate an estimate of the subjective probability distribution of the underlying indexes. They provide evidence that these subjective distributions are better forecasters of the underlying index returns. Alternatively, it is possible to combine estimates of the physical distribution generated from a time-series of returns, with estimates of the risk-neutral distribution inferred from option prices, and use equation (6) to infer something about the SDF  $M$ . For example, Jackwerth (2000) and Aït-Sahalia and Lo (2000) employ this approach to estimate empirical risk-aversion functions.

We take a slightly different approach. Specifically, we follow Eriksson, Ghysels and Wang (2009) and use a Normal Inverse Gaussian (NIG) approximation to generate an estimate of both the subjective and the risk-neutral probability distributions of the market portfolio. We use this information and equation (6) to compute  $M$ . The particular appeal to this approach is that the densities are characterized entirely by the first four moments of the distribution. Hence, given estimates of the mean, variance, skewness, and kurtosis, we can characterize assets' conditional densities. Importantly, the authors show that this method is particularly well suited when the distribution exhibits skewness, and fat tails, as it does in the returns distributions that we examine.

Since the results in the preceding subsection are little affected by our choice of benchmark portfolio, for convenience we focus on the SDF implied by the S&P 500. This choice allows us to easily compute the risk-neutral moments of the benchmark: options on this index are heavily traded, and we can compute these moments analogously to the procedure employed in Section III of the main paper for

individual assets. This contrasts with alternative SDFs, such as those implied by the industry index tangency portfolio or the Fama and French (1993) factors, for which options are not traded on the combination of the assets that generate the tangency portfolio.

The Bakshi, Kapadia and Madan (2003) procedure provides a straightforward approach for the computation of risk-neutral moments; computation of conditional physical moments is somewhat more problematic. While procedures exist for estimating conditional variance, econometric work surrounding the estimation of conditional skewness, and kurtosis is lacking. We follow Jackwerth (2000) and use four years of daily data through the first date of our option sample period to estimate sample variance, skewness, and kurtosis.

Finally, to estimate the conditional physical mean of the market  $\mu_t$ , we follow Jackwerth (2000) and add a risk premium of 8% to the risk-free rate observed at time  $t$ .<sup>3</sup> Once physical and risk-neutral distributions are estimated using the NIG method, the  $\tau$ -period SDF,  $M_t(\tau)$ , is computed as in equation (6) by taking the risk-free discounted ratio of the risk-neutral to physical distribution.

The time series average of SDF functions is depicted in Figure IA.I. In addition to the SDF obtained using the NIG approximation to the density, we also present averages of SDFs obtained by fitting linear, quadratic, and cubic functions of the S&P 500 return support to the NIG approximation each period. The figure shows that the linear and quadratic SDFs are downward sloping throughout their range, consistent with decreasing risk aversion over all levels of wealth. In contrast, the NIG SDF and, to a lesser extent the cubic SDF, are upward-sloping over some portion of the support. In particular, the NIG SDF has a segment in the mid-range of the graph that is increasing, consistent with the evidence in Jackwerth (2000) and Brown and Jackwerth (2001).<sup>4</sup>

While the NIG class is versatile (e.g., as Eriksson, Ghysels and Forsberg (2004) note, its domain is much wider than Gram-Charlier or Edgeworth expansions), there are some restrictions on its use. In particular, the parameters of the NIG approximation may become imaginary and so the distribution cannot be computed. This constraint does not arise in the case of three- and 12-month to maturity options, and arises in only one month for the 6-month maturity options. However, this condition is

frequently violated in the case of one-month to maturity options. As a result, we compute SDFs using only three-, six-, and 12-month-maturity options.

In Table IA-XVI, we report estimates of alphas (pricing errors) of the moment-sorted portfolios implied by the Euler equation calculated from each of the SDFs estimated above, using options closest to three, six, and 12 months to maturity. In general, across all specifications, precision of the estimates is quite poor; despite this, the results suggest that regardless of the specification of the SDF, the sign and the economic magnitudes of the alphas across volatility-, skewness-, and kurtosis-sorted portfolios after risk-adjustment remain similar to those observed in Table IA-II.

In all, the results of this section appear to corroborate the findings from the preceding sections. There is little evidence to suggest that the payoffs of moment-sorted portfolios are related to systematic exposure to a SDF. It is important to note, however, that our results do not necessarily imply that the alpha, or residual return, is an arbitrage profit. Related to the possibility of a misspecified SDF, the estimates of the SDF used to construct  $\alpha$  control only for non-diversifiable risk (including the risk of higher co-moments) in the context of a well diversified portfolio and investors with homogeneous beliefs. For example, if investors have a preference for individual securities' skewness, as in Brunnermeier, Gollier and Parker (2007), or have heterogeneous beliefs as in Chabi-Yo, Ghysels and Renault (2010), they may hold concentrated portfolios and push up the price of securities that are perceived to have a higher probability of an extremely good outcome. As a consequence, the lower subsequent returns of high-skew stocks may represent an equilibrium result.

#### **IV. Simulation study**

To conclude we report the results of a simulation study that compares the method proposed by Xing, Zhang and Zhao (2010) based on the slope of the implied volatility smile with our measure of skewness. We do this for a setting in which we know the closed-form solutions of the conditional

skewness as well as option prices. In particular, we look at the MSE of skewness estimators based on the Heston model. We specify

$$\begin{aligned} dY_t &= (r - \frac{1}{2}V_t)dt + \sqrt{V_t}dW_t^1 \\ dV_t &= \kappa(\theta - V_t)dt + \sigma\rho\sqrt{V_t}dW_t^1 + \sigma\sqrt{1-\rho^2}\sqrt{V_t}dW_t^2, \end{aligned}$$

where  $W^1, W^2$  are two independent Brownian motions. The values of the structural parameters we use are  $r = 5\%$ ,  $\kappa = 1.62$ ,  $\theta = 0.04$ ,  $\sigma = 0.44$ , and  $\rho = -0.76$ . These parameter values are taken from ?.

The density function of  $Y_T$  conditional on the information up to time  $t$ ,  $f(y; t, T, x_t)$ , can be derived from the conditional characteristic function using the inverse Fourier transform; consequently, we can compute the population conditional skewness by taking the third derivative of the characteristic function with respect to  $u$ . Specifically, for any  $u \in C$ , the conditional characteristic function of the log price over some horizon  $T - t$ ,  $E(e^{uY_T} | \mathcal{F}_t)$ , is

$$\Psi(u; t, T, x_t) \doteq \exp(\psi_1(u, T - t) + \psi_2(u, T - t)v_t + uy_t),$$

where  $x_t \doteq (y_t, v_t)$  and

- $\psi_1(u, \tau) = ru\tau - \kappa\theta \left( \frac{\gamma+b}{\sigma^2}\tau + \frac{2}{\sigma^2} \ln \left[ 1 - \frac{\gamma+b}{2\gamma}(1 - e^{-\gamma\tau}) \right] \right)$ ,
- $\psi_2(u, \tau) = -\frac{a(1-e^{-\gamma\tau})}{2\gamma-(\gamma+b)(1-e^{-\gamma\tau})}$ ,

with  $b = \sigma\rho u - \kappa$ ,  $a = u(1 - u)$ , and  $\gamma = \sqrt{b^2 + a\sigma^2}$  (see ?).

The population conditional skewness we calculate is then compared with the following estimators:

1. The Bakshi, Kapadia and Madan (2003) formulas appearing in the Appendix of the main paper taking discrete sum approximations similar to those applied to the sample data. We used two calls with moneyness of 0.8 and 0.95 and symmetrically chosen puts, giving us a total of four contracts to compute the discrete approximations.
2. The difference in the implied volatility of an ATM call and OTM put. We take moneyness to be 0.8 and 0.95 in the simulation study. This corresponds to the method used by Xing, Zhang and

Zhao (2010), and it also matches our choice of discrete points in the Bakshi, Kapadia and Madan (2003) formulas.

We report the results in the table appearing below. In Panel A, the simulations are started with  $Y_0 = 6.9$  and  $V_0 = 0.026$ . In Panel B, the simulations are run with the same starting values but the first 1,000 observations are dropped. We examine skewness estimators taken across three different option maturities. For each maturity, we conduct 500 simulations, with the sample size set to 500 days for each simulation. The table below reports the mean squared errors of the three conditional skewness estimators: (1) our option-based estimator using the formulas appearing in the Appendix of the main paper, and (2) the two implied volatility smile slope-based estimates with moneyness of 0.8 and 0.95.

	Bakshi et al.	Xing et al. M=0.8	Xing et al. M=0.95
Days to maturity			
Panel A			
10	0.13698	0.73807	0.78318
35	0.01003	1.33844	1.47169
60	0.03374	1.71996	1.86358
Panel B			
10	0.16302	0.75779	0.80260
35	0.01096	1.34114	1.47360
60	0.03498	1.70975	1.85193

The mean squared error results in the above table tell us that the range and discretization procedure, which we use in our paper, yield fairly accurate estimates of the conditional skewness, and that the approach of Xing, Zhang and Zhao (2010), which estimates skewness via two points on the implied volatility curve, is relatively noisy in comparison. The mean squared error is typically five to six times larger at the short maturity and even larger as maturities increase.

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**Table IA.I**  
**Descriptive Statistics: Risk Neutral Moment Portfolios**

Panels A and B present summary statistics for portfolios sorted on measures of firms' risk-neutral moments. Firms are sorted on average risk-neutral volatility, skewness, and kurtosis within each calendar quarter into terciles based on 30<sup>th</sup> and 70<sup>th</sup> percentiles. We then form equally weighted portfolios of these firms, holding the moment ranking constant for the subsequent calendar quarter. Risk-neutral moments are calculated using the procedure in Bakshi, Kapadia and Madan (2003); in Panel A we report results using options closest to one month to maturity, and in Panel B results with options closest to six months to maturity. The first column of each panel presents mean monthly returns. The second column presents characteristic-adjusted returns, calculated by determining, for each firm, the Fama and French (1993) 5X5 size- and book-to-market portfolio to which it belongs and subtracting that return. The next three columns present the average individual firm's risk-neutral volatility, skewness, and kurtosis of the stocks in the portfolio for the portfolio formation period. The final three columns display the beta, log market value, and book-to-market equity ratio of the portfolio. The final row of the table presents *t*-statistics of the null hypothesis that the difference in the third and first tercile are zero. Monthly return data cover the period April 1996 through December 2005, for a total of 117 monthly observations.

Panel A: One Month to Maturity								
Volatility								
Tercile	Mean Return	Char-Adj Return	Vol	Skew	Kurt	Beta	ln MV	B/M
1	1.281	0.365	16.450	-1.582	13.603	0.887	15.928	0.347
2	0.994	0.099	26.365	-1.360	11.208	1.388	14.637	0.337
3	0.856	0.134	46.337	-1.531	7.830	1.860	14.003	0.379
3-1	-0.425	-0.231	29.887	0.051	-5.773	0.973	-1.924	0.032
t(3-1)	-0.529	-0.364	46.074	0.716	-7.163	26.080	-29.044	3.237
Skewness								
Tercile	Mean Return	Char-Adj Return	Vol	Skew	Kurt	Beta	ln MV	B/M
1	1.356	0.459	28.017	-3.165	18.310	1.252	15.629	0.325
2	0.992	0.203	30.682	-1.292	7.173	1.431	14.696	0.353
3	0.788	-0.097	28.999	-0.042	8.542	1.385	14.239	0.381
3-1	-0.568	-0.556	0.983	3.123	-9.768	0.133	-1.391	0.056
t(3-1)	-1.426	-1.386	2.685	35.478	-7.754	5.027	-32.009	9.056
Kurtosis								
Tercile	Mean Return	Char-Adj Return	Vol	Skew	Kurt	Beta	ln MV	B/M
1	0.744	-0.082	35.731	-0.525	2.962	1.505	13.929	0.407
2	1.062	0.227	28.846	-1.250	7.363	1.391	14.770	0.345
3	1.304	0.417	23.749	-2.736	23.619	1.190	15.829	0.312
3-1	0.560	0.498	-11.982	-2.211	20.658	-0.315	1.900	-0.096
t(3-1)	1.646	1.575	-22.691	-24.737	16.283	-11.449	55.486	-11.274

Table continued on next page...

Panel B: Six Months to Maturity

Volatility								
Tercile	Mean Return	Char-Adj Return	Vol	Skew	Kurt	Beta	ln MV	B/M
1	1.287	0.353	15.667	-1.687	16.592	0.917	15.947	0.340
2	0.886	0.052	25.156	-1.492	10.134	1.380	14.654	0.334
3	0.997	0.219	44.579	-1.522	8.491	1.833	13.963	0.391
3-1	-0.290	-0.135	28.912	0.165	-8.102	0.917	-1.983	0.051
t(3-1)	-0.389	-0.231	45.589	2.280	-6.920	26.151	-30.090	4.999

Skewness								
Tercile	Mean Return	Char-Adj Return	Vol	Skew	Kurt	Beta	ln MV	B/M
1	1.283	0.383	26.131	-3.340	19.055	1.271	15.652	0.318
2	0.973	0.165	29.563	-1.352	7.738	1.427	14.693	0.353
3	0.885	0.042	28.216	-0.057	9.248	1.368	14.219	0.389
3-1	-0.398	-0.341	2.086	3.283	-9.807	0.097	-1.433	0.070
t(3-1)	-1.013	-0.904	6.552	35.630	-7.110	3.767	-33.819	10.825

Kurtosis								
Tercile	Mean Return	Char-Adj Return	Vol	Skew	Kurt	Beta	ln MV	B/M
1	0.952	0.167	34.978	-0.602	2.111	1.485	13.925	0.417
2	0.969	0.148	27.454	-1.311	7.899	1.374	14.750	0.343
3	1.220	0.299	22.202	-2.850	25.964	1.229	15.862	0.305
3-1	0.268	0.132	-12.775	-2.248	23.854	-0.256	1.937	-0.112
t(3-1)	0.847	0.442	-24.625	-24.446	17.192	-9.523	53.065	-13.250

**Table IA.II**  
**Risk Neutral Moment Double- and Triple-Sorted Portfolios**

The table presents the results of multi-way sorts on risk-neutral moments. We independently sort firms into tercile portfolios based on volatility, skewness, and kurtosis, and then form portfolios on the intersection of volatility and either skewness or kurtosis. For each of the nine portfolios formed, we report the average of subsequent returns. The results from sorting on volatility and skewness, for one-month and six-month options, are reported in Panel A, the results from sorting on volatility and kurtosis are reported in Panel B. We present results from sorting on medians of volatility, skewness, and kurtosis independently in Panel C. In Panels A and B, the number of firms in each portfolio are reported in parentheses below the returns.

Panel A: Volatility-Skewness Sorts								
	One Month to Maturity			Six Months to Maturity				
	S1	S2	S3	S1	S2	S3		
V1	1.197	1.253	0.979	V1	1.296	1.190	1.082	
N	(53)	(32)	(32)	N	(55)	(37)	(30)	
V2	1.147	1.413	0.723	V2	0.916	0.810	0.841	
N	(33)	(26)	(34)	N	(33)	(24)	(35)	
V3	1.313	0.463	0.634	V3	1.440	0.743	0.793	
N	(36)	(30)	(26)	N	(34)	(31)	(27)	
Panel B: Volatility-Kurtosis Sorts								
	One Month to Maturity			Six Months to Maturity				
	K1	K2	K3	K1	K2	K3		
V1	1.179	1.016	1.319	V1	1.234	0.976	1.379	
N	(33)	(38)	(52)	N	(31)	(37)	(54)	
V2	0.972	0.801	1.354	V2	0.954	0.406	1.045	
N	(37)	(29)	(27)	N	(38)	(29)	(26)	
V3	0.661	1.135	0.671	V3	0.845	1.147	0.836	
N	(53)	(26)	(13)	N	(54)	(26)	(12)	
Panel C: Volatility-Skewness-Kurtosis Sorts								
One Month to Maturity								
	V1S1K1	V1S1K2	V1S2K1	V1S2K2	V2S1K1	V2S1K2	V2S2K1	V2S2K2
Mean	1.385	1.241	1.348	0.789	0.853	1.097	0.562	0.487
N	(8)	(72)	(50)	(24)	(26)	(47)	(69)	(11)
Six Months to Maturity								
	V1S1K1	V1S1K2	V1S2K1	V1S2K2	V2S1K1	V2S1K2	V2S2K1	V2S2K2
Mean	0.268	1.191	1.319	0.756	1.048	1.195	0.641	0.410
N	(8)	(74)	(48)	(23)	(25)	(46)	(72)	(11)

**Table IA.III**  
**Fama and French Factor Risk Adjustment: Risk Neutral Moment-Sorted Portfolios**

The table presents the results of time series regressions of excess return differentials (High-Low) between portfolios ranked on risk-neutral volatility, skewness, and kurtosis on the three Fama and French (1993) factors MRP (the return on the value-weighted market portfolio in excess of a one-month T-Bill), SMB (the difference in returns on a portfolio of small capitalization and large capitalization stocks), and HML (the difference in returns on a portfolio of high and low book equity to market equity stocks). The moment-sorted portfolios are equally weighted, formed on the basis of terciles and re-formed each quarter. The table presents point estimates of the coefficients and t-statistics. In Panel A, we use options closest to Three Months to maturity to calculate risk-neutral moments; 12 month options are used in Panel B. Data cover the period April 1996 through December 2005 for 117 monthly observations.

Panel A: One Month to Maturity					
Volatility					
Rank	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	Adj. $R^2$
1	0.555	0.755	-0.461	-0.197	0.833
	2.941	18.150	-10.208	-3.668	
2	0.551	0.952	-0.390	-0.874	0.868
	1.845	14.475	-5.465	-10.276	
3	0.147	1.353	-0.208	-1.131	0.892
	0.380	15.836	-2.239	-10.241	
3-1	-0.408	0.598	0.253	-0.933	0.776
	-0.982	6.528	2.549	-7.888	
Skewness					
Rank	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	Adj. $R^2$
1	0.853	0.836	-0.243	-0.710	0.762
	2.290	10.184	-2.723	-6.688	
2	0.503	1.019	-0.392	-0.898	0.903
	1.894	17.401	-6.165	-11.873	
3	-0.081	1.184	-0.424	-0.587	0.905
	-0.324	21.465	-7.088	-8.237	
3-1	-0.934	0.347	-0.182	0.123	0.109
	-2.280	3.843	-1.851	1.053	
Kurtosis					
Rank	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	Adj. $R^2$
1	-0.127	1.209	-0.300	-0.678	0.904
	-0.466	20.067	-4.581	-8.700	
2	0.550	1.023	-0.424	-0.858	0.910
	2.216	18.717	-7.152	-12.138	
3	0.834	0.804	-0.323	-0.673	0.800
	2.688	11.762	-4.356	-7.617	
3-1	0.961	-0.405	-0.024	0.005	0.367
	3.254	-6.221	-0.335	0.054	

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Panel B: Six Months to Maturity

Volatility					
Rank	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	Adj. $R^2$
1	0.635	0.748	-0.458	-0.289	0.823
	3.091	16.518	-9.324	-4.941	
2	0.499	0.953	-0.468	-0.913	0.875
	1.716	14.868	-6.733	-11.021	
3	0.139	1.360	-0.106	-0.987	0.894
	0.373	16.529	-1.184	-9.283	
3-1	-0.496	0.612	0.353	-0.698	0.759
	-1.239	6.936	3.684	-6.123	

Skewness					
Rank	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	Adj. $R^2$
1	0.867	0.830	-0.293	-0.794	0.778
	2.358	10.243	-3.331	-7.581	
2	0.445	1.047	-0.446	-0.857	0.896
	1.636	17.476	-6.852	-11.068	
3	-0.017	1.152	-0.302	-0.558	0.907
	-0.070	21.286	-5.143	-7.986	
3-1	-0.884	0.322	-0.009	0.236	0.076
	-2.145	3.541	-0.092	2.007	

Kurtosis					
Rank	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	Adj. $R^2$
1	-0.033	1.194	-0.204	-0.550	0.902
	-0.126	20.507	-3.226	-7.304	
2	0.483	1.027	-0.473	-0.874	0.901
	1.844	17.797	-7.543	-11.723	
3	0.829	0.815	-0.355	-0.779	0.811
	2.589	11.542	-4.633	-8.542	
3-1	0.862	-0.380	-0.151	-0.229	0.279
	2.941	-5.875	-2.152	-2.746	

**Table IA.IV**  
**Fama and French and Liquidity Factor Risk Adjustment: Risk Neutral Moment-Sorted Portfolios**

The table presents the results of time series regressions of excess return differentials (High-Low) between portfolios ranked on risk-neutral volatility, skewness, and kurtosis on the three Fama and French (1993) factors MRP (the return on the value-weighted market portfolio in excess of a one-month T-Bill), SMB (the difference in returns on a portfolio of small capitalization and large capitalization stocks), and HML (the difference in returns on a portfolio of high and low book equity to market equity stocks). We also include the Pástor and Stambaugh (2003) liquidity factor, LIQ. The moment-sorted portfolios are equally weighted, formed on the basis of terciles and re-formed each quarter. The table presents point estimates of the coefficients and t-statistics. In Panel A, we use options closest to One Month to maturity to calculate risk-neutral moments; 3, 6, and 12 month options are used in Panels B-D. Data cover the period April 1996 through December 2005 for 117 monthly observations.

Panel A: One Month to Maturity						
Volatility						
Rank	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	$\beta_{LIQ}$	$\bar{R}^2$
1	-0.593	1.391	-0.224	0.282	0.181	0.570
	-1.146	10.332	-1.712	1.603	1.251	
2	-1.036	1.660	0.157	-0.197	0.204	0.642
	-1.647	10.142	0.990	-0.920	1.160	
3	-1.229	2.067	0.701	-0.220	0.136	0.694
	-1.652	10.680	3.730	-0.869	0.653	
3-1	-0.636	0.675	0.925	-0.502	-0.045	0.787
	-1.745	7.129	10.046	-4.051	-0.442	
Skewness						
Rank	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	$\beta_{LIQ}$	$\bar{R}^2$
1	-0.545	1.545	0.141	-0.267	0.211	0.718
	-1.075	11.718	1.102	-1.550	1.495	
2	-1.018	1.761	0.183	-0.121	0.171	0.636
	-1.548	10.294	1.103	-0.540	0.932	
3	-1.301	1.778	0.301	0.225	0.150	0.559
	-1.802	9.465	1.651	0.918	0.745	
3-1	-0.756	0.233	0.160	0.493	-0.061	0.077
	-1.894	2.242	1.587	3.628	-0.551	
Kurtosis						
Rank	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	$\beta_{LIQ}$	$\bar{R}^2$
1	-1.302	1.802	0.450	0.167	0.194	0.634
	-1.981	10.536	2.713	0.746	1.056	
2	-0.982	1.756	0.144	-0.110	0.138	0.616
	-1.454	9.991	0.845	-0.480	0.733	
3	-0.593	1.527	0.044	-0.223	0.212	0.676
	-1.105	10.946	0.322	-1.221	1.419	
3-1	0.710	-0.274	-0.407	-0.390	0.019	0.248
	2.446	-3.636	-5.552	-3.948	0.230	

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Panel B: Three Months to Maturity

Volatility						
Rank	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	$\beta_{LIQ}$	$\bar{R}^2$
1	-0.606	1.385	-0.237	0.383	0.142	0.544
	-1.147	10.083	-1.81	2.134	0.960	
2	-0.951	1.645	0.214	-0.187	0.214	0.659
	-1.569	10.435	1.401	-0.909	1.263	
3	-1.333	2.092	0.638	-0.333	0.161	0.695
	-1.748	10.547	3.311	-1.285	0.758	
3-1	-0.727	0.707	0.875	-0.716	0.020	0.812
	-1.972	7.369	9.390	-5.708	0.193	

  

Skewness						
Rank	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	$\beta_{LIQ}$	$\bar{R}^2$
1	-0.510	1.543	0.216	-0.171	0.178	0.727
	-1.053	12.240	1.764	-1.036	1.318	
2	-0.964	1.716	0.194	-0.195	0.196	0.623
	-1.426	9.746	1.135	-0.848	1.040	
3	-1.406	1.840	0.212	0.229	0.149	0.572
	-1.961	9.869	1.171	0.941	0.745	
3-1	-0.895	0.297	-0.004	0.400	-0.292	0.075
	-2.231	2.282	-0.037	2.907	-0.259	

  

Kurtosis						
Rank	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	$\beta_{LIQ}$	$\bar{R}^2$
1	-1.453	1.875	0.395	0.097	0.145	0.638
	-2.145	10.633	2.309	0.420	0.765	
2	-0.912	1.721	0.168	-0.133	0.187	0.616
	-1.354	9.819	0.985	-0.580	0.996	
3	-0.535	1.501	0.067	-0.122	0.196	0.680
	-1.054	11.361	0.524	-0.708	1.380	
3-1	0.918	-0.374	-0.328	-0.219	0.051	0.264
	2.975	-4.659	-4.208	-2.087	0.590	

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Panel C: Six Months to Maturity

Volatility						
Rank	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	$\beta_{LIQ}$	$\bar{R}^2$
1	-0.520	1.363	-0.246	0.309	0.166	0.549
	-0.992	9.989	-1.860	1.734	1.136	
2	-0.945	1.658	0.202	-0.115	0.186	0.646
	-1.546	10.432	1.307	-0.554	1.093	
3	-1.427	2.097	0.664	-0.357	0.174	0.699
	-1.864	10.528	3.435	-1.369	0.813	
3-1	-0.906	0.734	0.911	-0.666	0.008	0.797
	-2.336	7.272	9.293	-5.047	0.069	

Skewness						
Rank	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	$\beta_{LIQ}$	$\bar{R}^2$
1	-0.432	1.534	0.208	-0.212	0.222	0.711
	-0.847	11.554	1.610	-1.221	1.559	
2	-0.990	1.731	0.200	-0.090	0.143	0.630
	-1.524	10.241	1.220	-0.408	0.789	
3	-1.449	1.828	0.212	0.129	0.177	0.577
	-1.999	9.695	1.156	0.525	0.874	
3-1	-1.017	0.295	0.004	0.341	-0.045	0.054
	-2.522	2.809	0.041	2.487	-0.400	

Kurtosis						
Rank	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	$\beta_{LIQ}$	$\bar{R}^2$
1	-1.611	1.886	0.371	0.110	0.178	0.630
	-2.335	10.501	2.126	0.470	0.926	
2	-0.812	1.716	0.170	-0.097	0.132	0.623
	-1.248	10.130	1.034	-0.437	0.728	
3	-0.513	1.498	0.089	-0.184	0.236	0.671
	-0.965	10.821	0.664	-1.016	1.586	
3-1	1.098	-0.388	-0.281	-0.294	0.057	0.198
	3.333	-4.527	-3.382	-2.625	0.621	



Panel D: 12 Months to Maturity

Volatility						
Rank	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	$\beta_{LIQ}$	$\bar{R}^2$
1	-0.543	1.386	-0.216	0.326	0.176	0.553
	-1.028	10.081	-1.620	1.813	1.196	
2	-0.969	1.675	0.210	-0.101	0.165	0.652
	-1.599	10.621	1.371	-0.491	0.974	
3	-1.371	2.052	0.623	-0.392	0.193	0.692
	1.790	10.301	3.221	-1.504	0.901	
3-1	-0.827	-0.666	0.839	-0.718	0.016	0.789
	-2.165	6.699	8.693	-5.521	0.153	

Skewness						
Rank	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	$\beta_{LIQ}$	$\bar{R}^2$
1	-0.439	1.534	0.190	-0.217	0.240	0.711
	-0.861	11.555	1.471	-1.250	1.686	
2	-0.929	1.718	0.206	-0.074	0.141	0.628
	-1.439	10.234	1.265	-0.337	0.785	
3	-1.526	1.845	0.222	0.113	0.161	0.578
	-2.083	9.686	1.198	0.453	0.788	
3-1	-1.086	0.312	0.032	0.300	-0.079	0.048
	-2.647	2.9120	0.310	2.362	-0.691	

Kurtosis						
Rank	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	$\beta_{LIQ}$	$\bar{R}^2$
1	-1.599	1.891	0.373	0.082	0.186	0.632
	-2.299	10.446	2.125	0.347	0.960	
2	-0.841	1.731	0.155	-0.059	0.117	0.621
	-1.295	10.244	0.943	-0.268	0.643	
3	-0.489	1.472	0.107	-0.205	0.249	0.671
	-0.921	1.662	0.799	-1.138	1.678	
3-1	1.110	-0.418	-0.266	-0.288	0.062	0.203
	3.271	-4.736	-3.107	-2.490	0.657	

**Table IA.V**  
**Descriptive Statistics: Risk Neutral Co-moment Portfolios**

Panels A and B present summary statistics for portfolios sorted on measures of firms' risk-neutral moments. Firms are sorted on average risk-neutral covariance, co-skewness, and co-kurtosis within each calendar quarter into terciles based on 30<sup>th</sup> and 70<sup>th</sup> percentiles. We then form equally weighted portfolios of these firms, holding the moment ranking constant for the subsequent calendar quarter. The co-moments are calculated using firm risk-neutral moments and risk-neutral moments on the S&P 500 index. Specifically, we calculate the co-moments as  $COVAR_i^Q = \frac{S_{i,t}}{C_{i,t}} \mathcal{N}\left(\frac{\ln(S_{i,t}/K_i) + (r - \delta + 0.5\sigma^2)\tau}{\sigma\sqrt{\tau}}\right) \beta_i = b_i$ ,  $COSKEW_i^Q = b_i SKEW_{m,t}^Q(\tau) \frac{VAR_{i,t}^Q(\tau)}{\sqrt{VAR_{m,t}^Q(\tau)}}$ ,  $COKURT_i^Q = b_i \frac{KURT_{m,t}^Q(\tau)}{VAR_{i,t}^Q(\tau)VAR_{m,t}^Q(\tau)}$ . In these expressions,  $S_{i,t}$  is the stock price on date  $t$ ,  $C_{i,t}$  is the call price,  $K_i$  is the strike price,  $r$  is the risk-free rate,  $\delta$  is the dividend yield, and  $\beta_i$  is the Dimson beta calculated over the past 250 trading days. The subscript  $m$  refers to the S&P 500 index. Risk-neutral moments are calculated using the procedure in Bakshi, Kapadia and Madan (2003); in Panel A we report results using options closest to one month to maturity, and in Panel B results with options closest to six months to maturity. The first column of each panel presents mean monthly returns. The second column presents characteristic-adjusted returns, calculated by determining, for each firm, the Fama and French (1993) 5X5 size- and book-to-market portfolio to which it belongs and subtracting that return. The next three columns present the average risk-neutral volatility, skewness, and kurtosis of the portfolio for the portfolio formation period. The final three columns display the beta, log market value, and book-to-market equity ratio of the portfolio. Monthly return data cover the period April 1996 through December 2005, for a total of 117 monthly observations.

Panel A: One Month to Maturity								
Covariance								
Tercile	Mean Return	Char-Adj Return	Vol	Skew	Kurt	Beta	ln MV	B/M
1	0.892	0.263	27.303	-1.192	13.039	1.378	14.727	0.317
2	0.872	0.039	27.901	-1.398	9.258	1.351	14.801	0.344
3	1.255	0.214	32.632	-1.988	10.892	1.419	15.284	0.370
3-1	0.363	-0.050	5.329	-0.796	-2.147	0.040	0.557	0.053
t(3-1)	0.842	-0.127	6.131	-10.630	-2.176	1.141	5.596	6.107
Coskewness								
Tercile	Mean Return	Char-Adj Return	Vol	Skew	Kurt	Beta	ln MV	B/M
1	1.140	0.051	24.578	-1.966	11.993	1.167	15.585	0.361
2	1.040	0.294	30.523	-1.433	9.094	1.430	14.772	0.345
3	0.782	0.072	31.839	-1.169	12.147	1.535	14.459	0.324
3-1	-0.358	0.021	7.261	0.796	0.154	0.368	-1.126	-0.037
t(3-1)	-0.716	0.051	14.927	10.877	0.158	14.767	-10.467	-4.667
Cokurtosis								
Tercile	Mean Return	Char-Adj Return	Vol	Skew	Kurt	Beta	ln MV	B/M
1	0.930	0.185	37.079	-1.199	10.131	1.669	14.306	0.335
2	0.877	0.122	29.695	-1.440	10.245	1.435	14.728	0.344
3	1.204	0.171	20.464	-1.926	12.463	1.038	15.789	0.352
3-1	0.274	-0.014	-16.616	-0.727	2.332	-0.631	1.483	0.017
t(3-1)	0.462	-0.029	-34.253	-10.659	3.457	-20.624	14.611	2.081

Table continued on next page...

Panel B: Six Months to Maturity

Covariance								
Tercile	Mean Return	Char-Adj Return	Vol	Skew	Kurt	Beta	ln MV	B/M
1	0.665	-0.057	28.193	-1.296	12.999	1.596	14.363	0.303
2	1.021	0.252	27.467	-1.450	9.878	1.427	14.714	0.336
3	1.282	0.223	27.857	-2.118	12.475	1.128	15.750	0.393
3-1	0.617	0.280	-0.336	-0.823	-0.524	-0.468	1.387	0.089
t(3-1)	0.984	0.561	-0.421	-9.338	-0.426	-19.209	15.391	12.796

Co-skewness								
Tercile	Mean Return	Char-Adj Return	Vol	Skew	Kurt	Beta	ln MV	B/M
1	1.309	0.263	24.091	-2.112	13.193	1.031	15.892	0.373
2	1.037	0.234	28.478	-1.478	9.864	1.446	14.694	0.346
3	0.617	-0.071	30.595	-1.266	12.285	1.678	14.245	0.311
3-1	-0.691	-0.334	6.504	0.845	-0.908	0.648	-1.647	-0.063
t(3-1)	-1.062	-0.642	11.751	9.724	-0.747	26.973	-18.374	-9.476

Co-kurtosis								
Tercile	Mean Return	Char-Adj Return	Vol	Skew	Kurt	Beta	ln MV	B/M
1	0.747	0.040	33.062	-1.266	11.774	1.753	14.163	0.316
2	0.963	0.122	28.960	-1.474	9.861	1.456	14.695	0.349
3	1.274	0.299	20.993	-2.114	13.720	0.954	15.971	0.364
3-1	0.526	0.259	-12.069	-0.848	1.945	-0.799	1.808	0.048
t(3-1)	0.719	0.446	-32.640	-9.969	1.591	-29.149	20.184	6.466

**Table IA.VI**  
**Fama and French Factor Risk Adjustment: Risk Neutral Co-moment-Sorted Portfolios**

The table presents the results of time series regressions of excess return differentials (High-Low) between portfolios ranked on risk-neutral covariance, co-skewness, and co-kurtosis on the three Fama and French (1993) factors MRP (the return on the value-weighted market portfolio in excess of a one-month T-Bill), SMB (the difference in returns on a portfolio of small capitalization and large capitalization stocks), and HML (the difference in returns on a portfolio of high and low book equity to market equity stocks). The co-moments are calculated using firm risk-neutral moments and risk-neutral moments on the S&P 500 index. Specifically, we calculate the co-moments as  $COVAR_i^Q = \frac{S_{i,t}}{C_{i,t}} \mathcal{N} \left( \frac{\ln(S_{i,t}/K_i) + (r - \delta + 0.5\sigma^2)\tau}{\sigma\sqrt{\tau}} \right) \beta_i = b_i$ ,  $COSKEW_i^Q = b_i SKEW_{m,t}^Q(\tau) \frac{VAR_{i,t}^Q(\tau)}{\sqrt{VAR_{m,t}^Q(\tau)}}$ ,  $COKURT_i^Q = b_i \frac{KURT_{m,t}^Q(\tau)}{VAR_{i,t}^Q(\tau)VAR_{m,t}^Q(\tau)}$ . In these expressions,  $S_{i,t}$  is the stock price on date  $t$ ,  $C_{i,t}$  is the call price,  $K_i$  is the strike price,  $r$  is the risk-free rate,  $\delta$  is the dividend yield, and  $\beta_i$  is the Dimson beta calculated over the past 250 trading days. The subscript  $m$  refers to the S&P 500 index. The moment-sorted portfolios are equally weighted, formed on the basis of terciles and re-formed each quarter. The table presents point estimates of the coefficients and t-statistics. In Panel A, we use options closest to Three Months to maturity to calculate risk-neutral moments; 12 month options are used in Panel B. Data cover the period April 1996 through December 2005 for 117 monthly observations.

One Month to Maturity					
Covariance					
Rank	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	Adj. $R^2$
1	0.355	1.116	-0.366	-1.044	0.898
	1.142	16.301	-4.929	-11.807	
2	0.496	0.942	-0.366	-0.906	0.868
	1.634	14.088	-5.044	-10.480	
3	0.355	0.986	-0.380	-0.309	0.792
	1.196	15.084	-5.348	-3.655	
3-1	0.000	-0.129	-0.013	0.736	0.509
	0.001	-1.509	-0.143	6.632	
Coskewness					
Rank	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	Adj. $R^2$
1	0.317	0.893	-0.416	-0.238	0.778
	1.178	15.042	-6.450	-3.096	
2	0.634	1.003	-0.409	-0.909	0.872
	2.069	14.838	-5.582	-10.405	
3	0.201	1.128	-0.272	-1.110	0.898
	0.613	15.590	-3.459	-11.877	
3-1	-0.116	0.235	0.144	-0.873	0.675
	-0.306	2.815	1.590	-8.094	
Cokurtosis					
Rank	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	Adj. $R^2$
1	0.194	1.200	-0.215	-1.155	0.899
	0.556	15.577	-2.571	-11.605	
2	0.500	1.001	-0.459	-0.875	0.876
	1.704	15.467	-6.536	-10.458	
3	0.503	0.823	-0.406	-0.239	0.736
	1.793	13.307	-6.044	-2.985	
3-1	0.309	-0.377	-0.191	0.917	0.734
	0.797	-4.417	-2.060	8.313	

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Six Months to Maturity

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Covariance

Rank	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	Adj. $R^2$
1	0.332	1.127	-0.354	-1.246	0.880
	0.890	13.693	-3.968	-11.718	
2	0.500	0.964	-0.295	-0.895	0.861
	1.562	13.672	-3.859	-9.819	
3	0.372	0.946	-0.485	-0.121	0.852
	1.790	20.672	-9.766	-2.054	
3-1	0.039	-0.181	-0.131	1.125	0.729
	0.101	-2.114	-1.407	10.174	

Coskewness

Rank	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	Adj. $R^2$
1	0.402	0.863	-0.485	-0.122	0.834
	1.974	19.207	-9.940	-2.102	
2	0.561	1.003	-0.349	-0.913	0.869
	1.772	14.372	-4.610	-10.120	
3	0.218	1.157	-0.283	-1.221	0.877
	0.563	13.553	-3.052	-11.067	
3-1	-0.184	0.294	0.202	-1.099	0.744
	-0.453	3.272	2.072	-9.467	

Cokurtosis

Rank	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	Adj. $R^2$
1	0.274	1.219	-0.272	-1.233	0.900
	0.766	15.456	-3.173	-12.098	
2	0.465	0.984	-0.349	-0.889	0.864
	1.479	14.187	-4.630	-9.914	
3	0.472	0.827	-0.497	-0.143	0.798
	2.120	16.828	-9.320	-2.247	
3-1	0.198	-0.393	-0.225	1.091	0.799
	0.533	-4.789	-2.529	10.290	

**Table IA.VII**  
**Descriptive Statistics: Risk Neutral Idiosyncratic Moment Portfolios**

Panels A and B present summary statistics for portfolios sorted on measures of firms' risk-neutral moments. Firms are sorted on average risk-neutral idiosyncratic volatility, skewness, and kurtosis within each calendar quarter into terciles based on 30<sup>th</sup> and 70<sup>th</sup> percentiles. Idiosyncratic moments are calculated by regressing daily estimates of each firm's total moment on measures of the risk-neutral co-moment within a calendar quarter:  $\nu_{i,t}^Q = \kappa_{0i}^{\nu'} + \kappa_{1i}^{\nu'} COVAR_{i,t}^Q + \zeta_{i,t}^{\nu'}$ ,  $S_{i,t}^Q = \kappa_{0i}^S + \kappa_{1i}^S COSKEW_{i,t}^Q + \zeta_{i,t}^S$ , and  $\mathcal{X}_{i,t}^Q = \kappa_{0i}^{\mathcal{X}} + \kappa_{1i}^{\mathcal{X}} COKURT_{i,t}^Q + \zeta_{i,t}^{\mathcal{X}}$ . The average unexplained portion of the moments,  $\kappa_{0i}^{\nu'}$ ,  $\kappa_{0i}^S$ , and  $\kappa_{0i}^{\mathcal{X}}$ , are used as the measure of idiosyncratic moments. We then form equally weighted portfolios of these firms, holding the moment ranking constant for the subsequent calendar quarter. Risk-neutral moments are calculated using the procedure in Bakshi, Kapadia and Madan (2003); in Panel A we report results using options closest to one month to maturity, and in Panel B results with options closest to six months to maturity. The first column of each panel presents mean monthly returns. The second column presents characteristic-adjusted returns, calculated by determining, for each firm, the Fama and French (1993) 5X5 size- and book-to-market portfolio to which it belongs and subtracting that return. The next three columns present the average risk-neutral volatility, skewness, and kurtosis of the stocks in the portfolio for the portfolio formation period. The final three columns display the beta, log market value, and book-to-market equity ratio of the portfolio. Monthly return data cover the period April 1996 through December 2005, for a total of 117 monthly observations.

Panel A: One Month to Maturity								
Idiosyncratic Volatility								
Tercile	Mean Return	Char-Adj Return	Vol	Skew	Kurt	Beta	ln MV	B/M
1	1.168	0.195	17.813	-1.809	14.468	0.904	16.063	0.348
2	1.097	0.297	26.327	-1.386	11.173	1.390	14.739	0.329
3	0.674	-0.081	44.220	-1.389	6.888	1.894	14.009	0.359
3-1	-0.494	-0.276	26.407	0.420	-7.580	0.990	-2.054	0.011
t(3-1)	-0.575	-0.416	47.942	7.653	-11.778	25.883	-28.092	1.215
Idiosyncratic Skewness								
Tercile	Mean Return	Char-Adj Return	Vol	Skew	Kurt	Beta	ln MV	B/M
1	1.431	0.541	27.711	-2.966	17.194	1.275	15.699	0.318
2	0.824	0.056	30.426	-1.367	7.696	1.456	14.773	0.339
3	0.779	-0.102	28.841	-0.259	8.819	1.378	14.349	0.377
3-1	-0.652	-0.643	1.131	2.707	-8.375	0.103	-1.350	0.059
t(3-1)	-1.567	-1.577	3.209	45.279	-8.197	4.131	-33.327	8.773
Idiosyncratic Kurtosis								
Tercile	Mean Return	Char-Adj Return	Vol	Skew	Kurt	Beta	ln MV	B/M
1	0.735	-0.128	35.810	-0.714	4.004	1.543	14.028	0.388
2	0.926	0.164	28.484	-1.331	7.888	1.391	14.834	0.335
3	1.337	0.444	23.355	-2.556	21.740	1.201	15.925	0.313
3-1	0.602	0.572	-12.455	-1.842	17.737	-0.342	1.897	-0.075
t(3-1)	1.482	1.700	-23.825	-28.533	17.114	-13.024	53.150	-9.449

Table continued on next page...

Panel B: Six Months to Maturity

Idiosyncratic Volatility								
Tercile	Mean Return	Char-Adj Return	Vol	Skew	Kurt	Beta	ln MV	B/M
1	1.198	0.211	15.858	-1.786	16.850	0.905	16.061	0.336
2	0.901	0.129	25.069	-1.559	10.539	1.403	14.753	0.327
3	0.905	0.140	43.391	-1.481	7.732	1.868	13.994	0.374
3-1	-0.293	-0.071	27.533	0.305	-9.118	0.964	-2.067	0.039
t(3-1)	-0.373	-0.117	46.368	6.009	-9.735	26.124	-27.514	4.449

Idiosyncratic Skewness								
Tercile	Mean Return	Char-Adj Return	Vol	Skew	Kurt	Beta	ln MV	B/M
1	1.242	0.313	25.574	-3.227	18.434	1.287	15.719	0.312
2	0.968	0.218	29.101	-1.435	8.391	1.456	14.772	0.339
3	0.775	-0.069	28.282	-0.209	9.025	1.363	14.326	0.382
3-1	-0.467	-0.382	2.708	3.018	-9.409	0.076	-1.393	0.070
t(3-1)	-1.134	-0.961	9.136	41.162	-7.734	3.108	-30.343	9.522

Idiosyncratic Kurtosis								
Tercile	Mean Return	Char-Adj Return	Vol	Skew	Kurt	Beta	ln MV	B/M
1	0.892	0.096	34.789	-0.705	2.390	1.521	14.008	0.405
2	0.864	0.098	27.060	-1.421	8.520	1.373	14.851	0.330
3	1.264	0.336	21.815	-2.747	24.869	1.244	15.921	0.304
3-1	0.372	0.240	-12.975	-2.042	22.479	-0.277	1.913	-0.101
t(3-1)	1.007	0.715	-25.373	-28.240	17.507	-10.928	49.085	-11.347

**Table IA.VIII**  
**Fama and French Factor Risk Adjustment: Risk Neutral Idiosyncratic Moment-Sorted Portfolios**

The table presents the results of time series regressions of excess return differentials (High-Low) between portfolios ranked on idiosyncratic risk-neutral volatility, skewness, and kurtosis on the three Fama and French (1993) factors MRP (the return on the value-weighted market portfolio in excess of a one-month T-Bill), SMB (the difference in returns on a portfolio of small capitalization and large capitalization stocks), and HML (the difference in returns on a portfolio of high and low book equity to market equity stocks). Idiosyncratic moments are calculated by regressing daily estimates of each firm's total moment on measures of the risk-neutral co-moment within a calendar quarter:

$$\begin{aligned} \nu_{i,t}^Q &= \kappa_{0i}^{\nu} + \kappa_{1i}^{\nu} COVAR_{i,t}^Q + \zeta_{i,t}^{\nu} \\ s_{i,t}^Q &= \kappa_{0i}^s + \kappa_{1i}^s COSKEW_{i,t}^Q + \zeta_{i,t}^s \\ \chi_{i,t}^Q &= \kappa_{0i}^{\chi} + \kappa_{1i}^{\chi} COKURT_{i,t}^Q + \zeta_{i,t}^{\chi} \end{aligned}$$

We take the average unexplained portion of the moments,  $\kappa_{0i}^{\nu}$ ,  $\kappa_{0i}^s$ , and  $\kappa_{0i}^{\chi}$ , and use these as the measure of idiosyncratic moments. Moment-sorted portfolios are equally weighted, formed on the basis of terciles and re-formed each quarter. The table presents point estimates of the coefficients and t-statistics. In Panel A, we use options closest to One Month to maturity to calculate risk-neutral moments; Six Month options are used in Panel B. Data cover the period April 1996 through December 2005 for 117 monthly observations.

One Month to Maturity					
Idiosyncratic Volatility					
Tercile	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	Adj. $R^2$
1	0.576	0.788	-0.487	-0.238	0.835
	2.895	17.962	-10.236	-4.197	
2	0.524	0.956	-0.400	-0.842	0.872
	1.817	15.043	-5.803	-10.257	
3	0.092	1.295	-0.212	-1.199	0.873
	0.216	13.881	-2.089	-9.944	
3-1	-0.485	0.507	0.276	-0.961	0.732
	-1.068	5.073	2.541	-7.439	
Idiosyncratic Skewness					
Tercile	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	Adj. $R^2$
1	0.891	0.824	-0.294	-0.754	0.762
	2.389	10.021	-3.298	-7.097	
2	0.441	1.009	-0.333	-0.831	0.910
	1.780	18.468	-5.621	-11.772	
3	-0.108	1.188	-0.495	-0.698	0.897
	-0.398	19.786	-7.589	-8.993	
3-1	-1.000	0.364	-0.200	0.056	0.146
	-2.569	4.244	-2.151	0.509	
Idiosyncratic Kurtosis					
Tercile	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	Adj. $R^2$
1	-0.285	1.272	-0.394	-0.729	0.897
	-0.960	19.438	-5.550	-8.617	
2	0.571	0.973	-0.367	-0.823	0.897
	2.213	17.107	-5.946	-11.194	
3	0.892	0.788	-0.351	-0.736	0.809
	2.895	11.607	-4.758	-8.387	
3-1	1.177	-0.484	0.044	-0.007	0.377
	3.638	-6.793	0.565	-0.075	



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Six Months to Maturity

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Idiosyncratic Volatility

Tercile	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	Adj. $R^2$
1	0.595	0.743	-0.469	-0.230	0.818
	2.971	16.838	-9.792	-4.027	
2	0.489	0.981	-0.415	-0.857	0.883
	1.741	15.857	-6.176	-10.727	
3	0.120	1.307	-0.211	-1.188	0.857
	0.264	13.067	-1.945	-9.188	
3-1	-0.475	0.565	0.258	-0.958	0.717
	-0.983	5.304	2.228	-6.965	

Idiosyncratic Skewness

Tercile	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	Adj. $R^2$
1	0.905	0.827	-0.261	-0.755	0.758
	2.369	9.822	-2.855	-6.932	
2	0.406	1.023	-0.334	-0.840	0.900
	1.522	17.399	-5.238	-11.058	
3	-0.075	1.166	-0.527	-0.686	0.897
	-0.284	19.938	-8.293	-9.071	
3-1	-0.981	0.339	-0.265	0.069	0.121
	-2.334	3.657	-2.640	0.577	

Idiosyncratic Kurtosis

Tercile	$\alpha$	$\beta_{MRP}$	$\beta_{SMB}$	$\beta_{HML}$	Adj. $R^2$
1	-0.217	1.276	-0.430	-0.699	0.897
	-0.744	19.815	-6.145	-8.403	
2	0.517	0.977	-0.374	-0.844	0.895
	1.968	16.858	-5.941	-11.263	
3	0.896	0.779	-0.307	-0.738	0.790
	2.732	10.771	-3.908	-7.900	
3-1	1.114	-0.497	0.123	-0.039	0.330
	3.272	-6.629	1.509	-0.400	

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**Table IA.IX**  
**Descriptive Statistics: Risk Neutral Moment Portfolios without Volume Screens**

We present summary statistics for portfolios sorted on measures of firms' risk-neutral moments. Firms are sorted on average risk-neutral volatility, skewness, and kurtosis within each calendar quarter into terciles based on 30<sup>th</sup> and 70<sup>th</sup> percentiles. We then form equally weighted portfolios of these firms, holding the moment ranking constant for the subsequent calendar quarter. Risk-neutral moments are calculated using the procedure in Bakshi, Kapadia and Madan (2003); in Panels A-D we report results using options closest to one month to maturity, three, six, and 12 months to maturity. In calculating option moments, we do not require options to exhibit any volume over the calculation period. The first column of each panel presents mean monthly returns. The second column presents characteristic-adjusted returns, calculated by determining, for each firm, the Fama and French (1993) 5X5 size- and book-to-market portfolio to which it belongs and subtracting that return. The next three columns present the average individual firm's risk-neutral volatility, skewness, and kurtosis across the portfolio for the portfolio formation period. The final three columns display the beta, log market value, and book-to-market equity ratio of the portfolio. Monthly return data cover the period April 1996 through December 2005, for a total of 117 monthly observations.

Panel A: One Month to Maturity								
Volatility								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.22	0.27	16.49	-1.52	11.38	0.89	15.71	0.37
2	0.98	0.14	25.78	-1.04	7.50	1.28	14.31	0.39
3	0.87	0.15	44.84	-1.14	5.33	1.78	13.61	0.42

  

Skewness								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.23	0.38	26.47	-2.65	15.03	1.25	15.37	0.34
2	0.88	0.07	30.37	-1.03	5.78	1.35	14.38	0.40
3	0.99	0.15	28.74	-0.03	3.97	1.27	13.86	0.44

  

Kurtosis								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.07	0.11	34.0	-0.35	2.24	1.35	13.69	0.46
2	0.93	0.15	28.72	-1.00	5.81	1.32	14.37	0.39
3	1.15	0.32	22.62	-2.37	16.71	1.22	15.55	0.32

Table continued on next page...

Panel B: Three Months to Maturity								
Volatility								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.22	0.25	17.57	-1.33	9.39	0.84	15.68	0.38
2	1.08	0.21	27.27	-1.00	6.93	1.29	14.30	0.39
3	0.74	0.06	46.75	-1.15	5.31	1.83	13.64	0.40

  

Skewness								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.27	0.40	29.29	-2.52	13.20	1.24	15.34	0.35
2	0.93	0.12	31.43	-0.7	5.31	1.35	14.41	0.39
3	0.88	0.05	29.48	-0.05	3.66	1.29	13.86	0.43

  

Kurtosis								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	0.92	0.06	35.37	-0.33	2.18	1.37	13.68	0.45
2	0.89	0.08	30.26	-0.94	5.38	1.33	14.38	0.39
3	1.28	0.44	24.97	-2.24	14.58	1.18	15.54	0.34

Table continued on next page ...

Panel C: Six Months to Maturity								
Volatility								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.21	0.23	19.23	-0.86	5.57	0.82	15.62	0.40
2	1.14	0.28	29.46	-0.65	4.56	1.29	14.33	0.39
3	0.66	-0.01	48.62	-0.73	3.64	1.86	13.66	0.39

  

Skewness								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.37	0.50	32.39	-1.73	8.10	1.22	15.35	0.38
2	0.85	0.00	31.51	-0.59	3.45	1.30	14.48	0.39
3	0.88	0.12	32.73	0.07	2.58	1.37	13.75	0.41

  

Kurtosis								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	0.92	0.10	36.13	-0.18	1.71	1.38	13.70	0.43
2	0.91	0.07	32.46	-0.58	3.46	1.36	14.39	0.39
3	1.26	0.43	27.73	-1.50	8.96	1.14	15.52	0.36

Table continued on next page ...

Panel D: 12 Months to Maturity

Volatility								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.24	0.23	19.84	-0.84	5.40	0.82	15.46	0.40
2	1.06	0.20	30.36	-0.68	4.64	1.29	14.35	0.39
3	0.74	0.09	50.78	-0.80	3.77	1.85	13.81	0.38

Skewness								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.35	0.50	34.74	-1.84	8.42	1.21	15.42	0.38
2	0.85	0.00	32.32	-0.59	3.33	1.31	14.43	0.39
3	0.90	0.11	33.28	0.07	2.51	1.38	13.74	0.41

Kurtosis								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	0.90	0.06	36.49	-0.17	1.67	1.40	13.71	0.43
2	0.90	0.08	33.76	-0.57	3.35	1.36	14.36	0.39
3	1.30	0.45	29.60	-1.61	9.23	1.12	15.55	0.36

**Table IA.X**  
**Descriptive Statistics: Risk Neutral Moment Portfolios with Alternative Price Screen**

Table IA-X presents summary statistics for portfolios sorted on measures of firms' risk-neutral moments. Firms are sorted on average risk-neutral volatility, skewness, and kurtosis within each calendar quarter into terciles based on 30<sup>th</sup> and 70<sup>th</sup> percentiles. We then form equally weighted portfolios of these firms, holding the moment ranking constant for the subsequent calendar quarter. Risk-neutral moments are calculated using the procedure in Bakshi, Kapadia and Madan (2003); in Panels A-D we report results using options closest to one month to maturity, three, six, and 12 months to maturity. In calculating option moments, we delete observations with prices less than \$1. The first column of each panel presents mean monthly returns. The second column presents characteristic-adjusted returns, calculated by determining, for each firm, the Fama and French (1993) 5X5 size- and book-to-market portfolio to which it belongs and subtracting that return. The next three columns present the average individual firm's risk-neutral volatility, skewness, and kurtosis across the portfolio for the portfolio formation period. The final three columns display the beta, log market value, and book-to-market equity ratio of the portfolio. Monthly return data cover the period April 1996 through December 2005, for a total of 117 monthly observations.

Panel A: One Month to Maturity								
Volatility								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.23	0.30	18.41	-1.43	9.06	0.91	15.83	0.35
2	0.94	0.12	29.11	-1.15	7.28	1.34	14.51	0.34
3	0.89	0.25	48.68	-1.11	6.85	1.82	13.81	0.37

  

Skewness								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.28	0.40	29.83	-2.64	12.67	1.27	15.60	0.32
2	0.85	0.08	33.51	-1.08	5.21	1.39	14.57	0.36
3	0.96	0.21	31.40	0.00	6.01	1.34	13.97	0.39

  

Kurtosis								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.00	0.21	37.29	-0.33	2.03	1.41	13.79	0.41
2	0.91	0.13	31.94	-1.05	5.30	1.36	14.60	0.35
3	1.16	0.35	26.04	-2.33	16.51	1.24	15.74	0.31

Table continued on next page...

Panel B: Three Months to Maturity								
Volatility								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.20	0.26	19.22	-1.33	8.00	0.87	15.83	0.36
2	1.05	0.21	30.29	-1.12	6.68	1.35	14.48	0.35
3	0.77	0.16	50.11	-1.18	4.92	1.86	13.85	0.36

  

Skewness								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.30	0.46	31.98	-2.59	11.80	1.26	15.60	0.32
2	0.88	0.07	34.44	-1.06	4.94	1.39	14.58	0.35
3	0.89	0.16	31.82	-0.00	3.45	1.36	13.96	0.39

  

Kurtosis								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	0.93	0.14	37.67	-0.33	2.00	1.43	13.78	0.40
2	0.88	0.10	33.05	-1.02	5.08	1.38	14.59	0.35
3	1.27	0.44	28.00	-2.31	13.05	1.21	15.75	0.32

Table continued on next page...

Panel C: Six Months to Maturity								
Volatility								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.24	0.28	20.30	-0.91	5.17	0.85	15.80	0.37
2	1.09	0.27	31.74	-0.75	4.68	1.35	14.49	0.35
3	0.68	0.04	50.71	-0.80	3.63	1.89	13.86	0.35

  

Skewness								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.32	0.45	34.32	-1.88	7.73	1.23	15.63	0.34
2	0.88	0.04	33.65	-0.69	3.47	1.37	14.62	0.35
3	0.89	0.21	34.14	0.08	2.69	1.42	13.87	0.38

  

Kurtosis								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	0.88	0.14	37.60	-0.20	1.65	1.44	13.81	0.38
2	0.89	0.09	34.34	-0.66	3.53	1.39	14.56	0.35
3	1.31	0.46	29.93	-1.64	8.70	1.18	15.76	0.34

Table continued on next page...



Panel D: 12 Months to Maturity

Volatility								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.28	0.30	20.99	-0.88	4.94	0.86	15.65	0.37
2	0.94	0.11	32.82	-0.77	4.69	1.36	14.46	0.35
3	0.84	0.25	53.46	-0.89	3.75	1.86	14.06	0.34

Skewness								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.32	0.49	37.13	-1.98	7.95	1.22	15.70	0.34
2	0.87	0.07	34.81	-0.68	3.30	1.37	14.58	0.35
3	0.89	0.22	34.68	0.10	2.60	1.43	13.85	0.38

Kurtosis								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	0.92	0.18	38.26	-0.19	1.59	1.46	13.81	0.38
2	0.85	0.05	35.86	-0.64	3.36	1.40	14.53	0.35
3	1.32	0.47	32.14	-1.74	8.88	1.15	15.81	0.34

**Table IA.XI**  
**Descriptive Statistics: Risk Neutral Moment Portfolios with Alternative Price Screen 2**

Table IA-XI presents summary statistics for portfolios sorted on measures of firms' risk-neutral moments. Firms are sorted on average risk-neutral volatility, skewness, and kurtosis within each calendar quarter into terciles based on 30<sup>th</sup> and 70<sup>th</sup> percentiles. We then form equally weighted portfolios of these firms, holding the moment ranking constant for the subsequent calendar quarter. Risk-neutral moments are calculated using the procedure in Bakshi, Kapadia and Madan (2003); in Panels A-D we report results using options closest to one month to maturity, three, six, and 12 months to maturity. In calculating option moments, we delete observations with prices less than \$0.25. The first column of each panel presents mean monthly returns. The second column presents characteristic-adjusted returns, calculated by determining, for each firm, the Fama and French (1993) 5X5 size- and book-to-market portfolio to which it belongs and subtracting that return. The next three columns present the average individual firm's risk-neutral volatility, skewness, and kurtosis across the portfolio for the portfolio formation period. The final three columns display the beta, log market value, and book-to-market equity ratio of the portfolio. Monthly return data cover the period April 1996 through December 2005, for a total of 117 monthly observations.

Panel A: One Month to Maturity

Volatility								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.18	0.23	14.45	-1.56	14.40	0.88	15.63	0.38
2	1.04	0.19	22.61	-1.04	8.72	1.24	14.23	0.42
3	0.86	0.11	40.48	-1.19	6.10	1.77	13.47	0.46

  

Skewness								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.27	0.40	23.91	-2.68	18.22	1.27	15.16	0.37
2	0.89	0.06	26.81	-1.04	6.95	1.32	14.30	0.42
3	0.96	0.12	25.40	-0.08	4.64	1.21	13.86	0.46

  

Kurtosis								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.03	0.15	31.46	-0.41	2.67	1.31	13.66	0.49
2	0.93	0.09	25.55	-1.03	6.82	1.30	14.27	0.42
3	1.15	0.34	19.54	-2.36	20.37	1.21	15.41	0.34

Table continued on next page...

Panel B: Three Months to Maturity								
Volatility								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.19	0.20	15.92	-1.27	10.76	0.81	15.57	0.40
2	1.15	0.27	24.62	-0.98	7.72	1.24	14.24	0.42
3	0.70	-0.01	43.11	-1.14	5.75	1.84	13.50	0.43

  

Skewness								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.29	0.39	27.37	-2.42	14.72	1.25	15.11	0.39
2	0.97	0.14	28.26	-0.93	5.94	1.32	14.34	0.42
3	0.85	0.01	26.80	-0.05	4.16	1.24	13.86	0.45

  

Kurtosis								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	0.84	-0.01	32.42	-0.37	2.50	1.35	13.65	0.48
2	0.99	0.16	27.76	-0.93	5.89	1.32	14.29	0.42
3	1.26	0.39	22.42	-2.10	16.44	1.15	15.38	0.36

Table continued on next page...

Panel C: Six Months to Maturity								
Volatility								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.22	0.22	18.10	-0.83	6.18	0.79	15.48	0.42
2	1.15	0.29	27.70	-0.61	4.73	1.24	14.30	0.42
3	0.67	-0.03	46.46	-0.70	3.75	1.88	13.52	0.41

  

Skewness								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.29	0.39	30.92	-1.63	8.67	1.23	15.15	0.41
2	0.97	0.11	29.44	-0.55	3.62	1.27	14.40	0.41
3	0.84	0.06	31.32	0.03	2.74	1.33	13.73	0.44

  

Kurtosis								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	0.94	0.13	34.16	-0.19	1.84	1.35	13.68	0.45
2	0.92	0.62	31.09	-0.56	3.58	1.34	14.32	0.41
3	1.25	0.39	25.89	-.141	9.63	1.12	15.30	0.39

Table continued on next page...

Panel D: 12 Months to Maturity

Volatility								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.24	0.21	18.61	-0.81	6.06	0.80	15.34	0.42
2	1.13	0.26	28.52	-0.65	4.88	1.24	14.31	0.42
3	0.68	0.00	48.14	-0.76	3.93	1.87	13.65	0.41

Skewness								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.28	0.39	32.62	-1.74	9.13	1.22	15.22	0.41
2	0.97	0.12	30.19	-0.55	3.52	1.27	14.36	0.42
3	0.85	0.06	31.92	0.03	2.68	1.34	13.71	0.43

Kurtosis								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	0.88	0.08	34.61	-0.18	1.80	1.36	13.69	0.45
2	0.96	0.09	32.00	-0.55	3.49	1.35	14.29	0.42
3	1.26	0.41	27.51	-1.52	10.04	1.11	15.34	0.39

**Table IA.XII**

**Descriptive Statistics: Risk Neutral Moment Portfolios with Requirement of More OTM Options**

In Table IA-XII we present summary statistics for portfolios sorted on measures of firms' risk-neutral moments. Firms are sorted on average risk-neutral volatility, skewness, and kurtosis within each calendar quarter into terciles based on 30<sup>th</sup> and 70<sup>th</sup> percentiles. We then form equally weighted portfolios of these firms, holding the moment ranking constant for the subsequent calendar quarter. Risk-neutral moments are calculated using the procedure in Bakshi, Kapadia and Madan (2003); in Panels A to D we report results using options closest to one month to maturity, three, six, and 12 months to maturity. In calculating option moments, we require at least three out of the money (OTM) puts and three out of the money calls. The first column of each panel presents mean monthly returns. The second column presents characteristic-adjusted returns, calculated by determining, for each firm, the Fama and French (1993) 5X5 size- and book-to-market portfolio to which it belongs and subtracting that return. The next three columns present the average individual firm's risk-neutral volatility, skewness, and kurtosis across the portfolio for the portfolio formation period. The final three columns display the beta, log market value, and book-to-market equity ratio of the portfolio. Monthly return data cover the period April 1996 through December 2005, for a total of 117 monthly observations.

Panel A: One Month to Maturity								
Volatility								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.20	0.33	18.75	-1.33	9.74	0.97	16.15	0.33
2	0.81	0.01	29.60	-1.35	7.84	1.41	15.03	0.31
3	0.85	0.17	50.27	-1.23	5.49	1.89	14.24	0.34

  

Skewness								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.28	0.45	32.06	-2.73	13.96	1.35	15.92	0.30
2	0.84	0.04	33.33	-1.13	5.73	1.48	15.03	0.32
3	0.74	0.02	31.97	-0.12	4.11	1.39	14.51	0.35

  

Kurtosis								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	0.89	0.14	38.15	-0.44	2.55	1.53	14.29	0.35
2	0.84	0.06	32.15	-1.11	5.79	1.44	15.04	0.32
3	1.13	0.31	27.47	-2.43	15.42	1.27	16.12	0.30

Table continued on next page...

Panel B: Three Months to Maturity

Volatility								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.13	0.23	19.61	-1.20	8.48	0.94	16.18	0.33
2	0.96	0.14	30.93	-1.28	7.42	1.42	15.01	0.31
3	0.73	0.07	51.42	-1.22	5.38	1.93	14.24	0.33

Skewness								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.26	0.48	34.17	-2.61	12.77	1.33	15.95	0.31
2	0.82	0.00	34.21	-1.06	5.31	1.48	15.01	0.32
3	0.78	0.02	32.48	-0.10	3.93	1.41	14.50	0.34

Kurtosis								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	0.84	0.07	38.60	-0.41	2.46	1.56	14.24	0.35
2	0.76	0.01	33.33	-1.05	5.41	1.44	15.05	0.32
3	1.29	0.44	29.23	-2.32	14.09	1.24	16.14	0.30

Table continued on next page ...

Panel C: Six Months to Maturity								
Volatility								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.08	0.19	20.97	-0.93	6.14	0.91	16.16	0.34
2	1.08	0.28	32.97	-0.96	5.63	1.43	15.01	0.31
3	0.61	-0.08	53.07	-0.92	4.32	1.96	14.27	0.32

  

Skewness								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.21	0.37	36.48	-2.10	9.54	1.28	16.02	0.32
2	0.86	0.07	34.74	-0.76	4.03	1.47	15.04	0.32
3	0.77	0.02	35.19	-0.01	3.07	1.47	14.38	0.33

  

Kurtosis								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	0.87	0.07	39.35	-0.27	2.09	1.58	14.25	0.33
2	0.84	0.08	35.57	-0.75	4.10	1.48	15.02	0.32
3	1.15	0.32	31.21	-1.85	10.42	1.18	16.17	0.31

Table continued on next page ...



Panel D: 12 Months to Maturity								
Volatility								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.12	0.21	21.67	-0.94	6.15	0.92	16.03	0.34
2	0.96	0.15	34.11	-0.99	5.73	1.42	14.98	0.31
3	0.74	0.08	55.73	-1.02	4.52	1.95	14.44	0.32

Skewness								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.27	0.43	38.83	-2.23	9.99	1.27	16.11	0.32
2	0.86	0.07	36.10	-0.79	3.98	1.47	15.00	0.32
3	0.72	-0.04	35.91	-0.01	3.03	1.49	14.35	0.33

Kurtosis								
Rank	Mean Return	Char Adj Return	Vol	Skew	Kurt	Beta	ln MV	BM
1	0.90	0.12	40.16	-0.27	2.06	1.59	14.23	0.33
2	0.74	-0.012	37.22	-0.77	4.07	1.48	15.00	0.33
3	1.25	0.41	33.07	-1.98	10.84	1.16	16.22	0.31

**Table IA.XIII**  
**Time Series Regressions: Co-Moment Risk Adjustment**

The table presents the results of time series regressions of excess return differentials (Hi-Lo) between portfolios ranked on risk-neutral volatility, skewness, and kurtosis on portfolios representing co-moment risk. Excess portfolio returns are regressed on the excess return on stocks with high risk-neutral covariance with the S&P 500 and short low risk-neutral covariance (CV), the excess return on a portfolio long stocks with high risk-neutral co-skewness with the S&P 500 and short low risk-neutral co-skewness (CS), and the excess return on a portfolio long stocks with high risk-neutral co-kurtosis with the S&P 500 and short low risk-neutral co-kurtosis (CK). The moment-sorted portfolios are equally weighted, formed on the basis of terciles and re-formed each quarter. The table presents point estimates of the coefficients with  $t$ -statistics below the point estimates. Data cover the period April 1996 through December 2005 for 117 monthly observations.

	Panel A: One Month to Maturity					Panel B: Three Months to Maturity				
	$\alpha$	$\beta_{CV}$	$\beta_{CS}$	$\beta_{CK}$	$R^2$	$\alpha$	$\beta_{MRP}$	$\beta_{CS}$	$\beta_{CK}$	$\bar{R}^2$
Vol	-0.66 -1.84	0.66 7.50	0.92 10.12	-0.52 -4.40	0.79	-0.71 -1.97	0.71 8.02	0.88 9.55	-0.71 -5.95	0.81
Skew	-0.79 -2.00	0.21 2.20	0.15 1.53	0.47 3.64	0.08	-0.91 -2.28	0.29 2.94	-0.01 -0.08	0.39 2.98	0.08
Kurt	0.72 2.52	-0.27 -3.83	-0.40 -5.60	-0.38 -4.08	0.25	0.94 3.10	-0.36 -4.78	-0.32 -4.18	-0.20 -2.01	0.20
	Panel C: Six Months to Maturity					Panel D: 12 Months to Maturity				
	$\alpha$	$\beta_{CV}$	$\beta_{CS}$	$\beta_{CK}$	$R^2$	$\alpha$	$\beta_{CV}$	$\beta_{CS}$	$\beta_{CK}$	$\bar{R}^2$
Vol	-0.90 -2.36	0.74 7.87	0.91 9.44	-0.66 -5.29	0.80	-0.82 -2.18	0.67 7.28	0.84 8.84	-0.71 -5.77	0.79
Skew	-1.04 -2.62	0.28 2.87	-0.16 -0.02	0.32 2.49	0.06	-1.02 -2.47	0.30 2.95	0.03 0.25	0.29 2.14	0.05
Kurt	1.13 3.47	-0.37 -4.63	-0.27 -3.34	-0.27 -2.56	0.20	1.14 3.41	-0.40 -4.84	-0.26 -3.05	-0.26 -2.41	0.21

**Table IA.XIV**  
**Parametric Stochastic Discount Factor Risk Adjustments: S&P 500**

The table presents point estimates of the parameters of a SDF polynomial in the returns on the S&P 500 index. The SDF is specified as

$$m_t = d_0 + d_1 r_{T,t} + d_2 r_{T,t}^2 + d_3 r_{T,t}^3$$

where  $r_{T,t}$  is the return on the S&P 500 index (Panels A to D). The parameters are estimated via GMM using the sample moment restrictions

$$\hat{\alpha} = \frac{1}{T} \sum_{t=1}^T ((1+r_t)m_t - 1_N) = 0$$

where  $r_t$  is a  $10 \times 1$  vector of returns comprising 3 portfolios sorted on risk-neutral volatility, 3 portfolios sorted on risk-neutral skewness, 3 portfolios sorted on risk-neutral kurtosis, and a Treasury Bill. The column titled ' $J$ ' presents the test statistic for the overidentifying restrictions. In addition to point estimates, we present the pricing errors associated with high-low factor mimicking portfolios formed on volatility, skewness, and kurtosis in the columns  $\alpha_{vol}$ ,  $\alpha_{skew}$ , and  $\alpha_{kurt}$ , respectively. We examine three versions of the model above. The first restricts  $d_2 = d_3 = 0$ , representing a linear specification, the second restricts  $d_3 = 0$ , representing a quadratic specification, and the final, representing a cubic specification, is unrestricted. Panel A presents results for returns formed on the basis of options with one month to maturity; Panels B-D present complementary results for options based on three, six, and 12 months to maturity. Newey-West  $t$ -statistics are presented in below the point estimates and  $p$ -values for the  $J$ -statistic are presented in parentheses below the statistic. The data cover the period April 1996 through December 2005 for 117 monthly observations.

Panel A: $r_{T,t}$ One Month to Maturity								Panel B: $r_{T,t}$ Three Months to Maturity							
$d_0$	$d_1$	$d_2$	$d_3$	$J$	$\alpha_{vol}$	$\alpha_{skew}$	$\alpha_{kurt}$	$d_0$	$d_1$	$d_2$	$d_3$	$J$	$\alpha_{vol}$	$\alpha_{skew}$	$\alpha_{kurt}$
1.00	-4.49			10.36	-1.08	-0.22	0.14	1.00	-5.11			14.41	-1.40	-0.36	0.51
24.80	-1.98			(0.24)	-1.75	-0.57	0.42	21.78	-2.27			(0.07)	-1.95	-0.97	1.44
1.00	-2.52	8.97		11.25	-1.27	-0.39	0.51	1.00	-2.96	10.06		15.41	-1.77	-0.60	0.91
24.10	-0.47	0.66		(0.13)	-1.98	-0.96	1.72	21.36	-0.62	0.82		(0.03)	-2.34	-1.57	2.59
1.00	2.10	7.46	-6.85	10.97	-1.20	-0.44	0.55	1.00	14.27	-2.92	-27.73	12.01	-1.19	-0.91	0.94
19.69	0.10	0.50	-0.21	(0.09)	-2.07	-1.11	1.98	13.22	1.29	-0.23	-1.24	(0.06)	-2.28	-2.61	2.63

  

Panel C: $r_{T,t}$ Six Months to Maturity								Panel D: $r_{T,t}$ 12 Months to Maturity							
$d_0$	$d_1$	$d_2$	$d_3$	$J$	$\alpha_{vol}$	$\alpha_{skew}$	$\alpha_{kurt}$	$d_0$	$d_1$	$d_2$	$d_3$	$J$	$\alpha_{vol}$	$\alpha_{skew}$	$\alpha_{kurt}$
1.00	-5.75			22.77	-1.70	-0.92	1.01	1.00	-5.79			21.96	-1.68	-0.94	1.00
41.53	-2.05			(0.00)	-2.53	-2.48	-3.10	41.22	-2.05			(0.01)	-2.60	-2.48	3.10
1.00	-1.71	15.86		20.58	-1.81	-0.73	1.05	1.00	-3.21	11.47		21.18	-1.86	-0.77	0.99
14.57	-0.28	1.15		(0.00)	-1.93	-1.69	2.92	18.86	-0.53	0.81		(0.00)	2.35	-1.99	2.94
1.00	0.16	15.66	-2.64	20.15	-1.77	-0.74	1.06	1.00	19.79	2.40	-33.92	14.81	-1.13	-1.00	1.05
13.59	0.01	1.14	-0.08	(0.00)	-2.64	-1.68	2.81	9.23	1.45	0.15	-1.03	(0.02)	-2.07	-2.08	2.92

**Table IA.XV**  
**Parametric Stochastic Discount Factor Risk Adjustments: Industry Portfolios**

The table presents point estimates of the parameters of a SDF polynomial in the returns on the tangency portfolio that explains a set of industry portfolio returns. The SDF is specified as

$$m_t = d_0 + d_1 r_{T,t} + d_2 r_{T,t}^2 + d_3 r_{T,t}^3$$

where  $r_{T,t}$  is the industry tangency portfolio. The parameters are estimated via GMM using the sample moment restrictions

$$\hat{\alpha} = \frac{1}{T} \sum_{t=1}^T ((1+r_t)m_t - 1_N) = 0$$

where  $r_t$  is a  $10 \times 1$  vector of returns comprising 3 portfolios sorted on risk-neutral volatility, 3 portfolios sorted on risk-neutral skewness, 3 portfolios sorted on risk-neutral kurtosis, and a Treasury Bill. The column titled ' $J$ ' presents the test statistic for the overidentifying restrictions. In addition to point estimates, we present the pricing errors associated with high-low factor mimicking portfolios formed on volatility, skewness, and kurtosis in the columns  $\alpha_{vol}$ ,  $\alpha_{skew}$ , and  $\alpha_{kurt}$ , respectively. We examine three versions of the model above. The first restricts  $d_2 = d_3 = 0$ , representing a linear specification, the second restricts  $d_3 = 0$ , representing a quadratic specification, and the final, representing a cubic specification, is unrestricted. Panel A presents results for returns formed on the basis of options with one month to maturity; Panels B-D present complementary results for options based on three, six, and 12 months to maturity. Newey-West  $t$ -statistics are presented below the point estimates and  $p$ -values for the  $J$ -statistic are presented in parentheses below the statistic. The data cover the period April 1996 through December 2005 for 117 monthly observations.

Panel A: $r_{T,t}$ One Month to Maturity								Panel B: $r_{T,t}$ Three Months to Maturity							
$d_0$	$d_1$	$d_2$	$d_3$	$J$	$\alpha_{vol}$	$\alpha_{skew}$	$\alpha_{kurt}$	$d_0$	$d_1$	$d_2$	$d_3$	$J$	$\alpha_{vol}$	$\alpha_{skew}$	$\alpha_{kurt}$
1.00	-4.48			10.36	-1.08	-0.22	0.14	1.00	-4.73			14.79	-1.40	-0.44	0.53
24.80	-1.98			(0.24)	-1.75	-0.57	0.42	23.55	-2.14			(0.06)	-1.94	-1.15	1.46
1.00	-7.57	5.64		8.90	-0.98	-0.58	0.39	1.00	-3.12	-3.56		14.10	-1.52	-0.21	0.36
16.20	-1.41	1.04		(0.26)	-1.46	-2.12	1.45	19.15	-1.08	-0.98		(0.05)	-1.95	-0.56	1.04
1.00	-4.78	8.16	-4.87	8.73	-1.24	-0.56	0.51	1.00	2.54	4.35	-10.98	9.74	-1.89	-0.29	0.65
14.08	-0.98	0.91	-0.60	(0.19)	-2.12	-2.01	2.45	15.84	0.47	0.58	-1.19	(0.14)	-2.62	-0.90	2.31
Panel C: $r_{T,t}$ Six Months to Maturity								Panel D: $r_{T,t}$ 12 Months to Maturity							
$d_0$	$d_1$	$d_2$	$d_3$	$J$	$\alpha_{vol}$	$\alpha_{skew}$	$\alpha_{kurt}$	$d_0$	$d_1$	$d_2$	$d_3$	$J$	$\alpha_{vol}$	$\alpha_{skew}$	$\alpha_{kurt}$
1.00	-4.85			21.47	-1.50	-0.51	0.53	1.00	-5.21			18.36	-1.58	-0.60	0.62
22.95	-2.36			(0.01)	-2.12	-1.33	1.45	21.40	-2.36			(0.02)	-2.35	-1.48	1.70
1.00	-6.51	2.31		20.10	-1.55	-0.63	0.67	1.00	-3.92	-3.15		18.41	-1.71	-0.42	0.42
19.32	-1.43	0.56		(0.01)	-2.19	-1.84	2.56	18.04	-1.39	-1.06		(0.01)	-2.37	-1.00	1.27
1.00	-0.32	4.79	-8.09	19.14	-1.78	-0.51	0.70	1.00	4.23	3.69	-12.32	11.58	-1.89	-0.34	0.60
17.19	-0.04	0.76	-0.93	(0.00)	-2.62	-1.63	2.64	15.16	0.65	0.61	-1.28	(0.07)	-2.62	-0.85	2.01

**Table IA.XVI**  
**Parametric versus Non-Parametric Stochastic Discount Factor Risk Adjustments**

The table presents risk adjustments for the volatility, skewness, and kurtosis factor mimicking portfolios using SDFs implied by the S&P 500 risk-neutral and physical densities. The SDF is formed as a risk-free scaled ratio of the risk-neutral to physical probability measure

$$m_t(x, s, \tau) = e^{-r_t^f(\tau)} \frac{f_t^Q(x, s, \tau)}{f_t^P(x, s, \tau)}$$

where  $f_t^Q(\cdot)$  is the risk-neutral probability measure at time  $t$ ,  $f_t^P(\cdot)$  is the physical probability measure at time  $t$ , and  $\tau$  is the horizon. We approximate the risk-neutral and physical probability distributions using the Normal Inverse Gaussian (NIG) distribution. The risk-neutral measure is approximated using the risk-neutral moments calculated in the paper and the physical measure is calculated using returns data on the S&P 500 over the 1000 days prior to March, 31 1996. The table presents excess returns implied by discounting the factor mimicking portfolios by the SDF,

$$\hat{\alpha} = \frac{1}{T} \sum_{t=1}^T r_t(\tau) m_t(x_t, \tau)$$

where  $r_t(\tau)$  is the  $\tau$ -period return on the factor-mimicking portfolio at time  $t$ , and  $m_t(x_t, \tau)$  is the SDF evaluated at the observed  $\tau$ -period realization of the S&P 500 at time  $t$ . The column labeled “NIG” represents the discount factor implied by the NIG approximations to the densities. Columns “Linear,” “Quad,” and “Cubic” represent discount factors obtained by projecting the density-implied discount factor onto a linear, quadratic, and cubic polynomial, respectively. Panel A presents results for the volatility-sorted factor mimicking portfolio with rows representing portfolios formed on volatility estimated using options with one, three, six, and 12-months to maturity. Panels B and C present complementary results for skewness- and kurtosis-sorted factor mimicking portfolios. We separately examine SDFs based on options and returns with three, six, and 12 month horizons. Data for the three, six, and 12 month horizons begin in January, 1997, July, 1996, and April, 1996, respectively. All three horizons extend through December, 2005 for 106 (overlapping) observations. Point estimates are scaled to the monthly frequency, and Newey-West standard errors are presented in parentheses below the point estimates. Point estimates that are significantly different than zero at the 10% or higher significance level are presented in boldfaced type.

Panel A: Volatility

	Three Month				Six Month				Twelve Month			
	NIG	Linear	Quad	Cubic	NIG	Linear	Quad	Cubic	NIG	Linear	Quad	Cubic
One Month	-0.56	0.13	0.14	0.10	-1.29	-0.25	-0.30	-0.35	-1.04	-0.16	-0.17	-0.23
	-1.77	0.36	0.38	0.28	-1.39	-0.21	-0.26	-0.30	-1.89	-0.26	-0.28	-0.38
Three Month	-0.73	-0.04	-0.03	-0.07	-1.37	-0.32	-0.32	-0.38	-1.47	-0.40	-0.41	-0.48
	-1.83	-0.09	-0.07	-0.17	-1.88	-0.38	-0.37	-0.45	-1.96	-0.54	-0.55	-0.66
Six Month	-0.76	-0.04	-0.03	-0.07	-1.40	-0.35	-0.35	-0.41	-1.46	-0.40	-0.42	-0.49
	-1.90	-0.08	-0.06	-0.16	-1.96	-0.42	-0.41	-0.49	-1.97	-0.54	-0.56	-0.67
12 Month	-0.74	-0.08	-0.08	-0.12	-1.36	-0.42	-0.43	-0.48	-1.41	-0.40	-0.42	-0.49
	-1.86	-0.19	-0.17	-0.27	-1.87	-0.51	-0.51	-0.58	-1.91	-0.53	-0.55	-0.65

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Panel B: Skewness

	Three Month				Six Month				Twelve Month			
	NIG	Linear	Quad	Cubic	NIG	Linear	Quad	Cubic	NIG	Linear	Quad	Cubic
One Month	-0.23	-0.34	-0.33	-0.33	-0.82	-1.32	-1.27	-1.28	-0.11	-0.70	-0.68	-0.66
	-0.85	-0.88	-0.87	-0.86	-0.67	-0.97	-0.95	-0.96	-0.21	-0.98	-0.94	-0.94
Three Month	-0.32	-0.36	-0.36	-0.35	-0.72	-0.69	-0.67	-0.68	-0.44	-0.88	-0.87	-0.85
	-1.53	-1.19	-1.17	-1.18	-1.59	-1.16	-1.13	-1.16	-1.08	-1.43	-1.38	-1.41
Six Month	-0.44	-0.50	-0.50	-0.49	-0.91	-0.89	-0.88	-0.89	-0.65	-1.02	-1.02	-1.01
	-2.87	-2.02	-2.00	-2.03	-2.81	-1.95	-1.93	-1.98	-2.30	-2.17	-2.13	-2.18
12 Month	-0.44	-0.48	-0.48	-0.48	-0.92	-0.90	-0.88	-0.89	-0.65	-1.06	-1.06	-1.04
	-2.88	-1.93	-1.91	-1.94	-2.80	-1.90	-1.89	-1.93	-2.42	-2.08	-2.04	-2.09

Panel C: Kurtosis

	Three Month				Six Month				Twelve Month			
	NIG	Linear	Quad	Cubic	NIG	Linear	Quad	Cubic	NIG	Linear	Quad	Cubic
One Month	0.24	0.09	0.08	0.09	0.64	0.70	0.68	0.70	0.20	0.39	0.37	0.38
	1.46	0.46	0.44	0.48	0.88	0.90	0.89	0.93	0.63	0.99	0.93	0.98
Three Month	0.45	0.26	0.26	0.26	0.91	0.54	0.53	0.55	0.78	0.74	0.72	0.74
	3.52	1.61	1.59	1.66	3.14	1.68	1.67	1.76	3.16	2.23	2.21	2.32
Six Month	0.53	0.41	0.40	0.41	1.05	0.73	0.72	0.74	0.88	0.94	0.93	0.94
	4.09	2.29	2.29	2.34	4.01	1.96	1.96	2.06	3.61	2.40	2.39	2.52
12 Month	0.51	0.38	0.37	0.38	1.02	0.68	0.67	0.69	0.88	0.89	0.89	0.90
	4.12	2.34	2.34	2.39	3.87	2.07	2.07	2.18	3.53	2.60	2.59	2.72

**Table IA.XVII**  
**Industry Definitions**

Ticker	Description
BKX	KBW Bank Index
BTK	AMEX Biotechnology Index
CMR	Morgan Stanley Consumer Index
CYC	Morgan Stanley Cyclical Index
DRG	AMEX Pharmaceutcial Index
MSH	Morgan Stanley High-Technology Index
TXX	CBOE Technology Index
UTY	PHLX Utility Sector Index
XAL	AMEX Airline Index
XAU	PHLX Gold and Silver Sector Index
XBD	AMEX Securities Broker/Dealer Index
XCI	AMEX Computer Technology Index
XNG	AMEX Natural Gas Index
XOI	AMEX Oil Index

### Figure IA.I. Stochastic Discount Factors

The plots depict SDFs formed using risk-neutral moments of S&P 500 index options at the 12-month maturity. The plot labeled 'NIG' represents SDFs,  $m(x, s, \tau)$ , formed as

$$m(x, s, \tau) = e^{-r_f \tau} \frac{f^Q(x, s, \tau)}{f^P(x, s, \tau)}$$

where  $f(\cdot)$  is the NIG probability density function,  $Q$  denotes the risk-neutral probability measure, and  $P$  denotes the physical measure. The risk-neutral measure is calculated using risk-neutral moments retrieved from option prices and the physical measure using the historical moments of the S&P 500 index from 1992 through 1995. 'Linear,' 'Quadratic,' and 'Cubic' represent linear, quadratic, and cubic polynomial fits to the NIG kernel.

