Internet Appendix for "Human Capital, Bankruptcy, and Capital Structure"*

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In this online supplement, we show that the equation implicitly defining the competitive market wage in Proposition 2 has a unique solution between the values c_{nd} and c_{full}, where c_{nd} is the wage level that would prevail if there were no possibility of financial distress, and c_{full} is the wage level that gives the manager the entire value he is adding to the firm. We also derive an explicit expression for the value c_{nd}.

I. Existence and Uniqueness of the Optimal Wage

Proposition 2 and Appendix C in the paper define the competitive market wage implicitly via the equation

$$c^*(\overline{\phi}_t) \equiv \left\{ c \,\middle|\, \Delta(\overline{\phi}_t, D, c) = 0, \overline{\phi}_t + \frac{Dr\tau}{1-\tau} - \frac{\sigma}{\sqrt{2r}} \leq c < \overline{\phi}_t + \frac{Dr\tau}{1-\tau} \right\},$$

where

$$\begin{aligned} \Delta(\overline{\phi}, D, c) \equiv & \left(2\sqrt{2} \left(\frac{D-K}{1-\tau} \right) r^{3/2} + \left(e^{-\frac{\sqrt{2r}\,c}{\sigma}} - e^{\frac{\sqrt{2r}\,c}{\sigma}} \right) \sigma \right) e^{\frac{\sqrt{2r}\left(\left(\frac{K}{1-\tau} - D \right) r + \overline{\phi} \right)}{\sigma}} - \sigma - \qquad \text{(IA.1)} \\ & \sqrt{2r} \left(\overline{\phi} - c + \frac{Dr\tau}{1-\tau} \right) + e^{\frac{2\sqrt{2r}\left(\left(\frac{K}{1-\tau} - D \right) r + \overline{\phi} \right)}{\sigma}} \left(\sigma - \sqrt{2r} \left(\overline{\phi} - c + \frac{Dr\tau}{1-\tau} \right) \right). \end{aligned}$$

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Here we prove that this equation always has a unique solution between $c_{nd} \equiv \overline{\phi} + \frac{Dr\tau}{1-\tau} - \frac{\sigma}{\sqrt{2r}}$ and $c_{full} \equiv \overline{\phi} + \frac{Dr\tau}{1-\tau}$.[1] Note that $c_{full} > c_{nd} > 0$ because of our assumption that

$$\phi_0 > \frac{\sigma}{\sqrt{2r}} - \frac{Dr\tau}{1-\tau},$$

and the fact that $\overline{\phi} \geq \phi_0$. From equation (IA.1),

$$\Delta\left(c_{nd}\right) = \left[\frac{2\sqrt{2r}(D-K)r}{1-\tau} + \sigma\left(e^{-\frac{\sqrt{2r}c_{nd}}{\sigma}} - e^{\frac{\sqrt{2r}c_{nd}}{\sigma}}\right)\right] e^{\frac{\sqrt{2r}\left(\overline{\phi} - \left(D - \frac{K}{1-\tau}\right)r\right)}{\sigma}} - 2\sigma. \tag{IA.2}$$

Since $D \leq K$, and $e^{-x} - e^{x} < 0$ for all $x > 0$, the term in square brackets is strictly negative, and hence $\Delta\left(c_{nd}\right) < 0$. Now consider $\Delta\left(c_{full}\right)$. Define

$$\begin{aligned} x &= \frac{-\sqrt{2r}(D-K)r}{\sigma(1-\tau)}, \\ y &= \frac{\sqrt{2r}c_{full}}{\sigma}, \end{aligned}$$

and note that $x, y \geq 0$. We can rewrite equation (IA.1) as

$$\frac{\Delta\left(c_{full}\right)}{\sigma} = \left(e^{-y} - e^{y} - 2x\right)e^{x+y} + e^{2(x+y)} - 1 \equiv f(x,y). \tag{IA.3}$$

It is immediate that $f(0,y) = 0$ for all y. Now differentiate with respect to x to obtain

$$f_x(x,y) = e^{x+y}\left(2e^{x+y} + e^{-y} - e^{y} - 2x - 2\right) \equiv e^{x+y}g(x,y), \tag{IA.4}$$

[1] Section II of this Internet Appendix shows that c_{nd} is the optimal wage in the absence of financial distress. Since the possibility of financial distress makes the employee worse off, we are looking for a solution greater than this value. In addition, due to the insurance provided by the firm, the employee cannot be paid more than the full amount of value he is currently adding, c_{full}.

and note that f_x and g always have the same sign. When $x = 0$,

$$\begin{aligned} g(0,y) &= 2e^{y} + e^{-y} - e^{y} - 2, \\ &= e^{y} + e^{-y} - 2, \\ &\geq 0 \qquad \text{for all } y. \end{aligned}$$

Differentiating again, we obtain

$$\begin{aligned} g_x(x,y) &= 2e^{x+y} - 2, \\ &\geq 0 \qquad \text{for all } x, y \geq 0. \end{aligned}$$

Since $g(0,y) \geq 0$ and $g_x(x,y) \geq 0$ for all $x \geq 0$, this implies that $g(x,y)$ and $f_x(x,y)$ are nonnegative for all $x, y \geq 0$. This, combined with the fact that $f(0,y) = 0$ for all y, implies in turn that $f(x,y) \geq 0$ for all $x, y \geq 0$, and hence that

$$\Delta\left(c_{full}\right) \geq 0.$$

Since $\Delta\left(c_{nd}\right) < 0$ and $\Delta\left(c_{full}\right) \geq 0$, by continuity there must be at least one solution to equation (IA.1) between c_{nd} and c_{full}. To prove uniqueness, note that if there were more than one solution, there would have to be at least one value of c in this region at which $\Delta'(c) = 0$. But, differentiating equation (IA.1), the equation $\Delta'(c) = 0$ has exactly two solutions,

$$
\begin{aligned}
c_{min} &= \underline{\phi} - \overline{\phi}, \\
&\leq 0, \\
&< c_{nd}. \\
c_{max} &= \overline{\phi} - \underline{\phi}, \\
&= \overline{\phi} + \frac{Dr\tau}{1-\tau} + \frac{(K-D)r}{1-\tau}, \\
&\geq c_{full}
\end{aligned}
$$

Since neither of these values is between c_{nd} and c_{full}, we conclude that there must be exactly one solution to equation (IA.1) between c_{nd} and c_{full}.

II. Solution with No Distress

To derive a lower bound on the employee's promised wage, consider a simplified version of the model in which there is no financial distress or bankruptcy; the firm can continue to pay the employee's promised wage, regardless of how low productivity becomes. In this case, given the random walk assumption for ϕ_t, the manager's optimal compensation must be of the form

$$c(\overline{\phi}) = \overline{\phi} + \theta,$$

where θ is some constant (which depends on D and the other parameters of the model). Define

$$x_t \equiv \phi_t - \overline{\phi}_t.$$

By the structure of the optimal contract, for any Δ we have

$$\begin{aligned} V(\phi_t + \Delta, \overline{\phi}_t + \Delta) &= V(\phi_t, \overline{\phi}_t), \\ &= V(\phi_t - \overline{\phi}_t, 0), \\ &\equiv v(x_t). \end{aligned}$$

From equation (C4) in the paper, V solves the partial differential equation

$$\frac{1}{2}\sigma^2 V_{\phi\phi} - rV + Kr - Dr(1-\tau) + (\phi - c(\overline{\phi}))(1-\tau) = 0. \tag{IA.5}$$

In terms of x, this becomes the ordinary differential equation

$$\frac{1}{2}\sigma^2 v_{xx} - rv + (x-\theta)(1-\tau) + Kr - Dr(1-\tau) = 0, \tag{IA.6}$$

the general solution to which is

$$v(x) = Ae^{\sqrt{2r}\,x/\sigma} + Be^{-\sqrt{2r}\,x/\sigma} + \frac{(x-\theta)(1-\tau)}{r} + K - D(1-\tau). \tag{IA.7}$$

For any choice of θ, v must satisfy the two boundary conditions[2]

$$v'(0) = 0, \tag{IA.8}$$

$$\lim_{x \to -\infty} v'(x) = \frac{(1-\tau)}{r}. \tag{IA.9}$$

[2]The first boundary condition is a consequence of x_t possessing an upper reflecting boundary at 0 (see Dumas (1991)). The second boundary condition applies because, for very low x, hitting the upper boundary is irrelevant. Thus, an increase of \$1 in x today results in a permanent increase of \$$(1-\tau)$ in the dividend, with a present value of $(1-\tau)/r$.

These imply that

$$A = \frac{-\sigma(1-\tau)}{r\sqrt{2r}}, \tag{IA.10}$$

$$B = 0. \tag{IA.11}$$

To determine θ, note that we must have $v(0) = K - D$, which yields

$$\theta = \frac{Dr\tau}{1-\tau} - \frac{\sigma}{\sqrt{2r}}. \tag{IA.12}$$

In other words, the optimal compensation contract is to set

$$c(\overline{\phi}) = \overline{\phi} + \frac{Dr\tau}{1-\tau} - \frac{\sigma}{\sqrt{2r}}. \tag{IA.13}$$

REFERENCES

Dumas, Bernard, 1991, Super contact and related optimality conditions, *Journal of Economic Dynamics and Control* 15, 675–685.