Internet Appendix to "Financial Distress and the Cross-section of Equity Returns"

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This Internet Appendix provides additional proofs and results that are left out of the main text of the paper due to space limitation. Section I of this appendix collects proofs of the corollaries to Proposition 1 in the main text. Section II presents a general dynamic model of investment and capital structure decisions that allows for shareholder recovery in default. Section III describes the numerical procedure used to solve the general model and reports numerical results on expected returns, value spreads and momentum profits that complement those obtained from the simple model in the paper. Section IV contains additional empirical results mentioned in the main text.

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I. Additional Proofs

Proof of Corollary 1.1: Taking the limit $\zeta \to 0$ in (1) we obtain the Brownian motion case. The ordinary differential equation describing the limited liability option is

$$\frac{1}{2}\sigma^2 U''(p) + \mu^{\mathbb{Q}} U'(p) - rU(p) = 0. \tag{IA.1}$$

The general solution to (IA.1) is of the form $U(p_t) = Ae^{\phi p_t}$. After imposing the boundary condition $\lim_{p_t \to \infty} U(p_t) = 0$ and setting $\mu^{\mathbb{Q}} = r - \delta - \frac{1}{2}\sigma^2$, the equity value $V(p_t)$ is given by equation (9) with ϕ given in (10). Solving the value-matching and smooth pasting conditions (7) and (8) for $\zeta \to 0$ yields the expressions of the constant A and the default threshold $\underline{p}$ in (11). ∎

Proof of Corollary 1.2: The equity β_t of a firm that is still operating at time t, that is, for $p_t > \underline{p}$, is

$$\begin{aligned} \beta_t &= \frac{d\log V(p_t)}{dp_t} = \frac{1}{V(p_t)}\left(\frac{e^{p_t}}{\delta} + \pi_t\left(\eta b - \frac{1}{\delta}\right)e^{\underline{p}}\right) && \text{(IA.2)} \\ &= 1 + \frac{1}{V(p_t)}\left[\frac{c+l}{r} + \pi_t\left(1 - \frac{1}{\phi}\right)\left(\eta b - \frac{1}{\delta}\right)e^{\underline{p}}\right], && \text{(IA.3)} \end{aligned}$$

where (IA.2) follows by using the definition of risk-neutral probability of default (13) and (IA.3) obtains by isolating the expression of $V(p_t)$ in (12) for $p_t \geq \underline{p}$. The corollary follows after substituting the expression of $\underline{p}$ in (11) and rearranging terms. ∎

Proof of Corollary 1.3: Let us consider the case of $\eta = 0$ first. Because, from (13), π_t is inversely related to p_t, to show that $d\beta_t/d\pi_t > 0$ it is sufficient to show that $d\beta_t/dp_t < 0$. Using the expressions in equations (12) and (13) with $\eta = 0$, we obtain

$$\beta_t = \frac{d\log V(p_t)}{dp_t} = \frac{e^{p_t} - \pi_t e^{\underline{p}}}{\delta V(p_t)}. \tag{IA.4}$$

Hence,

$$
\begin{aligned}
\frac{d\beta_t}{dp_t} = \frac{d^2 \log V(p_t)}{dp_t^2} &= \frac{\delta V(p_t)(e^{p_t} - \phi \pi_t e^{\underline{p}}) - (e^{p_t} - \pi_t e^{\underline{p}})^2}{\delta^2 V^2(p_t)} \\
&= -e^{p_t + \underline{p}} \left(\frac{(\pi_t + \phi - 1) + \phi(\phi - 2)\pi_t - \phi(\phi - 1)\pi_t e^{\underline{p} - p_t}}{\phi \delta^2 V^2(p_t)} \right) \\
&= -e^{p_t + \underline{p}} \left(\frac{\phi - 1}{\phi} \right) \left(\frac{1 + (\phi - 1)\pi_t - \phi \pi_t e^{\underline{p} - p_t}}{\delta^2 V^2(p_t)} \right),
\end{aligned}
$$

where to arrive at the second equality, we use the expression $\frac{c+l}{r} = \frac{\phi - 1}{\phi} \frac{e^{\underline{p}}}{\delta}$ derived from equation (11) with η set to zero. Therefore, to show that $d^2 \log(V(p_t))/dp_t^2 < 0$, we only need to show that

$$
1 + (\phi - 1)\pi_t - \phi \pi_t e^{\underline{p} - p_t} > 0 \tag{IA.5}
$$

for all p_t. Because p_t is bounded from below by $\underline{p}$, we first check if the equality (IA.5) holds when $p_t \to \underline{p}$. Let $\epsilon = p_t - \underline{p} > 0$. Using a second-order Taylor expansion of $e^{-\epsilon}$ around ϵ, we obtain

$$
e^{\underline{p} - p_t} = e^{-\epsilon} \approx 1 - \epsilon + \frac{1}{2}\epsilon^2 + o(\epsilon^2), \tag{IA.6}
$$

$$
\pi_t = e^{-\phi\epsilon} \approx 1 + \phi\epsilon + \frac{1}{2}\phi^2\epsilon^2 + o(\epsilon^2). \tag{IA.7}
$$

Substituting the above expressions in (IA.5) and simplifying we have that the leading-order term is $\frac{1}{2}\phi(\phi - 1)\epsilon^2 > 0$. We then note that the derivative of the left-hand side of (IA.5) with respect to p_t is $\phi(\phi - 1)(1 - e^{\underline{p} - p_t}) > 0$. This proves that $d\beta/dp_t < 0$ and hence $d\beta/d\pi_t > 0$ when $\eta = 0$. In particular, when $\pi_t \to 1$, using a second-order Taylor expansion of $V(p_t)$,

$$
V(p_t) \approx -\frac{1}{2}\phi\,\epsilon^2\,\frac{c + l}{r} + o(\epsilon^2), \tag{IA.8}
$$

and substituting (IA.7) in (IA.4), we have $\beta_t \approx \frac{2}{\epsilon} + 1 + \phi \to +\infty$ as $\pi_t \to 1$.

To analyze the case of $\eta > 0$, let us rewrite the beta expression in Corollary 1.2 as

$$\beta_t = 1 + \frac{c + l - \pi_t(\eta ar + c + l)}{rV(p_t)}. \tag{IA.9}$$

Then,

$$\frac{d\beta_t}{d\pi_t} = \frac{-(\eta ar + c + l)rV(p_t) - (c + l - \pi_t(\eta ar + c + l))r\frac{dV(p_t)}{d\pi_t}}{(rV(p_t))^2}. \tag{IA.10}$$

Applying the chain rule,

$$\frac{dV(p_t)}{d\pi_t} = \frac{dV(p_t)}{dp_t}\left(\frac{d\pi_t}{dp_t}\right)^{-1} = \frac{dV(p_t)}{dp_t}\frac{1}{\phi\pi_t}, \tag{IA.11}$$

and the definition of $\beta_t = \frac{1}{V(p_t)}\frac{dV(p_t)}{dp_t}$ in (IA.10), we obtain

$$\frac{d\beta_t}{d\pi_t} = -\frac{1}{V(p_t)}\frac{(\eta ar + c + l)}{r} - \frac{(\beta_t - 1)\beta_t}{\phi\pi_t}. \tag{IA.12}$$

When $\pi_t \to 0$, that is, $p_t \gg \underline{p}$, the first term in (IA.12) goes to zero as $V(p_t) \to \infty$. The second term will be positive as $\beta_t > 1$ and $\phi < 0$. Therefore, β_t increases in π_t for small levels of π_t. When $\pi_t \to 1$, the first term in (IA.12) is negative because $V(p_t) > 0$. Furthermore, from (IA.9), $\beta_t - 1 \to -\frac{\eta a}{V(p_t)} < 0$. Because $\phi < 0$, the second term in (IA.12) is negative as well. Therefore, $\frac{d\beta_t}{d\pi_t} < 0$, that is, β_t decreases in π_t when $\pi \to 1$. ∎

Proof of Corollary 1.4: Because $\lambda > 0$, from (17), $AC(p_t) > 0$ if and only if $\theta_t \equiv \frac{1}{\beta_t}\frac{d\beta_t}{dp_t} > 0$. From (IA.12), we have that

$$\begin{aligned} \theta_t &= \frac{1}{\beta_t}\frac{d\beta_t}{d\pi_t}\frac{d\pi_t}{dp_t} \\ &= 1 - \beta_t - \frac{1}{\beta_t V(p_t)}\frac{\phi(\eta ar + c + l)}{r}\pi_t. \end{aligned} \tag{IA.13}$$

We now show that when $\eta = 0$, $AC_t < 0$. If $\lambda > 0$ and $\beta_t > 0$, then it suffices to show that $d^2\log(V(p_t))/dp_t^2 < 0$. This is true from the proof of Corollary 1.3.

For $\eta > 0$, when $\pi_t \to 0$, $\theta_t < 0$ because $\beta_t > 1$. When $\pi_t \to 1$, as shown in the proof of Corollary 1.3, $1 - \beta_t > 0$. Because $\phi < 0$, the second term in (IA.13) is also positive. Therefore, when $\eta > 0$ and $\pi_t \to 1$, $\theta_t > 0$.

We now show that the boundary for log-convexity of the equity value, $p^*(\eta)$, is increasing in η. From the expression of equity value (9) with $p_t > \underline{p}$, we have

$$\frac{d^2 \log V(p_t)}{dp_t^2} = \frac{\frac{(\phi-1)^2 A e^{(\phi+1)p_t}}{\delta} - \frac{c+l}{r}\left(\frac{e^{p_t}}{\delta} + \phi^2 A e^{\phi p_t}\right)}{V^2(p_t)}. \tag{IA.14}$$

Because $\frac{d^2 \log V(p^*(\eta))}{dp_t^2} = 0$, $p^*(\eta)$ satisfies

$$\frac{(\phi-1)^2 A e^{(\phi+1)p^*(\eta)}}{\delta} = \frac{c+l}{r}\left(\frac{e^{p^*(\eta)}}{\delta} + \phi^2 A e^{\phi p^*(\eta)}\right). \tag{IA.15}$$

Taking derivatives with respect to η on both sides of (IA.15), and denoting $X = e^{p^*(\eta)}$, we obtain

$$\begin{aligned}\frac{\partial X}{\partial \eta} &= \frac{\frac{\partial A}{\partial \eta}\left(\frac{(\phi-1)^2 X^{\phi}}{\delta} - \left(\frac{c+l}{r}\right)\phi^2 X^{\phi-1}\right)}{A\left((\phi-1)\phi^2\left(\frac{c+l}{r}\right)X^{\phi-2} - \frac{\phi(\phi-1)^2}{\delta}X^{\phi-1}\right)} \\ &= \frac{\frac{\partial A}{\partial \eta}X}{(1-\phi)A}\left(\frac{\frac{(\phi-1)^2}{\delta}X - \frac{c+l}{r}\phi^2}{\frac{\phi(\phi-1)}{\delta}X - \frac{c+l}{r}\phi^2}\right).\end{aligned} \tag{IA.16}$$

From the expressions in (11), $\frac{\partial A}{\partial \eta} > 0$. Moreover, because $\phi < 0$, $1-\phi > 1$, and $\frac{(\phi-1)^2}{\delta}X - \frac{c+l}{r}\phi^2 > \frac{\phi(\phi-1)}{\delta}X - \frac{c+l}{r}\phi^2$. Finally, because $p^* > \underline{p}$, it can be shown that $\frac{\phi(\phi-1)}{\delta}X - \frac{c+l}{r}\phi^2 > 0$. Hence

$$\frac{\partial X}{\partial \eta} > \frac{\frac{\partial A}{\partial \eta}X}{(1-\phi)A} > 0, \tag{IA.17}$$

implying that $\partial p^*(\eta)/\partial \eta > 0$. ∎

II. A General Model of Levered Equity Returns

In this section we generalize the model in the paper to account for endogenous investment and financing decisions. We adopt the neoclassical Lucas-Prescott framework and construct a stationary economic environment that can be used as a laboratory for analyzing the effect of shareholder recovery upon financial distress on the cross-section of equity returns. In this environment, each firm is characterized by a production technology generating cash flows that are subject to both economy-wide and firm-specific shocks. The firm's manager maximizes equity value by optimally choosing (i) the level of capital investment, (ii) financing through a mix of debt and equity, and (iii) whether to default.

The structure of the model closely follows that in Gomes and Schmid (2010), with the following exceptions: (i) we explicitly allow for shareholder recovery in the event of default; (ii) we do not impose that investment is irreversible; and (iii) we explicitly model capital adjustment costs (see also Li (2008), Livdan, Sapriza, and Zhang (2009), and Obreja (2006)).

A. *Production Technology*

The production output $Y_{i,t}$ of firm i at time t is

$$Y_{i,t} = K_{i,t}^{\alpha} e^{X_t + Z_{i,t}}, \quad \alpha \in (0,1), \tag{IA.18}$$

where $K_{i,t}$ is the firm's capital level at the beginning of period t, and α is the capital share in total output, chosen to be between zero and one in order to obtain decreasing return to scale. The variables X_t and $Z_{i,t}$ in (IA.18) represent, respectively, the aggregate and firm-specific shocks to output. These shocks are modeled as stationary Markov processes evolving according to the following autoregressive processes:

$$X_{t+1} = X_t + (1-\rho_x)(\overline{X} - X_t) + \sigma_x \varepsilon_{t+1}^{x}, \tag{IA.19}$$

$$Z_{i,t+1} = Z_{i,t} + (1-\rho_z)(\overline{Z} - Z_{i,t}) + \sigma_z \varepsilon_{i,t+1}^{z}, \tag{IA.20}$$

where $\overline{X}$ and $\overline{Z}$ are the long-run averages, ρ_x and ρ_z the autocorrelation coefficients, and σ_x and σ_z the volatility coefficients. The innovations ε^x_{t+1} and $\varepsilon^z_{i,t+1}$ are normally distributed with mean zero and unit variance, $E[\varepsilon^z_{i,t}\varepsilon^x_t] = 0$ for all i, and $E[\varepsilon^z_{i,t}\varepsilon^z_{j,t}] = 0$ for all $i \neq j$. In period t, the firm's after-tax profit is

$$\Pi_{i,t} = (1-\tau)(Y_{i,t} - fK_{i,t} - F), \tag{IA.21}$$

where τ is the corporate tax rate, and f and F are the proportional and fixed costs, respectively.

B. Investment

Each period, the firm makes an investment decision that affects its capital stock in the next period according to the capital accumulation equation

$$K_{i,t+1} = I_{i,t} + (1-\kappa)K_{i,t}, \tag{IA.22}$$

where $I_{i,t}$ is the amount of new investment at time t, and κ is the depreciation rate of the installed capital. Following Lucas (1967), we assume a quadratic adjustment cost for new investment, that is,

$$h(I_{i,t}) = \frac{\theta}{2}\left(\frac{I_{i,t}}{K_{i,t}}\right)^2 K_{i,t}, \tag{IA.23}$$

where $\theta > 0$ is the adjustment cost coefficient.

C. Financing

In order to finance new investment and distribution to shareholders, the firm chooses whether to issue new equity (that is, negative dividends), new debt, or a combination of both. As in Li (2008) and Gomes and Schmid (2009), we assume that the only debt instrument available to the firm is a one-period bond. At each date t the firm decides to issue a bond with promised principal $B_{i,t+1}$ and coupon $b_{i,t+1}$, to be repaid at time $t+1$. The debt is assumed to be issued at par, so at time t firm i raises an amount equal to $B_{i,t+1}$. The firm is implicitly allowed to refinance all its liability in each period. Accounting for tax deductibility of the coupon payment, we define firm i's *total debt commitment* at time t as

$$D_{i,t} = B_{i,t} + (1-\tau)b_{i,t}. \tag{IA.24}$$

This quantity represents the amount needed to service the debt issued by the firm in the *previous* period and coming due at time t.

If we assume that the net cash flow to the equity is paid out as dividends to equityholders, the dividend at time t is then

$$c_{i,t} = \Pi_{i,t} + \tau\kappa K_{i,t} - I_{i,t} - h(I_{i,t}) - D_{i,t} + B_{i,t+1}. \tag{IA.25}$$

If $c_{i,t} < 0$, the firm can raise external financing through a seasoned equity offering. Following Gomes (2001) and Hennessy and Whited (2007), we assume that it is costless to increase debt but costly to raise new equity. The cost of raising new financing through a seasoned equity offering is assumed to be

$$\Lambda(c_{i,t}) = (\lambda_0 + \lambda_1(-c_{i,t}))\mathbf{1}_{c_{i,t}<0}, \tag{IA.26}$$

where λ_0 is the fixed cost and λ_1 represents the proportional cost. Therefore, the net dividend is

$$d_{i,t} = c_{i,t} - \Lambda(c_{i,t}). \tag{IA.27}$$

D. Equity Valuation

A firm's equity value is the maximal present value of the discounted stream of dividends that the firm can achieve by altering its investment and financing policy. To evaluate cash flow, we assume a process for the pricing kernel $\mathbb{M}_{t,t+1}$ similar to that in Berk, Green, and Naik (1999) and Zhang (2005),

$$\mathbb{M}_{t,t+1} = \beta \exp \left\{ \Gamma_t (X_t - X_{t+1}) \right\}, \quad \Gamma_t = \gamma_0 + \gamma_1 (X_t - \overline{X}), \tag{IA.28}$$

where $0 < \beta < 0$ is the time discount factor, and γ_0 and γ_1 are constants.[1]

At any point in time a firm is entirely described by four state variables: the aggregate shock X_t, the firm-specific shock $Z_{i,t}$, the capital level K_t, and the total debt commitment $D_{i,t}$ defined in (IA.24). We denote by $S_{i,t} = \{X_t, Z_{i,t}, K_{i,t}, D_{i,t}\}$ the vector of state variables. Equity value, $V(S_t)$, is the solution to a dynamic programming problem with optimal investment financing and default choices. Unless the company optimally defaults at time t, these choices will result in a new level of capital $K_{i,t+1}$ and a new level of total debt commitment $D_{i,t+1}$. Hence, the future levels of capital $K_{i,t+1}$ and total debt commitment $D_{i,t+1}$ are *control* variables at time t, but become *state* variables at time $t+1$.

In the absence of shareholder recovery upon financial distress, the firm is financially viable as long as equity has a positive value, that is, $V(S_{i,t}) > 0$; default occurs when $V(S_{i,t}) = 0$. In the presence of shareholder recovery, equity can extract a fraction η of the residual asset value upon financial distress, $R(S_t)$. Following Hennessy and Whited (2007), we model the residual asset value as

$$R(S_{i,t}) = \max \left\{ \Pi_{i,t} + \tau\kappa K_{i,t} + \xi_1 (1-\kappa) K_{i,t} - \xi_0, 0 \right\}, \tag{IA.29}$$

where $\Pi_{i,t}$ is the after-tax profit defined in (IA.21), and $1 - \xi_1$ and ξ_0 are the proportional and fixed distress costs, respectively. Firm i's equity value is therefore determined by the solution to the following Bellman equation:

$$V(S_{i,t}) = \max \left\{ \eta R(S_{i,t}), \max_{\{K_{i,t+1}, D_{i,t+1}\}} \left\{ d(S_{i,t}) + \mathbb{E}_t \left[\mathbb{M}_{t,t+1} V(S_{i,t+1}) \right] \right\} \right\}. \tag{IA.30}$$

One potential difficulty in computing the net cash flow—$c_{i,t}$ in (IA.25), and in turn $d_{i,t}$ in (IA.27)—needed for the solution to the dynamic programming problem (IA.30) relates to the determi-

[1] The market price of risk is equal to $\lambda_{m,t} = Var_t[\mathbb{M}_{t,t+1}]/E_t[\mathbb{M}_{t,t+1}]$. Given the assumption of normality in the innovations of X_t, $\lambda_{m,t} = \beta(e^{\sigma_m^2} - 1)$, where $\sigma_m = \sigma_x[\gamma_0 + \gamma_1(X_t - \overline{X})]$.

nation of the face value $B_{i,t+1}$ (and the coupon $b_{i,t+1}$) of the newly issued debt. As argued in Li (2008), the use of the total debt commitment $D_{i,t}$ as a state variable simplifies the problem considerably because we can avoid having to keep track of the coupon $b_{i,t+1}$. Denoting by

$$\chi_{i,t+1} = \mathbf{1}_{\{V(S_{i,t+1})>\eta R(S_{i,t+1})\}} \tag{IA.31}$$

the indicator function for the firm's solvency, we can evaluate the market value of the bond as follows:

$$\begin{aligned} B_{i,t+1} &= \mathbb{E}_t\left[\mathbb{M}_{t,t+1}\left(\chi_{i,t+1}(b_{i,t+1}+B_{i,t+1}) + (1-\chi_{i,t+1})(1-\eta)R(S_{i,t+1})\right)\right] \\ &= \frac{\mathbb{E}_t\left[\mathbb{M}_{t,t+1}\left\{\chi_{i,t+1}\frac{D_{i,t+1}}{1-\tau} + (1-\chi_{i,t+1})(1-\eta)R(S_{i,t+1})\right\}\right]}{1+\frac{\tau}{1-\tau}\mathbb{E}_t\left[\mathbb{M}_{t,t+1}\chi_{i,t+1}\right]}, \end{aligned} \tag{IA.32}$$

where the first equality considers the debt value in the cases of solvency and default, respectively, and the last equation uses the definition of total debt commitment (IA.24) to express the coupon $b_{i,t+1}$ as a function of $D_{i,t+1}$ and $B_{i,t+1}$. The bond pricing equation (IA.32) involves only knowledge of the evolution of the state variables $S_{i,t}$ and will be used to determine the cash flows net of investment and financing defined in (IA.25).

We solve the model numerically by using value function iterations and discretization of the state space. The model contains a total of 19 parameters, summarized in Table IA.I. While it would be ideal to calibrate these parameters by matching relevant moments via, for example, a simulated method of moments (SMM) methodology, the large dimensionality of the state space makes this approach computationally infeasible.[2] To calibrate the model, we instead follow Livdan, Sapriza, and Zhang (2009) and Gomes and Schmid (2009), who base their parameter choice on the values used in the existing macro and finance literatures (e.g., Gomes (2001), Cooley and Quadrini (2001), Cooper and Ejarque (2003), Zhang (2005), and Hennessy and Whited (2005, 2007)). The model is solved on a monthly basis. Details of the solution methodology are provided in Section III of this Internet Appendix.

[2]The problem is characterized by four state variables. After discretization, the state space contains a total of 6,630,000 grid points.

Table IA.I
Parameters for the General Model

The table reports the parameters used in solving the model described in Section II of the Internet Appendix. Parameter values are consistent with those in Gomes and Schmid (2009), Li (2008), and Zhang (2005).

Parameter	Description	Value
$\overline{X}$	Long-run average of aggregate productivity	-3.100
σ_x	Conditional volatility of aggregate productivity	0.002
ρ_x	Persistence of aggregate productivity	0.983
$\overline{Z}$	Long-run average of firm-specific productivity	0.000
σ_z	Conditional volatility of firm-specific productivity	0.100
ρ_z	Persistence of firm-specific productivity	0.900
α	Capital share	0.650
δ	Capital depreciation	0.010
f	Variable cost of production	0.000
F	Fixed cost of production	0.034
θ	Adjustment cost	15.000
γ_0	Constant price of risk	50
γ_1	Time-varying price of risk	-1000
β	Time-preference coefficient	0.995
τ	Tax rate	0.350
ξ_0	Fixed bankruptcy cost	0.120
ξ_1	Liquidation value per unit of capital	0.900
λ_0	Fixed equity issuance cost	0.080
λ_1	Variable equity issuance cost	0.025

For each firm, the solution of the model consists of the firm's equity value and the associated optimal investment, financing, and default policies over the state space. Knowledge of these quantities allows us to analyze the cross-sectional properties of equity returns and the effect of shareholder recovery η under a stationary distribution of the underlying state variables. We construct the cross-section by bootstrapping expected returns from the stationary distribution and form portfolios according to the model-implied default probability. According to the intuition developed in Section I of the paper, the absence of shareholder recovery leads to expected returns that are increasing in default probability, while the presence of such recovery leads to expected returns that are hump-shaped in default probability.

Panel A: No shareholder recovery ($\eta = 0$) Panel B: Shareholder recovery ($\eta = 10\%$)

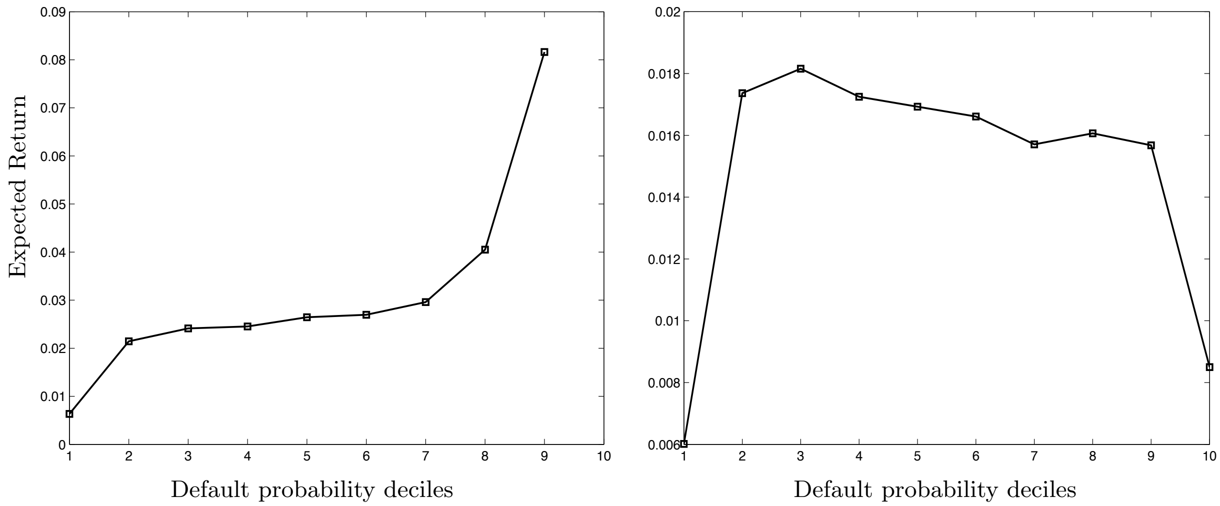

Figure IA.1. Expected return and default probability. The figure reports the monthly expected return as a function of default probability obtained from the general model of Section II of the Internet Appendix. Panel A refers to the case of no shareholder recovery, $\eta = 0$, while Panel B refers to the case in which shareholder recovery is set to $\eta = 10\%$ of the recovery value in (IA.29).

Figure IA.1 confirms the above intuition within the general model of this section.[3] Panel A presents the case of no shareholder recovery ($\eta = 0$), while Panel B depicts the case with expected shareholder recovery equal to 10% of the residual firm value defined in (IA.29). As can be clearly seen from the figure, the case of no recovery leads to a monotonically increasing relation between expected return and default probability that "explodes" in the highest decile when default becomes almost certain. In contrast, in the presence of expected shareholder recovery upon distress, the relation between expected return and default probability is humped, increasing at low levels of default probability and decreasing at high levels of default probability. These patterns are consistent with the implications of the simple equity valuation model as well as with the empirical results presented in Garlappi, Shu, and Yan (2008).

The simple model of Section I in the paper predicts that the value premium should be increasing in default probability in the absence of shareholder recovery and humped in default

[3]Numerical details for the figures reported in this section of the Internet Appendix are contained in Section III of this Appendix.

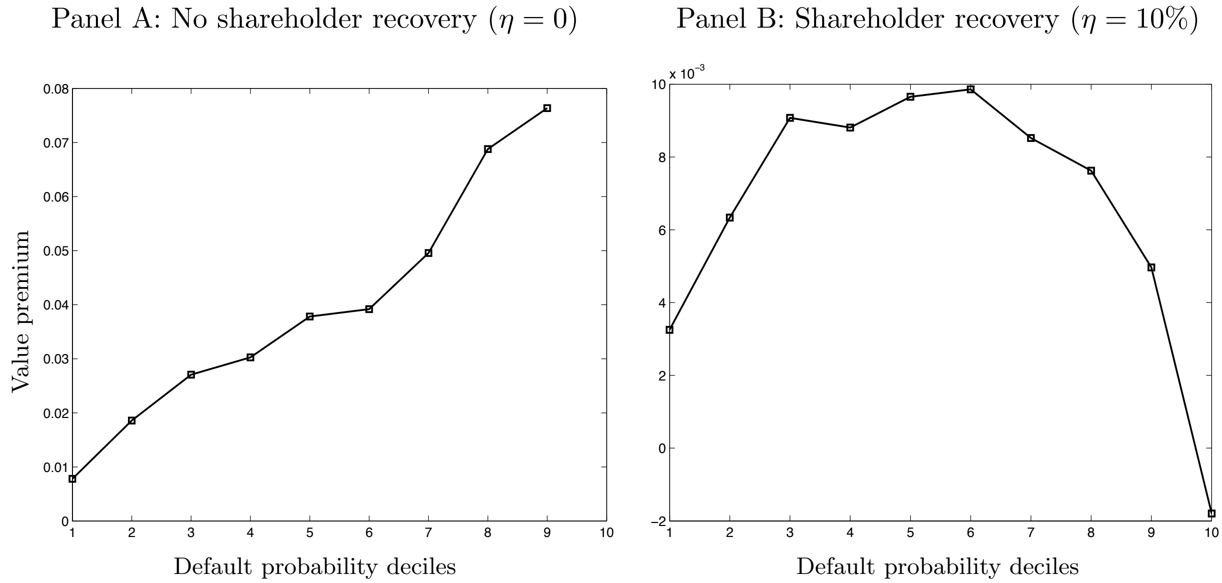

Figure IA.2. Value premium and default probability. The figure reports the monthly spread between the average expected returns of high book-to-market firms and those of low book-to-market firms within each decile of default probability in the cross-section of firms generated from the stationary solution of the general model of Section II of the Internet Appendix. Panel A corresponds to the case with $\eta = 0$, while Panel B refers to the case with $\eta = 10\%$.

probability in the presence of shareholder recovery. We construct the value premium in our stationary economy by following a similar bootstrapping methodology as the one used for expected returns. Figure IA.2 confirms that the prediction of the simple model is valid also for the general model of this section. Panel A plots the value premium on a monthly basis for the case of no shareholder recovery, while Panel B considers the case of expected shareholder recovery equal to 10% of the residual value defined in (IA.29). The presence of shareholder recovery substantially affects the pattern of the value premium conditional on default probability. The value premium is positive and increasing in the absence of shareholder recovery, while it is hump-shaped when shareholder recovery is present, turning negative in the highest default probability decile.

Finally, according to the simple model of Section I in the paper, in the presence of possible shareholder recovery, the humped relationship between expected returns and default probability implies the concentration of momentum profits in low-credit-quality firms, and no momentum in the absence of shareholder recovery. To verify whether this conjecture is also true in the

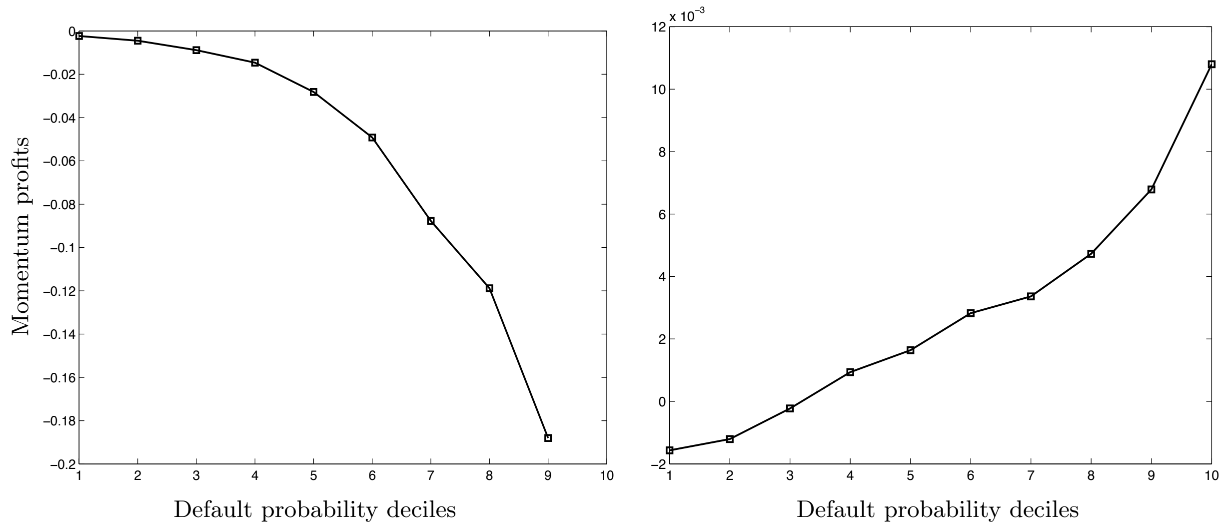

Figure IA.3. Momentum profits and default probability. The figure reports the monthly momentum profits as a function of default probability generated from the general model of Section II of the Internet Appendix. Panel A refers to the case of no shareholder recovery, $\eta = 0$, while Panel B refers to the case in which shareholder recovery is set to $\eta = 10\%$ of the recovery value in (IA.29).

general model of this section, we generate momentum profits by computing the spread between the model-implied expected returns of winners and losers, as described in the next section, and report the results in Figure IA.3. As before, Panel A considers the case of no shareholder recovery while Panel B examines the case of shareholder recovery equal to 10% of the residual value defined in (IA.29). The figure illustrates that momentum profits are positive and significant only for firms with shareholder recovery and with high default probability. The range of momentum profits runs from 2% to 12% annually, depending on the level of default probability, which is comparable to empirical estimates. For firms without shareholder recovery, the momentum strategy does not work, as it would only generate losses, as indicated in Panel A.[4]

In summary, the analysis in this section confirms that the intuition developed within the simple model of Section I in the paper is robust in a more general framework that allows firms to optimally choose their capital structure and investment levels.

[4]The large magnitude of losses is attributable to the explosive nature of expected returns as default probability approaches one.

III. Numerical Details of the General Model

A. Solution

We solve for the fixed point in the Bellman equation (IA.30) by using a standard value function iteration algorithm on a discretized grid space (see, for example, Judd (1998)). In particular, we discretize the four-dimensional state space by choosing (i) 100 equally spaced grid points between zero and 20 for both the capital level $K_{i,t}$ and the debt commitment level $D_{i,t}$, (ii) 17 grid points for the systematic shock $X_{i,t}$, and (iii) 39 grid points for the idiosyncratic shock $Z_{i,t}$. We use Tauchen's (1986) quadrature method to choose the grid points for the systematic and idiosyncratic shocks. As a result, the state space is discretized into $17 \times 39 \times 100 \times 100 = 6,630,000$ grid points.

B. Expected Equity Return

To simplify notation, in the following we drop the subscript i from firm-specific quantities. The expected equity return is defined as

$$E_t[R_{t+1}] = \frac{\mathbb{E}_t[V(S_{t+1})]}{V(S_t) - d(S_t)}, \tag{IA.33}$$

where the expectation $\mathbb{E}_t$ is taken with respect to the probability measure induced by the Markov processes (IA.19) and (IA.20) for the systematic and idiosyncratic shocks and by the optimal investment and financing policies. We subtract the dividend $d(S_t)$ from the equity value in the denominator of (IA.33) because equity value in (IA.30) is cum dividend. From the stationary solution we compute recursively the τ-month-ahead *probability of default* as follows:

$$p_\tau(S_t) = (1 - \chi(S_t)) + \chi(S_t) \cdot \mathbb{E}_t[p_{\tau-1}(S_{t+1})], \quad p_0(S_{t+1}) = 1 - \chi(S_{t+1}), \tag{IA.34}$$

where χ is the stationary default boundary (IA.31).

Figure IA.1 is obtained by bootstrapping expected returns from the stationary distribution. Specifically, according to the processes (IA.19), the unconditional distribution for X_t is normal with mean $\overline{X}$ and variance $\sigma_x^2/(1-\rho_x^2)$. Similarly, the unconditional distribution for Z_t is normal with mean $\overline{Z}$ and variance $\sigma_z^2/(1-\rho_z^2)$. A firm is characterized by a point $S_t = \{X_t, Z_t, K_t, Z_t\}$ in the state space. We construct three representative panels of 10,000 firms each. Each panel corresponds to a different realization of the systematic state variable X_t. We select these three points to be $\{\overline{X} - s_X, \overline{X}, \overline{X} + s_X\}$, where $\overline{X}$ is the long-run mean and $s_X = \sigma_x/(1-\rho_x^2)^{1/2}$ the long-run volatility of X_t. These values are chosen to represent three phases of the business cycle. For each realization of X_t we randomly select 100 points from the stationary distribution of Z_t and 10 points each for K_t and D_t, chosen uniformly from their respective support. After forming each panel, we sort firms into 10 portfolios according to their default probability (IA.34) and compute the equal-weighted *expected* returns of the portfolios thus obtained. We repeat this procedure 500 times for each panel and compute the average expected return conditional on the realization of X_t. Figure IA.1 reports the unconditional expected return, obtained by weighting each conditional expected return by the long-run probability of the chosen realization of X_t. Panel A presents the case of no shareholder recovery ($\eta = 0$), while Panel B considers the case with expected shareholder recovery equal to 10% of the residual value $R(S_t)$ defined in (IA.29).

C. Value Premium

From the stationary solution of the general model, we can construct the book-to-market ratio $BM(S_t)$ at each point S_t of the state space as

$$BM(S_t) = \frac{K_t - D_t}{V(S_t) - d(S_t)}. \tag{IA.35}$$

To study the structure of the value premium in the cross-section, we follow the bootstrap methodology in the previous subsection. For each realization of X_t we draw a panel of 10,000 firms by randomly selecting 100 points from the stationary distribution of Z_t and 10 points each for K_t and D_t chosen uniformly from their support. We then sort firms into 10 portfolios based on their default probability (IA.34). We finally sort each of these portfolios into five subportfolios according to the firms' book-to-market ratio. Within each default probability decile, we compute the value premium as the spread between the expected returns of the highest and lowest book-to-market quintiles. We repeat this procedure 500 times for each panel and aggregate the results from each panel by weighting them according to the long-run probability of the chosen realization of X_t. Figure IA.2 above reports the average value premium in each default probability decile across the 500 repetitions.

D. Momentum Profits

To construct momentum portfolios we generate a time series of realized returns that determines winners and losers in each period. For this purpose, we follow the bootstrapping procedure of the previous two subsections. For each realization of X_t we draw a panel of 10,000 firms by randomly selecting 100 points from the stationary distribution of Z_t and 10 points each for K_t and D_t, chosen uniformly from their support. Based on the dynamics of the state variables X_t and Z_t, and the optimal investment and financing strategies derived from the solution of the model, each state S_t will evolve to a *future* state S_{t+1}. The realized return is hence $\frac{V(S_{t+1})}{V(S_t)-d(S_t)}$, which is used to separate winners from losers. The expected return in the state S_{t+1} can be subsequently deduced directly from the stationary solution as discussed above in Subsection III.B.

We construct momentum profits by sorting the panel of firms in state S_{t+1} into 10 portfolios based on their default probability and, independently, into five portfolios based on the realized return from state S_t to state S_{t+1}. The bottom quintile represents the portfolio of *losers* and the top quintile the portfolio of *winners*. The expected momentum profits are calculated as the difference in the equal-weighted expected returns of winners and losers in each default probability decile. We repeat this procedure 500 times for each panel and aggregate the results from each panel by weighting them according to the long-run probability of the chosen realization of X_t. Figure IA.3 above reports the average monthly momentum profits in each default probability decile across the 500 repetitions.

IV. Additional Empirical Results

In this section we present additional empirical results that are mentioned in the main text of the paper, but omitted there because of space limitation. Table IA.II is equivalent to Table III in the paper except that portfolio returns are recorded for the second month after portfolio formation, instead of the subsequent month. Table IA.III reports the equal-weighted results that complement the value-weighted results contained in Table V in the paper.

Table IA.II
Value Premium and Default Probability: Skipping a month

Each month, stocks are sorted independently into terciles of book-to-market ratios (BM) and deciles of MKMV's EDF scores (EDF). The table reports the time-series average of value-weighted (VW) and equal-weighted (EW) returns of each portfolio obtained in the second month after portfolio formation. Portfolio returns are expressed in percentage per month. CAPM-alpha, FF-alpha, 4-Factor alpha, and 5-Factor alpha refer to the value premium after controlling for risk according to, respectively, the CAPM market factor, the Fama-French three-factor model, the Carhart four-factor model, and a five-factor model that also includes the liquidity factor of Pastor and Stambaugh (2003).

	Low EDF 1	2	3	4	5	6	7	8	9	High EDF 10
Panel A: Value-weighted										
Returns										
Low B/M	0.96	0.90	0.74	0.82	0.71	0.39	0.25	-0.05	-0.36	-0.24
Medium B/M	1.00	1.10	1.26	1.17	1.23	1.17	0.95	0.79	0.67	0.03
High B/M	0.96	1.19	1.28	1.42	1.39	1.32	1.47	1.41	1.12	0.84
Value Premium										
Raw	0.00	0.28	0.54	0.61	0.69	0.93	1.22	1.46	1.48	1.07
t-stat	-0.013	1.433	2.289	2.603	2.877	3.633	4.582	4.593	4.584	2.920
CAPM alpha	0.10	0.47	0.79	0.82	0.92	1.15	1.39	1.64	1.68	1.19
t-stat	0.531	2.581	3.754	3.874	4.312	4.842	5.417	5.329	5.381	3.232
FF alpha	-0.52	-0.07	0.15	0.23	0.39	0.58	0.96	1.03	1.22	0.85
t-stat	-3.611	-0.553	1.025	1.450	2.204	2.866	4.004	3.689	4.028	2.308
4-Factor alpha	-0.60	-0.20	-0.03	0.07	0.21	0.37	0.73	0.66	0.77	0.57
t-stat	-4.107	-1.484	-0.195	0.465	1.172	1.807	3.010	2.369	2.575	1.536
5-Factor alpha	-0.59	-0.21	0.00	0.13	0.17	0.27	0.73	0.76	0.75	0.62
t-stat	-3.977	-1.536	0.020	0.829	0.948	1.325	2.969	2.704	2.506	1.645
Panel B: Equal-weighted										
Returns										
Low B/M	1.10	0.98	0.89	0.78	0.71	0.56	0.46	0.42	0.33	1.27
Medium B/M	1.09	1.27	1.35	1.32	1.39	1.37	1.28	1.17	1.04	1.51
High B/M	1.21	1.21	1.53	1.58	1.59	1.57	1.65	1.69	1.60	1.84
Value Premium										
Raw	0.10	0.23	0.64	0.79	0.87	1.01	1.19	1.27	1.26	0.58
t-stat	0.599	1.250	3.272	3.931	4.358	4.926	5.998	5.1911	5.405	1.987
CAPM alpha	0.28	0.48	0.88	1.05	1.10	1.25	1.40	1.48	1.43	0.74
t-stat	1.859	3.130	5.506	6.345	6.393	7.090	7.923	6.564	6.389	2.648
FF alpha	-0.22	0.00	0.40	0.57	0.61	0.81	1.02	1.07	1.11	0.46
t-stat	-2.010	-0.027	3.784	5.009	5.002	5.789	6.789	5.224	5.195	1.677
4-Factor alpha	-0.24	-0.12	0.23	0.42	0.48	0.57	0.75	0.69	0.69	0.05
t-stat	-2.205	-1.224	2.236	3.696	3.904	4.193	5.157	3.521	3.394	0.180
5-Factor alpha	-0.23	-0.11	0.23	0.42	0.45	0.55	0.73	0.69	0.68	0.03
t-stat	-2.027	-1.150	2.159	3.663	3.663	3.968	4.982	3.488	3.299	0.107

Table IA.III

Momentum Profits and Default Probability: Equal-weighted returns

The column labeled "Uncond." reports momentum profits computed according to the "6-1-6" procedure in Jegadeesh and Titman (1993). The remaining columns report momentum profits similarly computed within EDF quintiles. To obtain these values, each month all stocks are sorted independently into quintiles of EDF scores and quintiles of winners/losers according to past six-month returns. We skip a month after portfolio formation. The equal-weighted returns of each portfolio for the subsequent six-month period are recorded and averaged over time. Portfolio returns are expressed in percentage per month. Momentum alphas are obtained after controlling for risk according to the Carhart four-factor model.

	Uncond.	EDF					Diff
		1	2	3	4	5	
Raw profits	1.00	0.99	1.01	1.17	1.25	0.98	-0.02
t-stat	4.508	4.612	5.009	6.286	6.537	4.448	-0.070
4-Factors alphas	0.26	0.15	0.20	0.42	0.53	0.35	0.20
t-statistic	1.899	1.178	1.861	3.876	4.264	1.941	0.878
				Factor loadings			
UMD	0.906	0.924	0.910	0.810	0.785	0.702	-0.222
t-stat	28.706	30.826	36.813	32.140	27.087	16.822	-4.277
MKT	0.028	0.013	0.013	0.043	0.043	0.013	0.000
t-stat	0.859	0.429	0.516	1.665	1.438	0.300	-0.007
HML	0.033	0.115	0.082	0.129	0.117	0.166	0.051
t-stat	0.687	2.496	2.164	3.328	2.618	2.583	0.633
SMB	-0.275	0.136	0.122	-0.015	-0.043	-0.210	-0.345
t-stat	-6.567	3.405	3.731	-0.435	-1.112	-3.792	-5.004

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