

Internet Appendix to “Global Currency Hedging”*

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A. Hedged Portfolio Return

Let $R_{c,t+1}$ denote the gross return in currency c from holding country c stocks from the beginning to the end of period $t + 1$, and let $S_{c,t+1}$ denote the spot exchange rate in dollars per foreign currency c at the end of period $t + 1$. By convention, we index the domestic country by $c = 1$ and the n foreign countries by $c = 2, \dots, n + 1$. Of course, the domestic exchange rate is constant over time and equal to one: $S_{1,t+1} = 1$ for all t .

At time t , the investor exchanges a dollar for $1/S_{c,t}$ units of currency c in the spot market, which she then invests in the stock market of country c . After one period, stocks from country c return $R_{c,t+1}$, which the U.S. investor can exchange for $S_{c,t+1}$ dollars, to earn an unhedged gross return of $R_{c,t+1}S_{c,t+1}/S_{c,t}$. For an arbitrarily weighted portfolio, the unhedged gross portfolio return is given by

$$R_{p,t+1}^{uh} = \mathbf{R}'_{t+1} \boldsymbol{\omega}_t (\mathbf{S}_{t+1} \div \mathbf{S}_t),$$

where $\boldsymbol{\omega}_t = \text{diag}(\omega_{1,t}, \omega_{2,t}, \dots, \omega_{n+1,t})$ is the $(n + 1 \times n + 1)$ diagonal matrix of weights on domestic and foreign stocks at time t , \mathbf{R}_{t+1} is the $(n + 1 \times 1)$ vector of gross nominal stock returns in local currencies, \mathbf{S}_{t+1} is the $(n + 1 \times 1)$ vector of spot exchange rates, and \div denotes the element-by-element ratio operator, so that the c -th element of $(\mathbf{S}_{t+1} \div \mathbf{S}_t)$ is $S_{c,t+1}/S_{c,t}$. The weights add up to one in each period t :

$$\sum_{c=1}^{n+1} \omega_{c,t} = 1 \quad \forall t. \tag{IA.1}$$

We next consider the hedged portfolio. Let $F_{c,t}$ denote the one-period forward exchange rate in dollars per foreign currency c , and $\theta_{c,t}$ the dollar value of the amount of forward exchange rate contracts for currency c the investor enters into at time t per dollar invested in her stock portfolio.¹ At the end of period $t + 1$, the investor gets to exchange $\theta_{c,t}/S_{c,t}$ units of the foreign currency-denominated return $R_{c,t+1}\omega_{c,t}/S_{c,t}$ back into dollars at an exchange rate $F_{c,t}$. She then exchanges the rest, which amounts to $(R_{c,t+1}\omega_{c,t}/S_{c,t} - \theta_{c,t}/S_{c,t})$ units of foreign currency c at the spot exchange rate $S_{c,t+1}$. Collecting returns for all countries leads to a hedged portfolio return $R_{p,t+1}^h$ of

$$R_{p,t+1}^h = \mathbf{R}'_{t+1} \boldsymbol{\omega}_t (\mathbf{S}_{t+1} \div \mathbf{S}_t) - \boldsymbol{\Theta}'_t (\mathbf{S}_{t+1} \div \mathbf{S}_t) + \boldsymbol{\Theta}'_t (\mathbf{F}_t \div \mathbf{S}_t), \tag{IA.2}$$

¹That is, at the end of month t , the investor can enter into a forward contract to sell one unit of currency c at the end of month $t + 1$ for a forward price of $F_{c,t}$ dollars.

where \mathbf{F}_t is the $(n+1 \times 1)$ vector of forward exchange rates and $\Theta_t = (\theta_{1,t}, \theta_{2,t}, \dots, \theta_{n,t}, \theta_{n+1,t})'$. Of course, since $S_{1t} = F_{1,t} = 1$ for all t , the choice of domestic hedge ratio $\theta_{1,t}$ is arbitrary. For convenience, we set it so that all hedge ratios add up to one:

$$\theta_{1,t} = 1 - \sum_{c=2}^{n+1} \theta_{c,t}. \quad (\text{IA.3})$$

Under covered interest parity, the forward contract for currency c trades at $F_{c,t} = S_{c,t}(1 + I_{1,t})/(1 + I_{c,t})$, where $I_{1,t}$ denotes the domestic nominal short-term riskless interest rate available at the end of period t , and $I_{c,t}$ is the corresponding country c nominal short-term interest rate. Thus, the hedged dollar portfolio return (IA.2) can be written as

$$R_{p,t+1}^h = \mathbf{R}'_{t+1} \boldsymbol{\omega}_t (\mathbf{S}_{t+1} \div \mathbf{S}_t) - \Theta'_t (\mathbf{S}_{t+1} \div \mathbf{S}_t) + \Theta'_t [(\mathbf{1} + \mathbf{I}_t^d) \div (\mathbf{1} + \mathbf{I}_t)], \quad (\text{IA.4})$$

where $\mathbf{I}_t = (I_{1,t}, I_{2,t}, \dots, I_{n+1,t})$ is the $(n + 1 \times 1)$ vector of nominal short-term interest rates and $\mathbf{I}_t^d = I_{1,t} \mathbf{1}$.

Equation (IA.4) shows that selling currency forward—that is, setting $\theta_{c,t} > 0$ —is analogous to a strategy of shorting foreign bonds and holding domestic bonds, that is, borrowing in foreign currency and lending in domestic currency.²

To capture the fact that the investor can alter the currency exposure implicit in her foreign stock position using forward contracts or lending and borrowing, we now define a new variable $\psi_{c,t}$ as $\psi_{c,t} \equiv \omega_{c,t} - \theta_{c,t}$. A fully hedged portfolio, in which the investor does not hold any exposure to currency c , corresponds to $\psi_{c,t} = 0$. A positive value of $\psi_{c,t}$ means that the investor wants to hold exposure to currency c , or equivalently that the investor does not want to fully hedge the currency exposure implicit in her stock position in country c . Of course, a completely unhedged portfolio corresponds to $\psi_{c,t} = \omega_{c,t}$. Thus, $\psi_{c,t}$ is a measure of currency demand or currency exposure. Accordingly, we refer to $\psi_{c,t}$ as currency demand or currency exposure indistinctly.

For convenience, we now rewrite equation (IA.4) in terms of currency demands:

$$\begin{aligned} R_{p,t+1}^h &= \mathbf{R}'_{t+1} \boldsymbol{\omega}_t (\mathbf{S}_{t+1} \div \mathbf{S}_t) - \mathbf{1}' \boldsymbol{\omega}_t [(\mathbf{S}_{t+1} \div \mathbf{S}_t) - (\mathbf{1} + \mathbf{I}_t^d) \div (\mathbf{1} + \mathbf{I}_t)] \\ &\quad + \boldsymbol{\Psi}'_t [(\mathbf{S}_{t+1} \div \mathbf{S}_t) - (\mathbf{1} + \mathbf{I}_t^d) \div (\mathbf{1} + \mathbf{I}_t)], \end{aligned} \quad (\text{IA.5})$$

²Note, however, that the two strategies are not completely equivalent except in the continuous time limit. We show in Internet Appendix B that, in continuous time, the two strategies are exactly equivalent.

where $\Psi_t = (\psi_{1,t}, \psi_{2,t}, \dots, \psi_{n+1,t})'$.

Note that $\Psi_t = \omega_t \mathbf{1} - \Theta_t$. Given the definition of $\psi_{c,t}$, equations (IA.1) and (IA.3) imply that

$$\psi_{1,t} = - \sum_{c=2}^{n+1} \psi_{c,t}, \quad (\text{IA.6})$$

or $\Psi_t' \mathbf{1} = \mathbf{0}$, so that $\psi_{1,t}$ indeed represents the domestic currency exposure. That currency demands must add to zero is intuitive. Since the investor is fully invested in stocks, she can achieve a long position in a particular currency c only by borrowing—or equivalently, by shorting bonds—in her own domestic currency, and investing the proceeds in bonds denominated in that currency. Thus, the currency portfolio is a zero investment portfolio.

B. Log Portfolio Returns Over Short Time Intervals

Assuming log-normality of the hedge returns, the derivation of the optimal Ψ requires an expression for the log-return on the hedged portfolio, $r_{p,t+1}^{\text{hedge}}$. We compute this log hedged return as a discrete-time approximation to its continuous-time counterpart. In order to do this, we need to specify, in continuous time, the return processes for stocks $P_{c,t}$, currencies $X_{c,t}$ and interest rates $B_{c,t}$. We assume that they all follow geometric brownian motions:

$$\frac{dP_{c,t}}{P_{c,t}} = \mu_{P_c} dt + (\sigma_{P_c})_t dW_t^{P_c}, \quad c = 1 \dots n + 1 \quad (\text{IA.7})$$

$$\frac{dB_{c,t}}{B_{c,t}} = \mu_{B_c} dt, \quad c = 1 \dots n + 1 \quad (\text{IA.8})$$

$$\frac{dX_{c,t}}{X_{c,t}} = \mu_{X_c} dt + (\sigma_{X_c})_t dW_t^{X_c}, \quad c = 1 \dots n + 1, \quad (\text{IA.9})$$

where $W_t^{P_c}$, $W_t^{B_c}$, and $W_t^{X_c}$ are diffusion processes. $\frac{dP_{c,t}}{P_{c,t}}$ represents the stock return, $\frac{dB_{c,t}}{B_{c,t}}$ the nominal return to holding a riskless bond from country, and $\frac{dX_{c,t}}{X_{c,t}}$ the return to holding foreign currency c .

For notational simplicity, in what follows, we momentarily drop time subscripts for the standard deviations.

Using Ito's lemma, the log returns on each asset are given by

$$\begin{aligned} d \log P_{c,t} &= \frac{dP_{c,t}}{P_{c,t}} - \frac{1}{2} \sigma_{P_c}^2 dt \\ d \log B_{c,t} &= \frac{dB_{c,t}}{B_{c,t}} - \frac{1}{2} \sigma_{B_c}^2 dt \\ d \log X_{c,t} &= \frac{dX_{c,t}}{X_{c,t}} - \frac{1}{2} \sigma_{X_c}^2 dt. \end{aligned}$$

Note that, because country 1 is the domestic country, which has a fixed exchange rate of one, we have $d \log X_{1,t} = 0$. This implies $\mu_{X_1} = \sigma_{X_1} = 0$.

The domestic currency return on foreign stock is then given by $\frac{dP_{c,t}X_{c,t}}{P_{c,t}X_{c,t}}$. To derive an expression for this return, we note that the return dynamics above, by standard calculations, imply

$$\begin{aligned} \log P_{c,t}X_{c,t} &= \log P_{c,0}X_{c,0} + \left(\mu_{P_c} + \mu_{X_c} - \frac{1}{2} \sigma_{P_c}^2 - \frac{1}{2} \sigma_{X_c}^2 \right) t \\ &\quad + \sigma_{P_c} (W_t^{P_c} - W_0^{P_c}) + \sigma_{X_c} (W_t^{X_c} - W_0^{X_c}). \end{aligned}$$

Differentiating, and then applying Ito's lemma, yields

$$\frac{dP_{c,t}X_{c,t}}{P_{c,t}X_{c,t}} = \frac{dP_{c,t}}{P_{c,t}} + \frac{dX_{c,t}}{X_{c,t}} + \sigma_{P_c} \sigma_{X_c} \rho_{P_c, X_c} dt \quad (\text{IA.10})$$

$$\frac{dP_{c,t}X_{c,t}}{P_{c,t}X_{c,t}} = d \log P_{c,t} + d \log X_{c,t} + \frac{1}{2} \text{Var}_t(p_{c,t} + x_{c,t}) dt, \quad (\text{IA.11})$$

where $x_{c,t} = d \log X_{c,t}$ and $p_{c,t} = d \log P_{c,t}$. Note that for $c=1$, the formula does yield the simple stock return as $\frac{dP_{1,t}X_{1,t}}{P_{1,t}X_{1,t}} = \frac{dP_{1,t}}{P_{1,t}} + \frac{dX_{1,t}}{X_{1,t}} + \sigma_{P_1} \sigma_{X_1} \rho_{P_1, X_1} dt = \frac{dP_{1,t}}{P_{1,t}}$.

A similar calculation yields the following dynamics for the return of the strategy consisting of holding the domestic bond and shorting the foreign one:

$$\frac{d(B_{1,t}/B_{c,t})}{B_{1,t}/B_{c,t}} = d \log B_{1,t} - d \log B_{c,t}. \quad (\text{IA.12})$$

The log return on the portfolio, by Ito's lemma, is

$$d \log V_t = \frac{dV_t}{V_t} - \frac{1}{2} \left(\frac{dV_t}{V_t} \right)^2,$$

where we use V_t to denote the value of the portfolio.

We can now derive each of the right-hand side terms:

$$\frac{dV_t}{V_t} = \sum_{c=1}^{n+1} \omega_{c,t} \left(\frac{dP_{c,t} X_{c,t}}{P_{c,t} X_{c,t}} \right) + \sum_{c=1}^{n+1} \theta_c \omega_{c,t} \frac{d(B_{1,t}/B_{c,t})}{B_{1,t}/B_{c,t}} - \sum_{c=1}^{n+1} \theta_c \omega_{c,t} \frac{dX_{c,t}}{X_{c,t}},$$

which follows from our convention regarding the domestic country.

Using expressions (IA.9), (IA.11), and (IA.12) to substitute and moving to matrix notation, we get

$$\begin{aligned} \frac{dV_t}{V_t} &= \mathbf{1}' \boldsymbol{\omega} (\mathbf{p}_{t+1} + \mathbf{x}_{t+1}) - \boldsymbol{\Theta}'_t (\mathbf{x}_{t+1} - \mathbf{b}_t^d + \mathbf{b}_t) \\ &\quad + \frac{1}{2} [\mathbf{1}' \boldsymbol{\omega}_t \text{diag} (\text{Var}_t (\mathbf{p}_{t+1} + \mathbf{x}_{t+1})) - \boldsymbol{\Theta}'_t \text{diag} (\text{Var}_t \mathbf{x}_{t+1})] dt \quad , \end{aligned}$$

where $\mathbf{p}_{t+1} = (d \log P_{1,t}, d \log P_{2,t}, \dots, d \log P_{n+1,t})'$, $\mathbf{x}_{t+1} = (d \log X_{1,t}, d \log X_{2,t}, \dots, d \log X_{n+1,t})'$, $\mathbf{b}_t^d = (d \log B_{1,t}) \mathbf{1}$, $\mathbf{b}_t = (d \log B_{1,t}, d \log B_{2,t}, \dots, d \log B_{n+1,t})'$, and $\text{diag} (X)$ denotes, for a symmetric $(n \times n)$ matrix X , the $(n \times 1)$ vector of its diagonal terms.

Then,

$$\begin{aligned} \left(\frac{dV_t}{V_t} \right)^2 &= \text{Var}_t [\mathbf{1}' \boldsymbol{\omega}_t (\mathbf{p}_{t+1} + \mathbf{x}_{t+1}) - \boldsymbol{\Theta}'_t (\mathbf{x}_{t+1} - \mathbf{b}_t^d + \mathbf{b}_t)] dt + o(dt) \\ &= \left[\begin{array}{c} \mathbf{1}' \boldsymbol{\omega}_t \text{Var}_t (\mathbf{p}_{t+1} + \mathbf{x}_{t+1}) \boldsymbol{\omega}_t \\ -2\mathbf{1}' \boldsymbol{\omega}_t \text{cov}_t (\mathbf{p}_{t+1} + \mathbf{x}_{t+1}, \mathbf{x}_{t+1} - \mathbf{b}_t^d + \mathbf{b}_t) \boldsymbol{\Theta}_t \\ + \boldsymbol{\Theta}' \boldsymbol{\omega}_t \text{Var}_t (\mathbf{x}_{t+1} - \mathbf{b}_t^d + \mathbf{b}_t) \boldsymbol{\Theta}_t \end{array} \right] dt + o(dt) . \end{aligned}$$

So, finally,

$$\begin{aligned} d \log V_t &= \mathbf{1}' \boldsymbol{\omega}_t (\mathbf{p}_{t+1} + \mathbf{x}_{t+1}) - \boldsymbol{\Theta}'_t (\mathbf{x}_{t+1} - \mathbf{b}_t^d + \mathbf{b}_t) \tag{IA.13} \\ &\quad + \frac{1}{2} [\mathbf{1}' \boldsymbol{\omega}_t \text{diag} (\text{Var}_t (\mathbf{p}_{t+1} + \mathbf{x}_{t+1})) - \boldsymbol{\Theta}'_t \text{diag} (\text{Var}_t \mathbf{x}_{t+1})] dt \\ &\quad - \frac{1}{2} \text{Var}_t [\mathbf{1}' \boldsymbol{\omega}_t (\mathbf{p}_{t+1} + \mathbf{x}_{t+1}) - \boldsymbol{\Theta}'_t (\mathbf{x}_{t+1} - \mathbf{b}_t^d + \mathbf{b}_t)] dt + o(dt) . \end{aligned}$$

We can now get the approximation for $r_{p,t+1}^h$ by computing the previous expression for $dt = 1$, replacing $d \log X_{c,t} = \Delta s_{c,t+1}$, $d \log P_{c,t} = r_{c,t+1}$, and $d \log B_{c,t} = i_{c,t}$, and

neglecting the higher-order terms. Noting, for any variable, \mathbf{z}_t , the $(n + 1 \times 1)$ vector $(z_{1,t}, z_{2,t} \dots z_{n+1,t})$, this is equivalent to replacing in equation (IA.13) \mathbf{p}_{t+1} by \mathbf{r}_{t+1} , \mathbf{x}_{t+1} by $\Delta \mathbf{s}_{t+1}$, \mathbf{b}_t^d by \mathbf{i}_t^d , and \mathbf{b}_t by \mathbf{i}_t :

$$r_{p,t+1}^h \simeq \mathbf{1}' \boldsymbol{\omega}_t (\mathbf{r}_{t+1} + \Delta \mathbf{s}_{t+1}) - \boldsymbol{\Theta}'_t (\Delta \mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t) + \frac{1}{2} \Sigma_t^h,$$

where Σ_{t+1}^h is equal to

$$\begin{aligned} \Sigma_t^h &= \mathbf{1}' \boldsymbol{\omega}_t \text{diag} (\text{Var}_t (\mathbf{r}_{t+1} + \Delta \mathbf{s}_{t+1})) - \boldsymbol{\Theta}'_t \text{diag} (\text{Var}_t \Delta \mathbf{s}_{t+1}) \\ &\quad - \text{Var}_t [\mathbf{1}' \boldsymbol{\omega}_t (\mathbf{r}_{t+1} + \Delta \mathbf{s}_{t+1}) - \boldsymbol{\Theta}'_t (\Delta \mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t)], \end{aligned}$$

where, for any variable z , \mathbf{z}_t denotes the vector of country observations $(z_{1,t}, z_{2,t} \dots z_{n+1,t})'$ and small case letters denote logs in the following fashion: $r_{c,t+1} = \log (R_{c,t+1})$, $s_{t+1} = \log (S_{t+1})$, $i_t^d = \log (1 + I_{1,t}) \mathbf{1}$ and $i_{c,t} = \log (1 + I_{c,t})$.

We can now rewrite the portfolio return as a function of $\boldsymbol{\Psi}_t$ by substituting for $\boldsymbol{\Theta}_t$. This yields

$$\begin{aligned} r_{p,t+1}^h &= \mathbf{1}' \boldsymbol{\omega}_t (\mathbf{r}_{t+1} + \mathbf{i}_t^d - \mathbf{i}_t) + \boldsymbol{\Psi}'_t (\Delta \mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t) + \frac{1}{2} \Sigma_t^h \\ &= i_{1,t}^d + \mathbf{1}' \boldsymbol{\omega}_t (\mathbf{r}_{t+1} - \mathbf{i}_t) + \boldsymbol{\Psi}'_t (\Delta \mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t) + \frac{1}{2} \Sigma_t^h, \end{aligned}$$

where

$$\begin{aligned} \Sigma_t^h &= \mathbf{1}' \boldsymbol{\omega}_t \text{diag} (\text{Var}_t (\mathbf{r}_{t+1} + \Delta \mathbf{s}_{t+1})) - (-\boldsymbol{\Psi}_t + \boldsymbol{\omega}_t \mathbf{1})' \text{diag} (\text{Var}_t (\Delta \mathbf{s}_{t+1})) \\ &\quad - \text{Var}_t (\mathbf{1}' \boldsymbol{\omega}_t (\mathbf{r}_{t+1} + \mathbf{i}_t^d - \mathbf{i}_t) + \boldsymbol{\Psi}'_t (\Delta \mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t)). \end{aligned} \tag{IA.14}$$

C. Equivalence Between Forward Contracts and Foreign Currency Borrowing and Lending

With the same notation and assumptions as above, when the investor uses forward contracts to hedge currency risk, the portfolio return is

$$R_{p,t+1}^h = \mathbf{R}'_{t+1} \boldsymbol{\omega}_t (\mathbf{S}_{t+1} \div \mathbf{S}_t) - \boldsymbol{\Theta}'_t [(\mathbf{S}_{t+1} \div \mathbf{S}_t) - (\mathbf{1} + \mathbf{I}_t^d) \div (\mathbf{1} + \mathbf{I}_t)].$$

Another natural view is one in which the investor borrows in foreign currency and lends in domestic currency to hedge currency risk. Then, the portfolio return is then

$$R_{p,t+1}^{BL} = \mathbf{R}'_{t+1} \boldsymbol{\omega}_t (\mathbf{S}_{t+1} \div \mathbf{S}_t) - \boldsymbol{\Theta}' (\mathbf{S}_{t+1} \div \mathbf{S}_t) (\mathbf{1} + \mathbf{I}_t) + \boldsymbol{\Theta}' (\mathbf{1} + \mathbf{I}_t^d)$$

Thus, with V_t^{BL} the value of the portfolio with borrowing and lending, we have in continuous time:

$$\begin{aligned} \frac{dV_t^{BL}}{V_t^{BL}} &= \sum_{c=1}^{n+1} \omega_{c,t} \left(\frac{dP_{c,t} X_{c,t}}{P_{c,t} X_{c,t}} \right) - \sum_{c=1}^{n+1} \Theta_{c,t} \frac{dX_{c,t} B_{c,t}}{X_{c,t} B_{c,t}} + \sum_{c=1}^{n+1} \Theta_{c,t} \frac{dB_{1,t}}{B_{1,t}} \\ &= \sum_{c=1}^{n+1} \omega_{c,t} \left(\log P_{c,t} + \log X_{c,t} + \frac{1}{2} \text{Var}_t (p_{c,t} + x_{c,t}) dt \right) \\ &\quad - \sum_{c=1}^{n+1} \Theta_{c,t} \left(\log (X_{c,t}) + \log (B_{c,t}) + \frac{1}{2} \text{Var}_t (x_{c,t}) dt \right) \\ &\quad + \sum_{c=2}^{n+1} \Theta_{c,t} \log (B_{1,t}) \\ &= \mathbf{1}' \boldsymbol{\omega}_t (\mathbf{p}_{t+1} + \mathbf{x}_{t+1}) - \boldsymbol{\Theta}' (\mathbf{x}_{t+1} + \mathbf{b}_t - \mathbf{b}_t^d) + \frac{1}{2} \mathbf{1}' \boldsymbol{\omega}_t \text{diag Var}_t (\mathbf{p}_{t+1} + \mathbf{x}_{t+1}) dt \\ &\quad - \frac{1}{2} \boldsymbol{\Theta}' \text{diag Var}_t (\mathbf{x}_{t+1}) dt, \end{aligned}$$

and

$$\left(\frac{dV_t^{BL}}{V_t^{BL}} \right)^2 = \text{Var}_t \left(\boldsymbol{\omega}'_t (\mathbf{p}_{t+1} + \mathbf{x}_{t+1}) - \boldsymbol{\Theta}' (\mathbf{x}_{t+1} + \mathbf{b}_t - \mathbf{b}_t^d) \right) dt + o(dt).$$

Therefore,

$$\begin{aligned}
d \log V_t^{BL} &= \frac{dV_t^{BL}}{V_t^{BL}} - \frac{1}{2} \left(\frac{dV_t^{BL}}{V_t^{BL}} \right)^2 \\
&= \boldsymbol{\omega}'_t (\mathbf{p}_{t+1} + \mathbf{x}_{t+1}) - \boldsymbol{\Theta}' (\mathbf{x}_{t+1} + \mathbf{b}_t - \mathbf{b}_t^d) + \frac{1}{2} \boldsymbol{\omega}'_t \text{diag Var}_t (\mathbf{p}_{t+1} + \mathbf{x}_{t+1}) dt \\
&\quad - \frac{1}{2} \boldsymbol{\Theta}' \text{diag Var}_t (\mathbf{x}_{t+1}) dt \\
&\quad - \frac{1}{2} \text{Var}_t (\boldsymbol{\omega}'_t (\mathbf{p}_{t+1} + \mathbf{x}_{t+1}) - \boldsymbol{\Theta}' (\mathbf{x}_{t+1} + \mathbf{b}_t - \mathbf{b}_t^d)) dt + o(dt).
\end{aligned}$$

We now go to the limit of $dt = 1$ and get

$$\begin{aligned}
r_{p,t+1}^{BL} &\simeq \mathbf{1}' \boldsymbol{\omega}_t (\mathbf{r}_{t+1} + \boldsymbol{\Delta} \mathbf{s}_{t+1}) - \boldsymbol{\Theta}' (\boldsymbol{\Delta} \mathbf{s}_{t+1} + \mathbf{i}_t - \mathbf{i}_t^d) + \frac{1}{2} \Sigma_t^h \\
&= r_{p,t+1}^h.
\end{aligned}$$

D. Mean-Variance Optimization

D.1. Unconstrained Hedge Ratio

In the general case, $r_{p,t+1}^h - i_{1,t}^d = \mathbf{1}' \boldsymbol{\omega}_t (\mathbf{r}_{t+1} - \mathbf{i}_t) + \Psi'_t (\boldsymbol{\Delta} \mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t) + \frac{1}{2} \Sigma_t^h$, and the Lagrangian is

$$\begin{aligned}
\mathcal{L}(\tilde{\Psi}) &= \frac{1}{2} (1 - \lambda) \text{Var}_t [\mathbf{1}' \boldsymbol{\omega}_t (\mathbf{r}_{t+1} - \mathbf{i}_t) + \Psi'_t (\boldsymbol{\Delta} \mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t)] \\
&\quad + \lambda \left[\mu_H - \text{E}_t (\mathbf{1}' \boldsymbol{\omega}_t (\mathbf{r}_{t+1} - \mathbf{i}_t) + \Psi'_t (\boldsymbol{\Delta} \mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t)) - \frac{1}{2} \Sigma_t^h \right].
\end{aligned}$$

Substituting for Σ_t^h using equation (IA.14), this expression is equivalent to

$$\begin{aligned}
\mathcal{L}(\tilde{\Psi}) &= \frac{1}{2} \text{Var}_t (\mathbf{1}' \boldsymbol{\omega}_t (\mathbf{r}_{t+1} - \mathbf{i}_t) + \Psi'_t (\boldsymbol{\Delta} \mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t)) \\
&\quad + \lambda [\mu_H - \text{E}_t (\mathbf{1}' \boldsymbol{\omega}_t (\mathbf{r}_{t+1} - \mathbf{i}_t) + \Psi'_t (\boldsymbol{\Delta} \mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t))] \\
&\quad - \frac{\lambda}{2} [\mathbf{1}' \boldsymbol{\omega}_t \text{diag} (\text{Var}_t (\mathbf{r}_{t+1} + \boldsymbol{\Delta} \mathbf{s}_{t+1})) - (\boldsymbol{\omega}_t \mathbf{1} - \Psi_t)' \text{diag} (\text{Var}_t (\boldsymbol{\Delta} \mathbf{s}_{t+1}))]
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}(\tilde{\Psi}) &= \frac{1}{2} \text{Var}_t(\Psi'_t(\Delta \mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t)) - \lambda \text{E}_t(\Psi'_t(\Delta \mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t)) \\
&\quad - \frac{\lambda}{2} \Psi'_t \text{diag}(\text{Var}_t(\Delta \mathbf{s}_{t+1})) \\
&\quad + \text{cov}_t(\mathbf{1}'\boldsymbol{\omega}_t(\mathbf{r}_{t+1} - \mathbf{i}_t), \Psi'_t(\Delta \mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t)) \\
&\quad + \frac{1}{2} \text{Var}_t(\mathbf{1}'\boldsymbol{\omega}_t(\mathbf{r}_{t+1} - \mathbf{i}_t)) - \lambda \text{E}_t(\mathbf{1}'\boldsymbol{\omega}_t(\mathbf{r}_{t+1} - \mathbf{i}_t)) \\
&\quad + \frac{\lambda}{2} \mathbf{1}'\boldsymbol{\omega}_t [\text{diag}(\text{Var}_t(\Delta \mathbf{s}_{t+1})) - \text{diag}(\text{Var}_t(\mathbf{r}_{t+1} + \Delta \mathbf{s}_{t+1}))] \\
&\quad + \lambda \mu_H
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}(\tilde{\Psi}) &= \frac{1}{2} \Psi'_t \text{Var}_t(\Delta \mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t) \Psi_t - \lambda \Psi'_t \begin{bmatrix} \text{E}_t(\Delta \mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t) \\ + \frac{1}{2} \text{diag}(\text{Var}_t(\Delta \mathbf{s}_{t+1})) \end{bmatrix} \\
&\quad + \text{cov}_t(\mathbf{1}'\boldsymbol{\omega}_t(\mathbf{r}_{t+1} - \mathbf{i}_t), (\Delta \mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t)) \Psi_t \\
&\quad + K(\lambda),
\end{aligned}$$

where

$$\begin{aligned}
K(\lambda) &= \lambda \mu_H + \frac{1}{2} \text{Var}_t(\mathbf{1}'\boldsymbol{\omega}_t(\mathbf{r}_{t+1} - \mathbf{i}_t)) - \lambda \text{E}_t(\mathbf{1}'\boldsymbol{\omega}_t(\mathbf{r}_{t+1} - \mathbf{i}_t)) \\
&\quad + \frac{\lambda}{2} \mathbf{1}'\boldsymbol{\omega}_t [\text{diag}(\text{Var}_t(\Delta \mathbf{s}_{t+1})) - \text{diag}(\text{Var}_t(\mathbf{r}_{t+1} + \Delta \mathbf{s}_{t+1}))].
\end{aligned}$$

Note that $K(\lambda)$ is independent of $\tilde{\Psi}_t$.

Now we need to solve only for $\tilde{\Psi}_t$, as Ψ_1 , the demand for domestic currency, is given once the other currency demands are determined. We rewrite the Lagrangian in terms of $\tilde{\Psi}_t$:

$$\begin{aligned}
\mathcal{L}(\tilde{\Psi}) &= \frac{1}{2} \tilde{\Psi}'_t \text{Var}_t(\tilde{\Delta \mathbf{s}}_{t+1} - \tilde{\mathbf{i}}_t^d + \tilde{\mathbf{i}}_t) \tilde{\Psi}_t - \lambda \tilde{\Psi}'_t \begin{bmatrix} \text{E}_t(\tilde{\Delta \mathbf{s}}_{t+1} - \tilde{\mathbf{i}}_t^d + \tilde{\mathbf{i}}_t) \\ + \frac{1}{2} \text{diag}(\text{Var}_t(\tilde{\Delta \mathbf{s}}_{t+1})) \end{bmatrix} \\
&\quad + \text{cov}_t(\mathbf{1}'\boldsymbol{\omega}_t(\mathbf{r}_{t+1} - \mathbf{i}_t), (\tilde{\Delta \mathbf{s}}_{t+1} - \tilde{\mathbf{i}}_t^d + \tilde{\mathbf{i}}_t)) \tilde{\Psi}_t \\
&\quad + K(\lambda).
\end{aligned}$$

The F.O.C. gives the following expression for the optimal $\tilde{\Psi}_t$:

$$0 = \text{cov}_t \left(\mathbf{1}' \boldsymbol{\omega}_t (\mathbf{r}_{t+1} - \mathbf{i}_t), \left(\tilde{\Delta} \mathbf{s}_{t+1} - \tilde{\mathbf{i}}_t^d + \tilde{\mathbf{i}}_t \right) \right) \\ + \text{Var}_t \left(\tilde{\Delta} \mathbf{s}_{t+1} - \tilde{\mathbf{i}}_t^d + \tilde{\mathbf{i}}_t \right) \tilde{\Psi}_t^* - \lambda \left[E_t \left(\tilde{\Delta} \mathbf{s}_{t+1} - \tilde{\mathbf{i}}_t^d + \tilde{\mathbf{i}}_t \right) + \frac{1}{2} \text{diag} \left(\text{Var}_t \left(\tilde{\Delta} \mathbf{s}_{t+1} \right) \right) \right].$$

Finally, the optimal vector of currency demands is

$$\tilde{\Psi}_t^* (\lambda) = \lambda \text{Var}_t^{-1} \left(\tilde{\Delta} \mathbf{s}_{t+1} - \tilde{\mathbf{i}}_t^d + \tilde{\mathbf{i}}_t \right) \left[E_t \left(\tilde{\Delta} \mathbf{s}_{t+1} - \tilde{\mathbf{i}}_t^d + \tilde{\mathbf{i}}_t \right) + \frac{1}{2} \text{diag} \left(\text{Var}_t \tilde{\Delta} \mathbf{s}_{t+1} \right) \right] \\ - \text{Var}_t^{-1} \left(\tilde{\Delta} \mathbf{s}_{t+1} - \tilde{\mathbf{i}}_t^d + \tilde{\mathbf{i}}_t \right) \left[\text{cov}_t \left(\mathbf{1}' \boldsymbol{\omega}_t (\mathbf{r}_{t+1} - \mathbf{i}_t), \left(\tilde{\Delta} \mathbf{s}_{t+1} - \tilde{\mathbf{i}}_t^d + \tilde{\mathbf{i}}_t \right) \right) \right].$$

D.2. Constrained Hedge Ratio

In the case where $\tilde{\Psi}_t = \psi_t \tilde{\mathbf{1}}$ (where $\tilde{\mathbf{1}}$ denotes an $n \times 1$ vector of ones), we let ψ_t^* be the optimal scalar constrained hedge ratio and we have

$$\begin{aligned} \mathcal{L}(\psi_t) &= \frac{1}{2} \psi_t^2 \tilde{\mathbf{1}}' \text{Var}_t \left(\tilde{\Delta \mathbf{s}}_{t+1} - \tilde{\mathbf{i}}_t^d + \tilde{\mathbf{i}}_t \right) \tilde{\mathbf{1}} - \lambda \psi_t \tilde{\mathbf{1}}' \left[\begin{array}{c} \text{E}_t \left(\tilde{\Delta \mathbf{s}}_{t+1} - \tilde{\mathbf{i}}_t^d + \tilde{\mathbf{i}}_t \right) \\ + \frac{1}{2} \text{diag} \left(\text{Var}_t \left(\tilde{\Delta \mathbf{s}}_{t+1} \right) \right) \end{array} \right] \\ &\quad + \psi_t \text{cov}_t \left(\mathbf{1}' \boldsymbol{\omega}_t (\mathbf{r}_{t+1} - \mathbf{i}_t), \left(\tilde{\Delta \mathbf{s}}_{t+1} - \tilde{\mathbf{i}}_t^d + \tilde{\mathbf{i}}_t \right) \right) \tilde{\mathbf{1}} \\ &\quad + K(\lambda) \end{aligned}$$

and

$$\begin{aligned} \psi_t^* &= \frac{\lambda \tilde{\mathbf{1}}' \left[\text{E}_t \left(\tilde{\Delta \mathbf{s}}_{t+1} - \tilde{\mathbf{i}}_t^d + \tilde{\mathbf{i}}_t \right) + \frac{1}{2} \text{diag} \left(\text{Var}_t \left(\tilde{\Delta \mathbf{s}}_{t+1} \right) \right) \right]}{\tilde{\mathbf{1}}' \text{Var}_t \left(\tilde{\Delta \mathbf{s}}_{t+1} - \tilde{\mathbf{i}}_t^d + \tilde{\mathbf{i}}_t \right) \tilde{\mathbf{1}}} \\ &\quad - \frac{\text{cov}_t \left(\mathbf{1}' \boldsymbol{\omega}_t (\mathbf{r}_{t+1} - \mathbf{i}_t), \left(\tilde{\Delta \mathbf{s}}_{t+1} - \tilde{\mathbf{i}}_t^d + \tilde{\mathbf{i}}_t \right) \right) \tilde{\mathbf{1}}}{\tilde{\mathbf{1}}' \text{Var}_t \left(\tilde{\Delta \mathbf{s}}_{t+1} - \tilde{\mathbf{i}}_t^d + \tilde{\mathbf{i}}_t \right) \tilde{\mathbf{1}}}. \end{aligned}$$

In this case, ψ_t^* can equivalently be written in terms of the full matrices:

$$\begin{aligned} \psi_t^* &= \frac{\lambda \mathbf{1}' \left[\text{E}_t \left(\Delta \mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t \right) + \frac{1}{2} \text{diag} \left(\text{Var}_t \left(\Delta \mathbf{s}_{t+1} \right) \right) \right]}{\mathbf{1}' \text{Var}_t \left(\Delta \mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t \right) \mathbf{1}} \\ &\quad - \frac{\mathbf{1}' \text{cov}_t \left(\boldsymbol{\omega}_t (\mathbf{r}_{t+1} - \mathbf{i}_t), \left(\Delta \mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t \right) \right) \mathbf{1}}{\mathbf{1}' \text{Var}_t \left(\Delta \mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t \right) \mathbf{1}}. \end{aligned}$$

This case corresponds to a domestic investor hedging the same ratio of his foreign stock holdings for all foreign currencies.

E. Invariance of Optimal Currency Demand With Respect to Base Country

In the system of n^2 bilateral exchange rates, there are really only n free parameters as all exchange rates can be backed out of the n bilateral rates for one base domestic

country. We use this fact to show that, for a portfolio of stocks from the $n + 1$ countries in our model, the optimal hedge ratios on stocks from country c , Ψ_c^{j*} , is the same for any base country j . Let us now use the subscript j to index the domestic country.

We assume for this derivation that weights on international stocks are the same for investors from all countries so that $\boldsymbol{\omega}_t^j = \boldsymbol{\omega}_t$. In terms of our empirical tests, this result will hence apply to the cases of an equally weighted or a value weighted world portfolio, in which weights do not vary with the base country. They do not hold for a home biased portfolio, in which weights vary with base country by definition.

Let us think of country 1 as our base country, and write the optimal vector of foreign currency demand assuming that $\lambda^j = 0$ for all values of j . We have

$$\begin{aligned}\tilde{\Psi}_{RM}^{1*} &= -\text{Var}_t \left(\tilde{\Delta} s_{t+1}^1 - \tilde{i}_t^{1,d} + \tilde{i}_t^1 \right)^{-1} \left[\text{cov}_t \left(\mathbf{1}' \boldsymbol{\omega}_t (\mathbf{r}_{t+1} - \mathbf{i}_t), \tilde{\Delta} s_{t+1}^1 - \tilde{i}_t^{1,d} + \tilde{i}_t^1 \right) \right] \\ &= -\text{Var}_t \left(\tilde{x}_{t+1}^1 \right)^{-1} \left[\text{cov}_t \left(\mathbf{y}_{t+1}^W, \tilde{x}_{t+1}^1 \right) \right],\end{aligned}$$

where $x_{t+1}^1 = \Delta s_{t+1}^1 - i_t^{1,d} + i_t^1$ and $\mathbf{y}_{t+1}^W = \mathbf{1}' \boldsymbol{\omega}_t (\mathbf{r}_{t+1} - \mathbf{i}_t)$.

Now, let us consider exchange rates from the perspective of country 2. By definition of the exchange rate between countries 1 and 2, it follows that $s_{t+1,1}^2 = -s_{t+1,2}^1$.

Also, by definition of the exchange rates, $S_{t+1,3}^2$ units of currency 2 can be exchanged into one unit of currency 3. And one unit of currency 3 is equivalent to $S_{t+1,3}^1$ units of currency 1, which is equivalent to $S_{t+1,3}^1/S_{t+1,2}^1$ units of currency 2. So, the absence of arbitrage implies the equality: $S_{t+1,3}^2 = S_{t+1,3}^1/S_{t+1,2}^1$. In logs, $s_{t+1,3}^2 = s_{t+1,3}^1 - s_{t+1,2}^1$. More generally, the following equality can be derived from the absence of arbitrage:

$$s_{t+1,c}^2 = s_{t+1,c}^1 - s_{t+1,2}^1 \quad c = 3 \dots n + 1.$$

In matrix notation, this amounts to a linear relationship between $\tilde{\Delta} \mathbf{s}_{t+1}^2$ and $\tilde{\Delta} \mathbf{s}_{t+1}^1$:

$$\tilde{\Delta} \mathbf{s}_{t+1}^2 = A_2 \cdot \tilde{\Delta} \mathbf{s}_{t+1}^1,$$

where

$$A_2 = \begin{pmatrix} -1 & 0 & \dots & \dots & 0 \\ -1 & 1 & 0 & \dots & \dots \\ -1 & 0 & 1 & 0 & \dots \\ -1 & 0 & 0 & \dots & 0 \\ -1 & 0 & \dots & 0 & 1 \end{pmatrix}.$$

Given our notation,

$$\tilde{\mathbf{i}}_t^{1,d} - \tilde{\mathbf{i}}_t^1 = (i_{t,2} - i_{t,1}, i_{t,3} - i_{t,1}, \dots, i_{t,n+1} - i_{t,1})'$$

and

$$\tilde{\mathbf{i}}_t^{2,d} - \tilde{\mathbf{i}}_t^2 = (i_{t,1} - i_{t,2}, i_{t,3} - i_{t,2}, \dots, i_{t,n+1} - i_{t,2})'.$$

It follows that $\tilde{\mathbf{i}}_t^{2,d} - \tilde{\mathbf{i}}_t^2 = A \left(\tilde{\mathbf{i}}_t^{1,d} - \tilde{\mathbf{i}}_t^1 \right)$.

Similarly, we have the following linear relationship between $\tilde{\mathbf{x}}_{t+1}^2$ and $\tilde{\mathbf{x}}_{t+1}^1$:

$$\tilde{\mathbf{x}}_{t+1}^2 = A \tilde{\mathbf{x}}_{t+1}^1. \quad (\text{IA.15})$$

Let us substitute equation (IA.15), the formula for $\tilde{\mathbf{x}}_{t+1}^2$, into the formula for the optimal hedge ratio given in the article. We use the properties of matrix second moments, that is, $\text{Var}(AX) = A \text{Var}(X) A'$, and $\text{cov}(AX, Y) = A \text{cov}(X, Y)$, and the property of inverse matrices, that is, $(AB)^{-1} = B^{-1}A^{-1}$. Also, we note that $A_2 = (A_2)^{-1}$ and $(A_2')^{-1} = A_2'$. Substitution yields

$$\begin{aligned} \tilde{\Psi}_{RM}^{2*} &= -\text{Var}_t(\tilde{\mathbf{x}}_{t+1}^2)^{-1} [\text{cov}_t(\mathbf{y}_{t+1}^W, \tilde{\mathbf{x}}_{t+1}^2)] \\ &= -(A_2')^{-1} \text{Var}_t(\tilde{\mathbf{x}}_{t+1}^1)^{-1} (A_2)^{-1} [A_2 \text{cov}_t(\tilde{\mathbf{x}}_{t+1}^1, \mathbf{y}_{t+1}^W)] \\ \tilde{\Psi}_{RM}^{2*}(\lambda^2) &= -(A_2')^{-1} \text{Var}_t(\tilde{\mathbf{x}}_{t+1}^1)^{-1} \text{cov}_t(\tilde{\mathbf{x}}_{t+1}^1, \mathbf{y}_{t+1}^W) \\ \tilde{\Psi}^{2*} &= A_2' \tilde{\Psi}^{1*}. \end{aligned}$$

We write out the vector $\tilde{\Psi}_{RM}^{2*}$:

$$\tilde{\Psi}_{RM}^{2*} = \left(-\sum_{c=2}^{n+1} \Psi_c^{1*}, \Psi_3^{1*}, \Psi_4^{1*}, \dots, \Psi_{n+1}^{1*} \right).$$

Given the property that $\sum_{c=1}^{n+1} \Psi_c^{j*} = 1$ for $j = 1..n+1$, $\Psi_1^{1*} = -\sum_{c=2}^{n+1} \Psi_c^{1*}$ so that $\widetilde{\Psi}_{RM}^{2*} = (\Psi_1^{1*}, \Psi_3^{1*}, \Psi_4^{1*}, \dots, \Psi_{n+1}^{1*})$. Applying this same property twice, $\Psi_2^{2*} = -\sum_{c \neq 2}^{n+1} \Psi_c^{2*} = -\sum_{c \neq 2}^{n+1} \Psi_c^{1*} = \Psi_2^{1*}$, so that $\Psi_{RM}^{2*} = (\Psi_1^{1*}, \Psi_2^{1*}, \Psi_3^{1*}, \Psi_4^{1*}, \dots, \Psi_{n+1}^{1*}) = \Psi_{RM}^{1*}$. Finally, the vector of optimal currency positions is the same for investors based in country 2 as that for country 1 investors.

Similar results hold for $j = 3..n+1$, where

$$A_3 = \begin{pmatrix} 1 & -1 & .. & .. & 0 \\ 0 & -1 & 0 & .. & .. \\ 0 & -1 & 1 & 0 & .. \\ 0 & .. & 0 & .. & 0 \\ 0 & -1 & .. & 0 & 1 \end{pmatrix},$$

$$A_4 = \begin{pmatrix} 1 & 0 & -1 & .. & 0 \\ 0 & 1 & .. & .. & .. \\ 0 & 0 & -1 & 0 & .. \\ 0 & .. & .. & 1 & 0 \\ 0 & -1 & -1 & 0 & 1 \end{pmatrix},$$

etc.

This analysis justifies dropping the base-country subscript j and interpreting the $(n+1 \times 1)$ vector $\Psi^* = \left(-\sum_{c=2}^{n+1} \Psi_c^{1*}, \Psi_2^{1*}, \Psi_3^{1*}, \dots, \Psi_{n+1}^{1*} \right)'$ as a common vector of foreign currency demands that is independent of the country of origin.

A situation in which investors from all countries are hedged perfectly corresponds to $\Psi^* = (0, 0, \dots, 0)'$.

A situation in which investors from country 1 are not hedged at all corresponds to $\Psi^* = (-1, \omega_2^1, \omega_3^1, \dots, \omega_{n+1}^1)'$. That is, investors from country i undo the hedge of the fully hedged portfolio by taking long positions in each foreign currency proportional to the weight of each foreign country in their stock portfolio. (The perfectly hedged portfolio obtains by shorting each foreign currency by that same amount.) They need to borrow one unit of domestic currency to finance that.

Finally, note that this proof relies on the fact that all relevant exchange rates for an investor in a given base country are linear combinations of the relevant exchange rates for each other base country. In other words, the assumption is that all investors optimize over the same set of currencies.

F. Computation of Sharpe Ratios

Table A14 reports in-sample Sharpe ratios generated by the set of currency hedging strategies for global portfolios of stocks and bonds considered in the paper. The denominator of the Sharpe ratio is given by the standard deviations of log portfolio returns reported in Table VII of the main text.

The numerator of the Sharpe ratio is given by the log of the mean gross return on each of the portfolios. We compute a time series of gross returns for each strategy using equation (IA.5), where Ψ_t is replaced by the vector of fixed or time-varying currency demands that corresponds to each currency hedging strategy—for example, Ψ_t is a vector of zeroes for the “Full Hedge” strategy. Next, we average the time series of gross returns, and take the natural log of the arithmetic mean.

Thus, our Sharpe ratio is computed as

$$\frac{\log(\mathbb{E}[R_{p,t+1}^h])}{\sqrt{\text{Var}(r_{p,t+1}^h)}},$$

which under high frequency returns or under lognormality is equivalent to

$$\frac{\mathbb{E}[r_{p,t+1}^h] + \frac{1}{2} \text{Var}(r_{p,t+1}^h)}{\sqrt{\text{Var}(r_{p,t+1}^h)}}.$$

Table IA.I
Currency Return Correlations

This table presents cross-country correlations of foreign currency log excess returns $sc,t+i,t-d,t$, where d indexes the base country. Correlations are presented separately for investors from each base country. They are computed using monthly returns.

	Euroland	Australia	Canada	Japan	Switzerland	U.K.	U.S.
<i>Base country: Euroland</i>							
Euroland	.						
Australia	.	1.00					
Canada	.	0.70	1.00				
Japan	.	0.35	0.35	1.00			
Switzerland	.	-0.09	-0.11	0.20	1.00		
U.K.	.	0.31	0.34	0.24	-0.02	1.00	
U.S.	.	0.63	0.87	0.40	-0.07	0.38	1.00
<i>Base country: Australia</i>							
Euroland	1.00						
Australia	.	.					
Canada	0.53	.	1.00				
Japan	0.67	.	0.46	1.00			
Switzerland	0.92	.	0.47	0.69	1.00		
U.K.	0.78	.	0.51	0.59	0.72	1.00	
U.S.	0.59	.	0.85	0.55	0.54	0.58	1.00
<i>Base country: Canada</i>							
Euroland	1.00						
Australia	0.23	1.00					
Canada	.	.	.				
Japan	0.59	0.25	.	1.00			
Switzerland	0.91	0.20	.	0.62	1.00		
U.K.	0.71	0.24	.	0.50	0.65	1.00	
U.S.	0.32	0.11	.	0.35	0.31	0.34	1.00
<i>Base Country: Japan</i>							
Euroland	1.00						
Australia	0.46	1.00					
Canada	0.55	0.74	1.00				
Japan			
Switzerland	0.87	0.33	0.39	.	1.00		
U.K.	0.72	0.49	0.56	.	0.60	1.00	
U.S.	0.55	0.67	0.90	.	0.41	0.58	1.00
<i>Base Country: Switzerland</i>							
Euroland	1.00						
Australia	0.47	1.00					
Canada	0.52	0.77	1.00				
Japan	0.31	0.46	0.47	1.00			
Switzerland		
U.K.	0.56	0.49	0.53	0.37	.	1.00	
U.S.	0.51	0.71	0.91	0.51	.	0.55	1.00
<i>Base Country: U.K.</i>							
Euroland	1.00						
Australia	0.35	1.00					
Canada	0.42	0.71	1.00				
Japan	0.50	0.42	0.44	1.00			
Switzerland	0.84	0.25	0.30	0.52	1.00		
U.K.	
U.S.	0.42	0.64	0.88	0.48	0.32	.	1.00
<i>Base Country: U.S.</i>							
Euroland	1.00						
Australia	0.26	1.00					
Canada	0.18	0.43	1.00				
Japan	0.55	0.25	0.10	1.00			
Switzerland	0.90	0.21	0.12	0.58	1.00		
U.K.	0.69	0.25	0.14	0.44	0.61	1.00	
U.S.

Table IA.II
Optimal Currency Exposure for an Equally Weighted Global Equity Portfolio: Single
Currency Case

This table considers an investor holding a portfolio composed of stocks from all countries, with equal weights, who chooses a position in one foreign currency at a time to minimize the variance of his portfolio. Rows indicate the base country of the investor, columns the currencies used to manage risk. Cells of Panel A are obtained by regressing the excess return to the global equity portfolio on the excess return of the column country currency to an investor based in the row country. All regressions include an intercept. Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio. We run monthly regressions on overlapping quarterly returns. Standard errors are corrected for autocorrelation due to overlapping intervals using the Newey-West procedure.

Base country	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	U.K.	U.S.
Euroland		-0.37*** (0.09)	-0.45*** (0.10)	-0.25*** (0.07)	0.28* (0.15)	-0.30*** (0.09)	-0.33*** (0.11)
Australia	0.37*** (0.09)		0.02 (0.08)	0.14* (0.07)	0.33*** (0.07)	0.21** (0.09)	0.16** (0.08)
Canada	0.45*** (0.10)	-0.02 (0.08)		0.15* (0.09)	0.38*** (0.09)	0.25** (0.11)	0.55*** (0.16)
Japan	0.25*** (0.07)	-0.14* (0.07)	-0.15* (0.09)		0.32*** (0.08)	0.05 (0.06)	-0.06 (0.09)
Switzerland	-0.28* (0.15)	-0.33*** (0.07)	-0.38*** (0.09)	-0.32*** (0.08)		-0.29*** (0.07)	-0.30*** (0.09)
U.K.	0.30*** (0.09)	-0.21** (0.09)	-0.25** (0.11)	-0.05 (0.06)	0.29*** (0.07)		-0.13 (0.11)
U.S.	0.33*** (0.11)	-0.16** (0.08)	-0.55*** (0.16)	0.06 (0.09)	0.30*** (0.09)	0.13 (0.11)	

Table IA.III
Optimal Currency Exposure for an Equally Weighted Global Equity Portfolio: Multiple Currency Case

This table considers an investor holding a portfolio composed of stocks from all countries, with equal weights, who chooses a vector of positions in all available foreign currencies to minimize the variance of his portfolio. In this case, the optimal currency positions do not depend on the investor's base country. Rows indicate the time-horizon T of the investor, columns the currencies used to manage risk. Rows are obtained by regressing the excess return to the global equity portfolio on the vector of all foreign currency excess returns. All regressions include an intercept. All returns considered are at the row time-horizon. Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio. We run monthly regressions on overlapping T -month returns, T varying from one month to 12 months. Standard errors are corrected for autocorrelation due to overlapping intervals using the Newey-West procedure.

Time horizon	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	U.K.	U.S.
Panel A : 7 country optimization							
1 month	0.17 (0.15)	-0.16 (0.11)	-0.61* (0.14)	-0.11 (0.07)	0.23 (0.12)	-0.11 (0.08)	0.60* (0.15)
2 months	0.29 (0.15)	-0.13 (0.09)	-0.63* (0.15)	-0.19* (0.07)	0.26 (0.13)	-0.11 (0.09)	0.51* (0.15)
3 months	0.32 (0.17)	-0.11 (0.09)	-0.61* (0.16)	-0.17 (0.09)	0.27 (0.15)	-0.10 (0.11)	0.40* (0.18)
6 months	0.20 (0.26)	-0.05 (0.14)	-0.38 (0.25)	-0.25* (0.12)	0.35 (0.20)	-0.06 (0.16)	0.19 (0.28)
12 months	-0.20 (0.40)	0.21 (0.20)	-0.22 (0.36)	-0.41* (0.17)	0.67* (0.30)	-0.20 (0.21)	0.15 (0.37)
Panel B : 5 country optimization							
1 month	0.37* (0.11)	-0.29* (0.11)		-0.08 (0.07)		-0.10 (0.08)	0.11 (0.08)
2 months	0.50* (0.11)	-0.27* (0.09)		-0.15* (0.07)		-0.09 (0.09)	0.01 (0.11)
3 months	0.56* (0.11)	-0.27* (0.10)		-0.14 (0.08)		-0.09 (0.11)	-0.06 (0.14)
6 months	0.53* (0.14)	-0.21 (0.13)		-0.21* (0.10)		-0.02 (0.15)	-0.09 (0.18)
12 months	0.44* (0.19)	0.05 (0.17)		-0.34* (0.15)		-0.16 (0.19)	0.01 (0.22)

Table IA.IV
Subperiod Analysis
Equally Weighted Global Equity Portfolio: Multiple Currency Case

This table replicates Table V for two subperiods, respectively extending from 1975:7 to 1989:12 and from 1990:1 to 2005:12. Time horizons include one, three, and 12 months only.

Time horizon	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	U.K.	U.S.
Panel A : 7 country optimization							
<i>Subperiod I : 1975-1989</i>							
1 month	0.15 (0.20)	-0.11 (0.16)	-0.73*** (0.23)	-0.06 (0.12)	0.08 (0.13)	-0.06 (0.11)	0.73*** (0.24)
3 months	0.14 (0.21)	-0.05 (0.12)	-0.63** (0.26)	-0.20 (0.14)	0.22 (0.18)	-0.09 (0.15)	0.62* (0.35)
12 months	-0.62 (0.45)	0.23 (0.22)	-0.15 (0.61)	-0.31 (0.23)	0.57* (0.33)	-0.04 (0.23)	0.33 (0.61)
<i>Subperiod II : 1990-2005</i>							
1 month	0.10 (0.27)	-0.25** (0.12)	-0.49*** (0.18)	-0.15 (0.09)	0.51** (0.23)	-0.20 (0.13)	0.48*** (0.18)
3 months	0.44 (0.28)	-0.17 (0.14)	-0.65*** (0.21)	-0.08 (0.10)	0.37 (0.23)	-0.12 (0.14)	0.22 (0.19)
12 months	0.56 (0.52)	-0.17 (0.29)	-0.31 (0.37)	-0.23 (0.23)	0.47 (0.49)	-0.22 (0.25)	-0.11 (0.37)
Panel B : 5 country optimization							
<i>Subperiod I : 1975-1989</i>							
1 month	0.21 (0.19)	-0.22 (0.16)		-0.06 (0.12)		-0.06 (0.11)	0.13 (0.10)
3 months	0.35** (0.17)	-0.15 (0.11)		-0.15 (0.11)		-0.10 (0.15)	0.05 (0.20)
12 months	-0.10 (0.22)	0.14 (0.20)		-0.20 (0.15)		-0.02 (0.21)	0.18 (0.24)
<i>Subperiod II : 1990-2005</i>							
1 month	0.56*** (0.12)	-0.40*** (0.12)		-0.08 (0.08)		-0.20* (0.11)	0.12 (0.13)
3 months	0.79*** (0.13)	-0.47*** (0.12)		-0.06 (0.11)		-0.11 (0.13)	-0.15 (0.17)
12 months	1.02*** (0.21)	-0.40* (0.23)		-0.22 (0.19)		-0.20 (0.25)	-0.20 (0.32)

Table IA.V
**Optimal Currency Exposure for a Value Weighted Global Equity Portfolio: Multiple
Currency Case**

This table considers an investor holding a portfolio composed of stocks from all countries, with constant value weights (reflecting the end-of-period 2005:12 weights as reported in Table VII), who chooses a vector of positions in all available foreign currencies to minimize the variance of his portfolio. In this case, the optimal currency positions do not depend on the investor's base country. Rows indicate the time-horizon T of the investor, columns the currencies used to manage risk. Rows are obtained by regressing the excess return to the global equity portfolio on the vector of all foreign currency excess returns. All regressions include an intercept. All returns considered are at the row time-horizon. Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio. We run monthly regressions on overlapping T -month returns, T varying from one month to 12 months. Standard errors are corrected for autocorrelation due to overlapping intervals using the Newey-West procedure.

Time horizon	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	U.K.	U.S.
Panel A : 7 country optimization							
1 month	0.13 (0.17)	-0.09 (0.10)	-0.70*** (0.15)	-0.13* (0.08)	0.22* (0.13)	-0.09 (0.08)	0.66*** (0.15)
2 months	0.22 (0.16)	-0.07 (0.09)	-0.73*** (0.15)	-0.22*** (0.08)	0.26* (0.13)	-0.06 (0.09)	0.60*** (0.16)
3 months	0.22 (0.17)	-0.04 (0.09)	-0.76*** (0.17)	-0.23** (0.10)	0.30** (0.15)	-0.03 (0.11)	0.55*** (0.19)
6 months	0.11 (0.24)	0.01 (0.14)	-0.60*** (0.22)	-0.32** (0.12)	0.39** (0.19)	0.03 (0.15)	0.39 (0.26)
12 months	-0.29 (0.39)	0.25 (0.22)	-0.49 (0.36)	-0.46** (0.18)	0.72** (0.30)	-0.09 (0.21)	0.36 (0.37)
Panel B : 5 country optimization							
1 month	0.29*** (0.11)	-0.25*** (0.09)		-0.08 (0.07)		-0.08 (0.08)	0.12 (0.09)
2 months	0.42*** (0.11)	-0.25** (0.10)		-0.16** (0.08)		-0.05 (0.09)	0.04 (0.11)
3 months	0.46*** (0.11)	-0.24** (0.10)		-0.17* (0.09)		-0.03 (0.11)	-0.03 (0.14)
6 months	0.45*** (0.13)	-0.20 (0.13)		-0.24** (0.11)		0.06 (0.15)	-0.07 (0.18)
12 months	0.39* (0.20)	0.01 (0.20)		-0.32** (0.16)		-0.08 (0.21)	0.00 (0.23)

Table IA.VI
Optimal Currency Exposure for a Home-biased Global Equity Portfolio: Single and Multiple Currency Cases

This table considers an investor holding a home-biased portfolio of global equity. The portfolio is constructed by assigning a 75% weight to the home country of the investor, and distributing the remaining 25% over the four other countries according to their value weights. The investor chooses a foreign currency position to minimize the variance of his portfolio. Panel A allows the investor to use only one foreign currency. Panel B allows her to choose a vector of positions in all available foreign currencies. Rows indicate the base country of the investor, columns the currencies used to manage risk. Cells of Panel A are obtained by regressing the excess return to the row country home-biased global equity portfolio on the excess return to the column country currency. Rows of panel B (excluding diagonal terms) are obtained by regressing the excess return to the row country portfolio on the vector of all foreign currency excess returns. All regressions include an intercept. Diagonal terms in panel B are obtained by computing the opposite of the sum of other terms and the corresponding standard deviation. Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio. We run monthly regressions on overlapping quarterly returns. Standard errors are corrected for autocorrelation due to overlapping intervals using the Newey-West procedure.

Base country	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	U.K.	U.S.
Panel A : Single currency							
Euroland		-0.40*** (0.10)	-0.52*** (0.12)	-0.31*** (0.08)	0.34* (0.18)	-0.32*** (0.11)	-0.45*** (0.13)
Australia	0.37*** (0.10)		0.09 (0.11)	0.16* (0.09)	0.31*** (0.09)	0.17 (0.12)	0.25** (0.10)
Canada	0.42*** (0.10)	-0.01 (0.10)		0.12 (0.09)	0.35*** (0.09)	0.17 (0.11)	0.88*** (0.19)
Japan	0.31*** (0.10)	-0.09 (0.09)	-0.08 (0.10)		0.35*** (0.10)	0.15* (0.08)	0.02 (0.11)
Switzerland	-0.45*** (0.15)	-0.35*** (0.08)	-0.42*** (0.09)	-0.29*** (0.09)		-0.29*** (0.08)	-0.38*** (0.10)
U.K.	0.25** (0.11)	-0.24** (0.10)	-0.30*** (0.12)	-0.10 (0.07)	0.25*** (0.09)		-0.21* (0.12)
U.S.	0.23** (0.11)	-0.14* (0.08)	-0.71*** (0.16)	-0.01 (0.09)	0.22** (0.09)	0.11 (0.11)	
Panel B : Multiple currencies at once							
Euroland	0.36 (0.23)	-0.08 (0.11)	-0.50** (0.21)	-0.20* (0.11)	0.33* (0.18)	-0.08 (0.13)	0.17 (0.22)
Australia	0.47** (0.20)	-0.16 (0.12)	-0.68*** (0.17)	-0.14 (0.11)	0.15 (0.19)	-0.24* (0.15)	0.60*** (0.22)
Canada	0.30 (0.20)	-0.05 (0.10)	-0.94*** (0.21)	-0.22** (0.10)	0.31 (0.20)	-0.23* (0.13)	0.83*** (0.21)
Japan	0.34* (0.17)	-0.14 (0.13)	-0.63*** (0.21)	-0.25** (0.12)	0.20 (0.16)	0.01 (0.12)	0.48** (0.21)
Switzerland	0.14 (0.21)	-0.12 (0.09)	-0.35* (0.19)	-0.07 (0.11)	0.37** (0.17)	-0.02 (0.13)	0.04 (0.21)
U.K.	0.30 (0.21)	-0.11 (0.10)	-0.56*** (0.19)	-0.20** (0.09)	0.28 (0.18)	-0.02 (0.13)	0.30 (0.20)
U.S.	0.15 (0.18)	0.00 (0.09)	-0.83*** (0.17)	-0.22** (0.09)	0.30* (0.15)	-0.03 (0.11)	0.62*** (0.19)

Table IA.VII
Optimal Currency Exposure for Single-country Bond Portfolios: Single and Multiple
Currency Cases

This table considers an investor holding a portfolio composed of long-term bonds from his own country, who chooses a foreign currency position to minimize the variance of his portfolio. Panel A allows the investor to use only one foreign currency. Panel B allows her to choose a vector of positions in all available foreign currencies. Rows indicate the bond being held (as well as the base country), columns the currencies used to manage risk. Cells of Panel A are obtained by regressing the hedged excess return to the row country bond on the excess return to the column country currency. Rows of Panel B (excluding diagonal terms) are obtained by regressing the excess return to the row country stock bond on the vector of all foreign currency excess returns. All regressions include an intercept. Diagonal terms in Panel B are obtained by computing the opposite of the sum of other terms in the same row and the corresponding standard deviation. Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio. We run monthly regressions on overlapping quarterly returns. Standard errors are corrected for autocorrelation due to overlapping intervals using the Newey-West procedure.

Bond market	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	U.K.	U.S.
Panel A : Single currency							
Euroland		0.04 (0.03)	0.05 (0.03)	-0.03 (0.04)	-0.02 (0.06)	0.09** (0.04)	0.06* (0.04)
Australia	-0.02 (0.05)		0.04 (0.07)	0.00 (0.05)	-0.01 (0.05)	0.02 (0.04)	0.06 (0.05)
Canada	-0.07 (0.05)	0.12** (0.05)		-0.07 (0.05)	-0.08* (0.04)	-0.07 (0.06)	0.24*** (0.08)
Japan	0.05 (0.05)	0.09** (0.04)	0.14*** (0.05)		0.01 (0.05)	0.10** (0.05)	0.16*** (0.05)
Switzerland	0.08 (0.07)	0.02 (0.02)	0.05 (0.03)	-0.02 (0.04)		0.09** (0.04)	0.05 (0.03)
U.K.	0.22*** (0.05)	0.07** (0.04)	0.11** (0.05)	0.02 (0.04)	0.13*** (0.05)		0.12*** (0.04)
U.S.	-0.21*** (0.05)	0.03 (0.05)	-0.21** (0.09)	-0.15*** (0.05)	-0.18*** (0.04)	-0.09* (0.05)	
Panel B : Multiple currencies							
Euroland	-0.10 (0.08)	0.01 (0.03)	-0.01 (0.07)	-0.07* (0.04)	0.03 (0.07)	0.08* (0.05)	0.07 (0.06)
Australia	-0.13 (0.15)	-0.02 (0.08)	-0.07 (0.13)	0.00 (0.06)	0.03 (0.12)	0.06 (0.07)	0.14 (0.10)
Canada	0.03 (0.12)	0.18*** (0.05)	-0.35*** (0.11)	-0.08 (0.06)	-0.08 (0.11)	-0.07 (0.08)	0.36*** (0.08)
Japan	-0.05 (0.10)	-0.02 (0.05)	0.00 (0.10)	-0.12* (0.06)	-0.07 (0.08)	0.07 (0.05)	0.18* (0.09)
Switzerland	-0.03 (0.08)	-0.03 (0.04)	0.05 (0.08)	-0.06 (0.05)	-0.04 (0.08)	0.10** (0.05)	0.01 (0.07)
U.K.	0.28** (0.13)	0.01 (0.06)	-0.05 (0.14)	-0.10 (0.06)	-0.04 (0.11)	-0.23*** (0.06)	0.13 (0.10)
U.S.	-0.22* (0.11)	0.19*** (0.06)	-0.30** (0.12)	-0.10 (0.06)	-0.02 (0.09)	0.09 (0.07)	0.36*** (0.09)

Table IA.VIII
Optimal Currency Exposure for an Equally Weighted Global Bond Portfolio: Multiple Currency Case

This table considers an investor holding a portfolio composed of bonds from all countries, with equal weights, who chooses a vector of positions in all available foreign currencies to minimize the variance of his portfolio. In this case, the optimal currency positions do not depend on the investor's base country. Rows indicate the time-horizon T of the investor, columns the currencies used to manage risk. Rows are obtained by regressing the excess return to the global bond portfolio on the vector of all foreign currency excess returns. All regressions include an intercept. All returns considered are at the row time-horizon. Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio. We run monthly regressions on overlapping T -month returns, T varying from one month to 12 months. Standard errors are corrected for autocorrelation due to overlapping intervals using the Newey-West procedure.

Time horizon	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	U.K.	U.S.
Panel A : 7 country optimization							
1 month	0.02 (0.05)	0.00 (0.02)	-0.12* (0.05)	-0.06* (0.03)	-0.04 (0.04)	-0.01 (0.03)	0.22* (0.05)
2 months	-0.01 (0.07)	0.03 (0.03)	-0.14* (0.06)	-0.08* (0.03)	-0.03 (0.05)	0.00 (0.04)	0.23* (0.05)
3 months	-0.03 (0.07)	0.04 (0.04)	-0.10 (0.08)	-0.07 (0.04)	-0.03 (0.07)	0.01 (0.05)	0.18* (0.06)
6 months	-0.08 (0.11)	0.13* (0.05)	-0.05 (0.10)	-0.10 (0.06)	0.00 (0.10)	0.06 (0.07)	0.05 (0.08)
12 months	-0.26 (0.17)	0.17 (0.09)	0.03 (0.16)	-0.11 (0.08)	0.11 (0.13)	0.14 (0.11)	-0.08 (0.11)
Panel B : 5 country optimization							
1 month	-0.02 (0.03)	-0.03 (0.02)		-0.07* (0.03)		-0.01 (0.03)	0.13* (0.03)
2 months	-0.04 (0.04)	-0.01 (0.03)		-0.09* (0.03)		-0.01 (0.03)	0.15* (0.04)
3 months	-0.06 (0.05)	0.01 (0.03)		-0.08 (0.04)		0.01 (0.05)	0.11* (0.04)
6 months	-0.08 (0.08)	0.11* (0.04)		-0.10 (0.06)		0.06 (0.07)	0.01 (0.06)
12 months	-0.14 (0.12)	0.18* (0.06)		-0.10 (0.07)		0.12 (0.11)	-0.06 (0.07)

Table IA.IX Subperiod Analysis
Optimal Currency Exposure for an Equally Weighted Global Bond Portfolio: Multiple
Currency Case

This table considers an investor holding a portfolio composed of bonds from all countries, with equal weights, who chooses a vector of positions in all available foreign currencies to minimize the variance of his portfolio. In this case, the optimal currency positions do not depend on the investor's base country. Rows indicate the time-horizon T of the investor, columns the currencies used to manage risk. Rows are obtained by regressing the excess return to the global equity portfolio on the vector of all foreign currency excess returns. All regressions include an intercept. All returns considered are at the row time-horizon. Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio. We run monthly regressions on overlapping T -month returns, T varying from one month to 12 months. Standard errors are corrected for autocorrelation due to overlapping intervals using the Newey-West procedure.

Time horizon	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	U.K.	U.S.
Panel A : 7 country optimization							
<i>Subperiod 1: 1975-1989</i>							
1 month	0.08 (0.07)	0.01 (0.04)	-0.24** (0.10)	-0.11*** (0.04)	-0.06 (0.05)	0.00 (0.03)	0.33*** (0.09)
3 months	-0.02 (0.10)	0.04 (0.05)	-0.30** (0.14)	-0.16*** (0.06)	-0.01 (0.08)	0.02 (0.06)	0.43*** (0.12)
12 months	-0.29* (0.16)	0.24** (0.11)	-0.14 (0.19)	-0.25*** (0.08)	0.12 (0.10)	0.20** (0.09)	0.13 (0.14)
<i>Subperiod 2: 1990-2005</i>							
1 month	-0.07 (0.09)	0.00 (0.03)	-0.05 (0.06)	-0.04 (0.03)	0.04 (0.07)	0.00 (0.04)	0.12** (0.06)
3 months	-0.17 (0.12)	0.13** (0.06)	-0.08 (0.11)	-0.01 (0.05)	0.06 (0.11)	0.04 (0.06)	0.04 (0.06)
12 months	-0.42** (0.21)	0.24** (0.12)	-0.14 (0.22)	0.04 (0.09)	0.25* (0.15)	0.12 (0.11)	-0.10 (0.10)
Panel B : 5 country optimization							
<i>Subperiod 1: 1975-1989</i>							
1 month	0.02 (0.06)	-0.04 (0.03)		-0.11** (0.04)		-0.01 (0.03)	0.15*** (0.04)
3 months	-0.03 (0.08)	-0.04 (0.04)		-0.14** (0.07)		0.01 (0.05)	0.20*** (0.06)
12 months	-0.18 (0.12)	0.16* (0.09)		-0.21*** (0.07)		0.20** (0.09)	0.03 (0.11)
<i>Subperiod 2: 1990-2005</i>							
1 month	-0.05 (0.04)	-0.01 (0.03)		-0.04 (0.03)		0.00 (0.04)	0.10** (0.04)
3 months	-0.13** (0.06)	0.12*** (0.04)		0.00 (0.05)		0.03 (0.06)	-0.02 (0.05)
12 months	-0.16 (0.13)	0.20*** (0.05)		0.04 (0.08)		0.08 (0.11)	-0.16* (0.10)

Table IA.X - Subperiod I

Optimal Conditional Currency Exposure for an Equally Weighted Global Portfolio:
Single and Multiple Currency Case

This table reports optimal currency exposure conditional on interest rate. For each base country-currency pair, we now let the optimal currency position vary with the log interest rate differential (interest rate of the foreign country minus that of the base currency). The "Single Currency" columns consider the case of an investor using one currency at a time to manage risk, but still constrain the slopes to be the same across foreign currencies. Resulting slope coefficients from a SUR estimation are reported for each base currency. The "Multiple Currency" columns consider the case of an investor using all foreign currencies simultaneously to manage risk, but still constrain the slopes to be the same across foreign currencies. Resulting slope coefficients are reported for each base currency.

Base Currency	Equity				Bonds			
	Single Currency		Multiple Currencies		Single Currency		Multiple Currencies	
	Slope	<i>p</i> -value	Slope	<i>p</i> -value	Slope	<i>p</i> -value	Slope	<i>p</i> -value
Euroland	-0.10 (0.94)	1.00	-7.57 (7.55)	0.11	0.10 (0.34)	0.99	1.92 (3.62)	0.00
Australia	-0.08 (0.51)	1.00	4.19 (2.58)	0.32	0.01 (0.28)	1.00	2.75*** (0.84)	0.61
Canada	-0.02 (0.49)	1.00	5.72 (3.83)	0.83	0.04 (0.36)	1.00	3.22 (2.92)	0.54
Japan	0.00 (0.07)	1.00	2.20 (4.81)	0.07	0.05 (0.44)	1.00	-0.07 (2.07)	0.03
Switz.	0.24 (0.87)	1.00	-8.22 (5.53)	0.04	0.11 (0.35)	1.00	1.74 (2.92)	0.03
U.K.	-0.05 (0.38)	1.00	2.77 (3.75)	0.11	0.12 (0.32)	1.00	-1.21 (1.90)	0.06
U.S.	0.02 (0.34)	1.00	-1.88 (4.48)	0.21	0.07 (0.57)	1.00	-0.69 (1.99)	0.09

Table IA.X - Subperiod II
Optimal Conditional Currency Exposure for an Equally Weighted Global Portfolio:
Single and Multiple Currency Case

This table reports optimal currency exposure conditional on interest rate. For each base country-currency pair, we now let the optimal currency position vary with the log interest rate differential (interest rate of the foreign country minus that of the base currency). The "Single Currency" columns consider the case of an investor using one currency at a time to manage risk, but still constrain the slopes to be the same across foreign currencies. Resulting slope coefficients from a SUR estimation are reported for each base currency. The "Multiple Currency" columns consider the case of an investor using all foreign currencies simultaneously to manage risk, but still constrain the slopes to be the same across foreign currencies. Resulting slope coefficients are reported for each base currency.

Base Currency	Equity				Bonds			
	Single Currency		Multiple Currencies		Single Currency		Multiple Currencies	
	Slope	<i>p</i> -value	Slope	<i>p</i> -value	Slope	<i>p</i> -value	Slope	<i>p</i> -value
Euroland	-1.24 (2.90)	0.99	8.48** (3.62)	0.23	0.04 (0.39)	1.00	-3.32* (1.95)	0.58
Australia	-0.71 (1.79)	1.00	8.96** (3.81)	0.91	0.00 (0.30)	1.00	0.42 (1.50)	0.41
Canada	-1.37 (3.19)	0.95	15.96** (6.44)	0.42	0.04 (0.21)	1.00	-0.79 (2.79)	0.14
Japan	0.10 (1.31)	1.00	3.59 (4.53)	0.52	-0.01 (0.15)	1.00	-0.03 (2.48)	0.89
Switz.	-1.49 (2.56)	0.97	8.40*** (2.70)	0.11	0.00 (0.24)	1.00	-0.37 (1.08)	0.41
U.K.	-0.71 (2.34)	0.99	8.14 (7.16)	0.03	0.06 (0.20)	1.00	-0.89 (3.27)	0.16
U.S.	-0.77 (1.78)	0.98	1.90 (4.46)	0.19	0.03 (0.20)	1.00	-1.84 (1.74)	0.42

Table IA.XI

Optimal Synthetic Carry Trade Currency Exposure for Equally Weighted Global Equity and Bond Portfolios

The first eight columns of this table consider an investor holding a global, equally weighted stock (Panel A) or bond portfolio (Panel B) who chooses a vector of positions in available currencies to minimize the variance of his portfolio. Available currencies include all foreign currencies as well as a synthetic currency. At each point in time, the synthetic currency return is the average of the return of holding the currencies of the three highest-interest-rate countries and financing the position using the currencies of the three lowest-interest-rate countries. The time t return is based on currencies chosen using time $t-1$ interest rates. The last column considers the same investor now choosing an optimal position in only one currency, the synthetic currency, to minimize the variance of his portfolio.

	Multiple Currencies							Single currency	
	Euroland	Australia	Canada	Japan	Switz.	U.K.	U.S.	Synthetic	Synthetic
Panel A: Stocks									
Full period	0.33**	-0.17*	-0.68***	-0.08	0.34**	-0.18	0.27*	0.27*	-0.23**
	(0.16)	(0.09)	(0.17)	(0.10)	(0.15)	(0.13)	(0.14)	(0.14)	(0.12)
Subperiod	0.22	-0.25	-0.91***	0.00	0.48**	-0.23	0.69**	0.73*	-0.13
	(0.21)	(0.18)	(0.22)	(0.20)	(0.22)	(0.14)	(0.33)	(0.38)	(0.14)
Subperiod	0.36	-0.14	-0.57***	-0.17*	0.38*	0.02	0.12	-0.26	-0.37*
	(0.30)	(0.14)	(0.21)	(0.10)	(0.23)	(0.20)	(0.21)	(0.17)	(0.20)
Panel B: Bonds									
Full period	-0.02	0.02	-0.14	-0.04	0.00	-0.02	0.20***	0.11	0.13***
	(0.07)	(0.04)	(0.09)	(0.05)	(0.07)	(0.06)	(0.07)	(0.08)	(0.05)
Subperiod	0.00	-0.02	-0.39***	-0.10	0.08	-0.03	0.45***	0.24*	0.19***
	(0.09)	(0.07)	(0.15)	(0.07)	(0.09)	(0.07)	(0.12)	(0.14)	(0.07)
Subperiod	-0.16	0.12**	-0.10	0.02	0.07	0.00	0.06	0.07	0.07
	(0.12)	(0.06)	(0.13)	(0.06)	(0.11)	(0.08)	(0.08)	(0.09)	(0.06)

Table IA.XII
Standard Deviations of Hedged Global Equity and Bond Portfolios

This table reports the standard deviation of portfolios featuring different uses of currency for risk management. We present results for equally weighted global portfolios, for equity and bonds as respectively described in Table IV and Table VI. Within each panel, rows represent base countries and columns represent the risk management strategy. "No hedge" refers to the simple equity portfolio. "Half hedge" refers to a portfolio in which half of the implicit currency risk is neutralized. "Full hedge" refers to a portfolio in which all of the implicit currency risk is neutralized. "Optimal hedge" refers to a portfolio in which the currency position is chosen optimally to minimize variance. Reported standard deviations are annualized and measured in percentage points. All results presented are computed considering returns at a quarterly horizon.

Base country	No hedge	Half hedge	Full hedge	Optimal hedge			Tests of significance					
				Baseline	Conditional hedging (constrained)	Synthetic currency	Baseline vs. full hedge		Baseline vs. no hedge		Conditional vs. Baseline	
							F-Stat	p-value	F-Stat	p-value	F-Stat	p-value
Panel A: Full period												
Equity												
Euroland	17.67	15.47	13.86	12.51	12.45	12.43	7.98	0.00	33.36	0.00	3.55	6.05
Australia	15.00	13.52	13.86	12.51	12.51	12.43	7.98	0.00	20.09	0.00	0.04	84.37
Canada	13.74	13.22	13.86	12.51	12.50	12.43	7.98	0.00	6.49	0.00	0.44	50.85
Japan	17.08	14.67	13.86	12.51	12.50	12.43	7.98	0.00	31.32	0.00	0.10	74.89
Switzerland	19.19	16.09	13.86	12.51	12.40	12.43	7.98	0.00	41.75	0.00	5.54	1.91
U.K.	16.78	14.74	13.86	12.51	12.50	12.43	7.98	0.00	25.47	0.00	0.15	69.90
U.S.	15.05	13.91	13.86	12.51	12.45	12.43	7.98	0.00	15.20	0.00	3.67	5.63
Bonds												
Euroland	8.39	6.10	5.40	5.21	5.21	5.19	2.76	1.23	54.14	0.00	0.42	51.99
Australia	12.08	7.85	5.40	5.21	5.19	5.19	2.76	1.23	210.02	0.00	6.57	1.08
Canada	10.18	7.12	5.40	5.21	5.21	5.19	2.76	1.23	85.17	0.00	0.03	86.00
Japan	10.86	6.85	5.40	5.21	5.21	5.19	2.76	1.23	87.07	0.00	0.30	58.52
Switzerland	9.93	6.52	5.40	5.21	5.21	5.19	2.76	1.23	85.62	0.00	0.40	52.89
U.K.	10.35	6.98	5.40	5.21	5.19	5.19	2.76	1.23	87.03	0.00	2.53	11.23
U.S.	10.53	7.36	5.40	5.21	5.20	5.19	2.76	1.23	127.73	0.00	1.68	19.53

Table IA.XII (continued)
Standard Deviations of Hedged Global Equity and Bond Portfolios

This table reports the standard deviation of portfolios featuring different uses of currency for risk management. We present results for equally weighted global portfolios, for equity and bonds as respectively described in Table IV and Table VI. Within each panel, rows represent base countries and columns represent the risk management strategy. "No hedge" refers to the simple equity portfolio. "Half hedge" refers to a portfolio in which half of the implicit currency risk is neutralized. "Full hedge" refers to a portfolio in which all of the implicit currency risk is neutralized. "Optimal hedge" refers to a portfolio in which the currency position is chosen optimally to minimize variance. Reported standard deviations are annualized and measured in percentage points. All results presented are computed considering returns at a quarterly horizon.

Base country	Optimal hedge						Tests of significance					
	No hedge	Half hedge	Full hedge	Baseline	Conditional hedging (constrained)	Synthetic currency	Baseline vs. full hedge		Baseline vs. no hedge		Conditional vs. Baseline	
							F-Stat	p-value	F-Stat	p-value	F-Stat	p-value
Panel B: Subperiod I												
Equity												
Euroland	16.79	14.89	13.74	13.12	13.08	12.85	2.84	1.18	14.87	0.00	1.00	31.76
Australia	16.49	14.09	13.74	13.12	13.03	12.85	2.84	1.18	14.33	0.00	2.64	10.63
Canada	15.31	13.90	13.74	13.12	13.07	12.85	2.84	1.18	6.14	0.00	2.23	13.74
Japan	16.53	14.42	13.74	13.12	13.11	12.85	2.84	1.18	16.94	0.00	0.21	64.78
Switzerland	18.46	15.47	13.74	13.12	13.04	12.85	2.84	1.18	20.15	0.00	2.21	13.94
U.K.	16.85	14.51	13.74	13.12	13.09	12.85	2.84	1.18	11.63	0.00	0.55	46.13
U.S.	16.16	14.40	13.74	13.12	13.11	12.85	2.84	1.18	8.04	0.00	0.18	67.56
Bonds												
Euroland	8.84	6.48	5.76	5.20	5.19	5.13	3.51	0.27	34.23	0.00	0.28	59.67
Australia	13.15	8.45	5.76	5.20	5.10	5.13	3.51	0.27	104.09	0.00	10.58	0.14
Canada	11.27	7.85	5.76	5.20	5.16	5.13	3.51	0.27	48.74	0.00	1.22	27.08
Japan	9.77	6.40	5.76	5.20	5.20	5.13	3.51	0.27	38.76	0.00	0.00	97.45
Switzerland	10.50	6.80	5.76	5.20	5.19	5.13	3.51	0.27	50.64	0.00	0.35	55.31
U.K.	11.63	7.65	5.76	5.20	5.19	5.13	3.51	0.27	68.36	0.00	0.41	52.45
U.S.	11.75	8.23	5.76	5.20	5.20	5.13	3.51	0.27	67.75	0.00	0.12	72.89
Panel C: Subperiod II												
Equity												
Euroland	18.38	15.94	13.92	11.25	11.16	11.14	10.70	0.00	37.18	0.00	5.49	2.02
Australia	13.46	12.92	13.92	11.25	11.12	11.14	10.70	0.00	9.92	0.00	5.54	1.97
Canada	12.07	12.50	13.92	11.25	11.04	11.14	10.70	0.00	3.34	0.38	6.15	1.41
Japan	17.72	15.02	13.92	11.25	11.24	11.14	10.70	0.00	32.12	0.00	0.63	42.94
Switzerland	19.71	16.54	13.92	11.25	11.09	11.14	10.70	0.00	44.78	0.00	9.66	0.22
U.K.	16.46	14.80	13.92	11.25	11.21	11.14	10.70	0.00	28.00	0.00	1.29	25.69
U.S.	13.94	13.41	13.92	11.25	11.25	11.14	10.70	0.00	9.98	0.00	0.18	67.05
Bonds												
Euroland	7.69	5.42	4.82	4.67	4.64	4.66	1.88	8.58	45.10	0.00	2.91	8.98
Australia	11.07	7.23	4.82	4.67	4.67	4.66	1.88	8.58	159.78	0.00	0.08	77.85
Canada	9.03	6.24	4.82	4.67	4.67	4.66	1.88	8.58	67.75	0.00	0.08	77.78
Japan	11.54	7.06	4.82	4.67	4.67	4.66	1.88	8.58	78.46	0.00	0.00	99.12
Switzerland	9.07	5.93	4.82	4.67	4.67	4.66	1.88	8.58	75.94	0.00	0.12	73.43
U.K.	8.68	6.00	4.82	4.67	4.67	4.66	1.88	8.58	74.16	0.00	0.07	78.60
U.S.	9.21	6.32	4.82	4.67	4.66	4.66	1.88	8.58	100.90	0.00	1.11	29.25

Table IA.XIII

**Optimal Conditional Currency Exposure for an Equally Weighted Global Portfolio:
Single and Multiple Currency Case Using the Real Interest Rate Differential**

This table reports optimal currency exposure conditional on real interest rates. For each base country-currency pair, we now let the optimal currency position vary with the log real interest rate differential (ex-post real interest rate of the foreign country minus that of the base country), and we impose the constraints that the slopes of the optimal positions with respect to the interest rate differential be equal across foreign currencies. The "Single Currency" columns consider the case of an investor using one currency at a time to manage risk, but still constrain the slopes to be the same across foreign currencies. Resulting slope coefficients from a SUR estimation are reported for each base country, followed by the p -value of a test of the constraint. A p -value of $x\%$ indicates that the constraint can be rejected at the $x\%$ level. The "Multiple Currency" columns consider the case of an investor using all foreign currencies simultaneously to manage risk, but still constrain the slopes to be the same across foreign currencies. Resulting slope coefficients are reported for each base country, followed by the p -value of a test of the constraint. A p -value of $x\%$ indicates that the constraint can be rejected at the $x\%$ level.

Base Currency	Equity				Bonds			
	Single Currency		Multiple Currencies		Single Currency		Multiple Currencies	
	Slope	p -value	Slope	p -value	Slope	p -value	Slope	p -value
Euroland	-0.23 (0.62)	1.00	0.73 (2.22)	0.38	0.02 0.11	1.00	-0.24 0.94	0.09
Australia	0.00 (0.33)	1.00	0.41 (0.68)	0.10	0.00 0.07	1.00	-0.21 0.28	0.20
Canada	0.03 (0.37)	1.00	-2.60 (1.98)	0.06	-0.02 0.12	1.00	-1.55 0.88	0.19
Japan	-0.05 (0.17)	0.99	-0.64 (1.45)	0.06	-0.02 0.09	1.00	-0.27 0.49	0.00
Switz.	-0.09 (0.48)	1.00	0.48 (1.86)	0.37	0.00 0.10	1.00	-1.01 0.73	0.66
U.K.	0.01 (0.36)	1.00	2.59*** (0.97)	0.63	0.02 0.07	1.00	0.17 0.43	0.00
U.S.	0.05 (0.34)	1.00	2.17 (2.22)	0.18	0.00 0.11	1.00	0.56 0.98	0.49

**Table IA.XIV
Sharpe Ratios**

This table reports Sharpe ratios of portfolios featuring different uses of currency for risk management. Please refer to Table VII for a detailed description of these portfolios. The Sharpe ratio of each portfolio is calculated as the ratio of the log mean gross return on the portfolio divided by the standard deviation of the log return on the portfolio. Please see Section F of the Internet Appendix for a detailed description of the calculation of the Sharpe ratio. All results presented are computed considering returns at a quarterly horizon.

Base country	No hedge	Half hedge	Full hedge	Optimal hedge		
				Baseline	Conditional hedging (constrained)	Synthetic currency
Panel A: Full period						
Equity						
Euroland	0.41	0.46	0.51	0.47	0.51	0.54
Australia	0.47	0.50	0.47	0.47	0.47	0.54
Canada	0.48	0.50	0.47	0.47	0.49	0.54
Japan	0.42	0.47	0.48	0.47	0.49	0.54
Switzerland	0.45	0.49	0.52	0.47	0.53	0.54
U.K.	0.38	0.45	0.49	0.47	0.48	0.54
U.S.	0.53	0.52	0.48	0.48	0.55	0.54
Bonds						
Euroland	0.31	0.41	0.44	0.43	0.45	0.50
Australia	0.25	0.35	0.46	0.44	0.41	0.50
Canada	0.26	0.36	0.45	0.43	0.44	0.50
Japan	0.26	0.38	0.43	0.43	0.46	0.50
Switzerland	0.39	0.48	0.44	0.44	0.45	0.50
U.K.	0.20	0.32	0.45	0.43	0.51	0.50
U.S.	0.36	0.43	0.46	0.44	0.49	0.50

Table IA.XIV (continued)
Sharpe Ratios

Base country	No hedge	Half hedge	Full hedge	Optimal hedge		
				Baseline	Conditional hedging (constrained)	Synthetic currency
Panel B: Subperiod I						
Equity						
Euroland	0.51	0.57	0.61	0.53	0.49	0.73
Australia	0.61	0.64	0.59	0.54	0.56	0.73
Canada	0.54	0.59	0.59	0.53	0.63	0.73
Japan	0.41	0.52	0.59	0.53	0.58	0.72
Switzerland	0.55	0.60	0.62	0.54	0.41	0.73
U.K.	0.52	0.59	0.60	0.53	0.59	0.73
U.S.	0.60	0.62	0.61	0.54	0.50	0.73
Bonds						
Euroland	0.14	0.18	0.19	0.05	0.08	0.20
Australia	0.23	0.25	0.20	0.06	0.08	0.21
Canada	0.12	0.16	0.20	0.05	0.19	0.20
Japan	-0.04	0.04	0.16	0.04	0.04	0.19
Switzerland	0.25	0.27	0.18	0.06	0.13	0.21
U.K.	0.14	0.18	0.19	0.06	-0.01	0.21
U.S.	0.22	0.23	0.22	0.05	0.02	0.20
Panel C: Subperiod II						
Equity						
Euroland	0.32	0.37	0.41	0.39	0.46	0.34
Australia	0.32	0.36	0.35	0.39	0.51	0.33
Canada	0.42	0.40	0.35	0.39	0.50	0.34
Japan	0.43	0.43	0.39	0.40	0.44	0.34
Switzerland	0.37	0.40	0.42	0.40	0.48	0.34
U.K.	0.26	0.33	0.39	0.39	0.45	0.33
U.S.	0.45	0.43	0.37	0.40	0.43	0.34
Bonds						
Euroland	0.50	0.68	0.74	0.80	0.74	0.84
Australia	0.27	0.47	0.77	0.80	0.81	0.84
Canada	0.42	0.60	0.76	0.80	0.79	0.84
Japan	0.50	0.67	0.75	0.81	0.80	0.85
Switzerland	0.55	0.72	0.75	0.81	0.79	0.84
U.K.	0.28	0.50	0.75	0.79	0.78	0.84
U.S.	0.53	0.68	0.76	0.81	0.73	0.85