Internet Appendix for "Nonbinding Voting for Shareholder Proposals"*

DORON LEVIT and NADYA MALENKO

The Internet Appendix contains the omitted proofs from the main appendix (Section I) and the analysis corresponding to two extensions of the model (Section II) - endogenous proposal submission and the presence of managerial retaliation.

I. Supplemental Proofs

A. Supplemental Proofs for Section I

Proof of Lemma A.1: The shareholder's benefit from voting for the proposal depends on his beliefs about the state of the world. Since $\omega_g > \omega_b$, other shareholders' votes are informative about their signals and could be used to update the shareholder's beliefs. However, when voting, the shareholder does not observe the votes of other shareholders and therefore has to consider N possible situations, corresponding to the number of affirmative votes among the other $N-1$ shareholders, T_{N-1}, being $0, 1, ...$ or $N-1$. Let us denote by $\Pi^s(T_{N-1})$ the shareholder's expected utility from voting "for" relative to his expected utility from voting "against," given T_{N-1} affirmative votes among other shareholders and his own signal s. Then the shareholder's expected benefit from voting for the proposal relative to voting against the proposal, $\Phi_{\omega,T^*}(s)$, is given by

$$\Phi_{\omega,T^*}(s) = \sum_{T=0}^{N-1} \Pi^s(T) \Pr(T_{N-1} = T|s),$$

and the shareholder prefers to vote affirmatively if and only if $\Phi_{\omega,T^*}(s) \geq 0$.

When the shareholder votes strategically, he takes into account the fact that his decision affects his utility only in certain events, namely, those when the shareholder's vote is pivotal,

*Citation format: Levit, Doron, and Nadya Malenko, 2011, Internet Appendix for "Nonbinding Voting for Shareholder Proposals," *Journal of Finance* 66, 1579-1614, http://www.afajof.org/supplements.asp. Please note: Wiley-Blackwell is not responsible for the content or functionality of any supporting information supplied by the authors. Any queries (other than missing material) should be directed to the authors of the article.

that is, changes the final outcome. Therefore, each shareholder conditions his decision not only on his private signal, but also on the information that must be true when he is pivotal. In the current setting, the shareholder is only pivotal if he changes the manager's decision to accept the proposal. The shareholder anticipates that the manager follows the decision rule T^* and understands that his vote changes the manager's decision if and only if the number of affirmative votes among the other $N-1$ shareholders, T_{N-1}, is exactly T^*. Thus, whenever $T_{N-1} \neq T^*$, the shareholder's vote does not change the manager's decision, and hence the shareholder is indifferent with respect to his vote, which implies $\Pi^s(T_{N-1}) = 0$. If $T_{N-1} = T^*$, then the shareholder's vote becomes pivotal for the manager because the proposal is accepted if and only if the shareholder votes affirmatively. Let $\Pr(\theta | T_{N-1} = T^*, s)$ be the posterior belief that the state is θ conditional on being pivotal for the manager and the signal s. Then the expected value that a shareholder with signal s gains by voting for the proposal in this event is given by

$$\Pi^s(T^*) = \Pr(G | T_{N-1} = T^*, s) - \Pr(B | T_{N-1} = T^*, s). \tag{IA.1}$$

Combining these arguments, the expected relative benefit from voting for the proposal is given by

$$\Phi_{\omega,T^*}(s) = \Pi^s(T^*) \Pr(T_{N-1} = T^* | s). \tag{IA.2}$$

We argue that $\Phi_{\omega,T^*}(g) > \Phi_{\omega,T^*}(b)$. Note that $\Pr(T_{N-1} = T^*|s) > 0$ since $T^* \in [0, N-1]$. Then plugging (IA.1) into (IA.2) and using Bayes' rule, we can rewrite $\Phi_{\omega,T^*}(s)$ as

$$\begin{aligned}\Phi_{\omega,T^*}(s) &= \Pr(G, T_{N-1} = T^*|s) - \Pr(B, T_{N-1} = T^*|s)\\ &= \Pr(T_{N-1} = T^*, s|G) - \Pr(T_{N-1} = T^*, s|B).\end{aligned}$$

Writing out the probabilities $\Pr(T_{N-1} = T^*, s|\theta)$ explicitly, we get

$$\Phi_{\omega,T^*}(s) = C_T^{N-1} \begin{cases} \pi_G^{T^*}(1-\pi_G)^{N-1-T^*}\rho - \pi_B^{T^*}(1-\pi_B)^{N-1-T^*}(1-\rho) & \text{if } s = g \\ \pi_G^{T^*}(1-\pi_G)^{N-1-T^*}(1-\rho) - \pi_B^{T^*}(1-\pi_B)^{N-1-T^*}\rho & \text{if } s = b \end{cases} \tag{IA.3}$$

and thus

$$\Phi_{\omega,T^*}(g) > \Phi_{\omega,T^*}(b) \Leftrightarrow \pi_G^{T^*}(1-\pi_G)^{N-1-T^*}(2\rho-1) > \pi_B^{T^*}(1-\pi_B)^{N-1-T^*}(1-2\rho),$$

which is true since $\rho > 0.5$.

Last, from (IA.3) it immediately follows that

$$\begin{aligned}\Phi_{\omega,T^*}(g) > 0 &\Leftrightarrow \frac{\pi_B^{T^*}(1-\pi_B)^{N-T^*}}{\pi_G^{T^*}(1-\pi_G)^{N-T^*}} < \frac{\rho}{1-\rho}\frac{1-\pi_B}{1-\pi_G}\\ \Phi_{\omega,T^*}(b) > 0 &\Leftrightarrow \frac{\pi_B^{T^*}(1-\pi_B)^{N-T^*}}{\pi_G^{T^*}(1-\pi_G)^{N-T^*}} < \frac{1-\rho}{\rho}\frac{1-\pi_B}{1-\pi_G},\end{aligned}$$

and noting that $\frac{\Pr\left[T_{N,\omega}=T^*|B\right]}{\Pr\left[T_{N,\omega}=T^*|G\right]} = \frac{\pi_B^{T^*}(1-\pi_B)^{N-T^*}}{\pi_G^{T^*}(1-\pi_G)^{N-T^*}}$ completes the proof. ■

Proof of Theorem 1, Case 2 - bad type mixing equilibria: This equilibrium exists and is responsive if and only if (1) $T_\omega^* \in [0, N-1]$ and (2) $\Phi_{\omega,T_\omega^*}(b) = 0$. Note that in this case, $\frac{1-\pi_B}{1-\pi_G} = \frac{\rho}{1-\rho}$ and according to (A3),

$$\Phi_{\omega,T_\omega^*}(b) = 0 \Leftrightarrow \frac{\Pr\left[T_{N,\omega} = T_\omega^*|B\right]}{\Pr[T_{N,\omega} = T_\omega^*|G]} = 1 \Leftrightarrow T_\omega^* = \frac{N \log \frac{\rho}{1-\rho}}{\log \frac{\pi_G}{\pi_B} + \log \frac{\rho}{1-\rho}}.$$

Since T_ω^* is an integer, this is only possible if the right-hand side is an integer. Note that when ω_b spans the interval $(0,1)$, $\frac{\pi_G}{\pi_B}$ spans the interval $(1, \frac{\rho}{1-\rho})$. Therefore, for any integer $I \in \left(\frac{N}{2}, N\right)$ there exists some $\omega_b \in (0,1)$ such that the right-hand side of the above equation equals I. Moreover, if $T_\omega^* \in \left(\frac{N}{2}, N\right)$, then the condition $T_\omega^* \in [0, N-1]$ (condition (1) above) is satisfied. Thus, this type of mixed strategy equilibria exists if and only if there exists $\omega_b \in (0,1)$ such that both the left-hand side, T_ω^*, and the right-hand side are equal to some integer $I \in \left(\frac{N}{2}, N\right)$. Recall that $T_\omega^* = \lfloor \tau_\omega(\beta^*) \rfloor$ and $\tau_\omega(\beta^*)$ is given by (A2). Thus, when $\omega_g = 1$,

$$T_\omega^* = \left\lfloor \frac{N \log \frac{\rho}{1-\rho}}{\log \frac{\pi_G}{\pi_B} + \log \frac{\rho}{1-\rho}} + \frac{\log \frac{\beta^*}{1-\beta^*}}{\log \frac{\pi_G}{\pi_B} + \log \frac{\rho}{1-\rho}} \right\rfloor.$$

Since $\frac{N \log \frac{\rho}{1-\rho}}{\log \frac{\pi_G}{\pi_B} + \log \frac{\rho}{1-\rho}} = I = T_\omega^*$ and I is an integer, it must be that $\frac{\log \frac{\beta^*}{1-\beta^*}}{\log \frac{\pi_G}{\pi_B} + \log \frac{\rho}{1-\rho}} \in [0,1)$. Expressing $\log \frac{\pi_G}{\pi_B}$ through I, this is equivalent to the requirement that $\frac{I}{N} \frac{\log \frac{\beta^*}{1-\beta^*}}{\log \frac{\rho}{1-\rho}} \in [0,1)$ for some integer $I \in \left(\frac{N}{2}, N\right)$. Hence, a responsive equilibrium exists if and only if there exists $I \in \left(\frac{N}{2}, N\right)$ such that $\frac{I}{N} \frac{\log \frac{\beta^*}{1-\beta^*}}{\log \frac{\rho}{1-\rho}} \in [0,1)$, which is equivalent to $\log \frac{\beta^*}{1-\beta^*} \in \frac{N}{I}[0, \log \frac{\rho}{1-\rho})$. Clearly, the equivalent requirement is that the last inequality is satisfied for the lowest integer I in the interval $\left(\frac{N}{2}, N\right)$, which is $\frac{N}{2}+1$ when N is even and $\frac{N+1}{2}$ when N is odd. Plugging these values into the last inequality, we conclude that responsive equilibria with a bad type mixing exist if and only if (A5) holds. ■

Proof of Theorem 1, Case 3 - good type mixing equilibria: This equilibrium exists and is responsive if and only if (1) $T_\omega^* \in [0, N-1]$ and (2) $\Phi_{\omega,T_\omega^*}(g) = 0$. Note that in this case, $\frac{\pi_G}{\pi_B} = \frac{\rho}{1-\rho}$ and according to (A3),

$$\Phi_{\omega,T_\omega^*}(g) = 0 \Leftrightarrow \frac{\Pr\left[T_{N,\omega} = T_\omega^*|B\right]}{\Pr[T_{N,\omega} = T_\omega^*|G]} = \frac{\rho}{1-\rho}\frac{1-\pi_B}{1-\pi_G} \Leftrightarrow T_\omega^* = \frac{N \log \frac{1-\pi_B}{1-\pi_G}}{\log \frac{\rho}{1-\rho} + \log \frac{1-\pi_B}{1-\pi_G}} - 1.$$

Since T_ω^* is an integer, this is only possible if the right-hand side is an integer. Note that when ω_g spans the interval $(0,1)$, $\frac{1-\pi_B}{1-\pi_G}$ spans the interval $(1, \frac{\rho}{1-\rho})$. Therefore, for any integer $I \in \left[0, \frac{N}{2}-1\right)$ there exists some $\omega_g \in (0,1)$ such that the right-hand side of the above equation equals I. Moreover, if $T^*_\omega \in \left[0, \frac{N}{2}-1\right)$, then the condition $T^*_\omega \in [0, N-1]$ (condition (1) above) is satisfied. Thus, this type of mixed strategy equilibria exists if and only if there exists $\omega_g \in (0,1)$ such that both the left-hand side T^*_ω, and the right-hand side are equal to some integer $I \in [0, \frac{N}{2}-1)$. Recall that $T^*_\omega = \lfloor \tau_\omega(\beta^*) \rfloor$ and $\tau_\omega(\beta^*)$ is given by (A2). Thus, when $\omega_b = 0$,

$$T^*_\omega = \left\lfloor \frac{N \log \frac{1-\pi_B}{1-\pi_G}}{\log \frac{\rho}{1-\rho} + \log \frac{1-\pi_B}{1-\pi_G}} + \frac{\log \frac{\beta^*}{1-\beta^*}}{\log \frac{\rho}{1-\rho} + \log \frac{1-\pi_B}{1-\pi_G}} \right\rfloor.$$

Since $\frac{N \log \frac{1-\pi_B}{1-\pi_G}}{\log \frac{\rho}{1-\rho} + \log \frac{1-\pi_B}{1-\pi_G}} - 1 = I = T^*_\omega$ and I is an integer, it must be that $\frac{\log \frac{\beta^*}{1-\beta^*}}{\log \frac{\rho}{1-\rho} + \log \frac{1-\pi_B}{1-\pi_G}} + 1 \in [0,1)$. Expressing $\log \frac{1-\pi_B}{1-\pi_G}$ through I, this is equivalent to the requirement that $\frac{N-I-1}{N} \frac{\log \frac{\beta^*}{1-\beta^*}}{\log \frac{\rho}{1-\rho}} + 1 \in [0,1)$ for some integer $I \in \left[0, \frac{N}{2}-1\right)$. Hence, a responsive equilibrium exists if and only if there exists $I \in \left[0, \frac{N}{2}-1\right)$ such that $\frac{N-I-1}{N} \frac{\log \frac{\beta^*}{1-\beta^*}}{\log \frac{\rho}{1-\rho}} \in [-1, 0)$, which is equivalent to $\log \frac{\beta^*}{1-\beta^*} \in \frac{N}{N-I-1} \left[\log \frac{1-\rho}{\rho}, 0\right)$. Clearly, the equivalent requirement is that the last inequality is satisfied for the highest integer I in the interval $\left[0, \frac{N}{2}-1\right)$, which is $\frac{N}{2}-2$ when N is even and $\frac{N-1}{2}-1$, when N is odd. Plugging these values into the last inequality, we conclude that responsive equilibria with a good type mixing exist if and only if (A6) holds. ■

B. Supplemental Definitions and Proofs for Section II

Full Description of Preferences. We first formalize the preferences of the shareholders, the manager, and the activist described in Section II of the main text for a given profile of strategies (ω, d_M, e) of the shareholders, manager, and activist, respectively. Note that the number of affirmative votes T, given that shareholders follow voting strategies ω and the state of the world is θ, follows a binomial distribution with parameters (N, π_θ), where π_θ is defined by (A1). Let $E^T_\omega[\cdot|\theta]$ be the conditional expectation operator over the number of affirmative votes under this distribution. Denote by $U_i(d, \theta)$ each agent's utility from the ultimate decision $d \in \{A, R\}$ when the state of the world is θ. As defined in the setup,

$$\begin{aligned} U_S(d,\theta) &= v(d,\theta) \\ U_M(d,\theta) &= k_M v(d,\theta) - (1-k_M) 1_{\{d=A\}} \\ U_A(d,\theta) &= k_A v(d,\theta) + (1-k_A) 1_{\{d=A\}}, \end{aligned}$$

where $v(d,\theta)$ is the value to the firm from accepting (rejecting) the proposal for $d = A\,(R)$. Last, if d is the action taken by the manager, denote the opposite action by $\hat{d}$, that is, $\widehat{d} = \{A, R\} \setminus d$.

Then shareholders' expected utility given state of the world θ and the profile of strategies (ω, d_M, e) is given by

$$W_S(\omega, d_M, e; \theta) = E_\omega^T \left[\begin{array}{c} (1 - e(T, d_M(T)))\, U_S(d_M(T), \theta) + \\ e(T, d_M(T)) \left(\lambda U_S(\widehat{d}_M(T), \theta) + (1-\lambda)\, U_S(d_M(T), \theta) \right) \end{array} \middle| \, \theta \right].$$

Similarly, but with additional costs c_M incurred by the manager if the proxy fight is successful, the manager's expected utility is given by

$$W_M(\omega, d_M, e; \theta) = E_\omega^T \left[\begin{array}{c} (1 - e(T, d_M(T)))\, U_M(d_M(T), \theta) + \\ e(T, d_M(T)) \left(\lambda \left[U_M(\widehat{d}_M(T), \theta) - c_M \right] + (1-\lambda)\, U_M(d_M(T), \theta) \right) \end{array} \middle| \, \theta \right].$$

Last, taking into account solicitation costs incurred by the activist, the activist's expected utility is given by

$$W_A(\omega, d_M, e; \theta) = E_\omega^T \left[\begin{array}{c} (1 - e(T, d_M(T)))\, U_A(d_M(T), \theta) + \\ e(T, d_M(T)) \left(\lambda U_A(\widehat{d}_M(T), \theta) + (1-\lambda)\, U_A(d_M(T), \theta) - c_A \right) \end{array} \middle| \, \theta \right].$$

Proof of Lemma B.1: Given ω and T, the benefit of the activist from accepting the proposal relative to rejecting it is $(1 - k_A) - k_A(1 - 2\beta_\omega(T))$. Thus, if the manager accepts the proposal,

$$e(T, A) = 1 \Leftrightarrow \lambda \left[-(1 - k_A) + k_A(1 - 2\beta_\omega(T)) \right] \geq c_A \Leftrightarrow \beta_\omega(T) \leq \beta^A.$$

Similarly, if the manager rejects the proposal,

$$e(T, R) = 1 \Leftrightarrow \lambda \left[(1 - k_A) - k_A(1 - 2\beta_\omega(T)) \right] \geq c_A \Leftrightarrow \beta_\omega(T) \geq \beta^R.$$

The last inequality of the statement follows from an explicit comparison of β^A, β^R, and $\frac{1}{2}$. ■

Proof of Lemma B.2: Given posterior belief $\beta_\omega(T)$, the manager's benefit from accepting the proposal relative to rejecting it when no proxy fight is organized is given by

$$X(\beta_\omega(T)) \equiv k_M(2\beta_\omega(T) - 1) - (1 - k_M), \tag{IA.4}$$

where $X(\beta_\omega(T)) > 0 \Leftrightarrow \beta_\omega(T) > \beta^{NP}$. The superscript NP shows that this threshold belief corresponds to the case of no proxy fight.

We start with the argument that the manager does not accept the proposal if his posterior beliefs are below β^A. Intuitively, if $\beta_\omega(T) < \beta^A$, then the activist prefers that the proposal be rejected and hence the manager, who is more biased against the proposal than the activist, also prefers that the proposal be rejected. To show this formally, note that according to Lemma B.1, when $\beta_\omega(T) \leq \beta^A$ the activist initiates a proxy fight if and only if the manager accepts the proposal. Hence, in this range,

$$d_M(T) = A \Leftrightarrow (1-\lambda)\,X(\beta_\omega(T)) \ > \lambda c_M.$$

Note that $\beta^{NP} \geq \frac{1}{2} \geq \beta^A$ and hence the inequality $\beta_\omega(T) \leq \beta^A$ implies that $\beta_\omega(T) \leq \beta^{NP}$, which implies that $X(\beta_\omega(T)) \leq 0$ according to (IA.4). Thus, the left-hand side of the inequality above is nonpositive, while the right-hand side is positive. Hence, $d_M(T) = R$ in that range, which implies that $\beta^* > \beta^A$.

Next, we consider two cases:

Case I - $\beta^R < \beta^{NP}$: Because β^P is also smaller than β^{NP}, then according to condition (A8) we need to show that $\beta^* = \max\{\beta^P, \beta^R\}$. First, consider T such that $\beta_\omega(T) \in (\beta^A, \beta^R]$. According to Lemma B.1, the activist never initiates a proxy fight in this range and thus

$$d_M(T) = A \Leftrightarrow X(\beta_\omega(T)) > 0 \Leftrightarrow \beta_\omega(T) > \beta^{NP}.$$

Therefore, since $\beta^R < \beta^{NP}$, the manager rejects the proposal if $\beta_\omega(T) \leq \beta^R$. Next, consider T such that $\beta_\omega(T) > \beta^R$. Then the activist initiates a proxy fight only when the manager rejects the proposal and hence

$$d_M(T) = A \Leftrightarrow X(\beta_\omega(T)) \ > \lambda\left[X(\beta_\omega(T)) - c_M\right] \Leftrightarrow \beta_\omega(T) > \beta^P.$$

The superscript P shows that this threshold belief corresponds to the case of a proxy fight. Combining the two ranges, we conclude that the manager accepts the proposal if and only if $\beta_\omega(T) > \max\{\beta^P, \beta^R\}$, that is, $\beta^* = \max\{\beta^P, \beta^R\}$, as required.

Case II - $\beta^R \geq \beta^{NP}$ (and hence $\beta^{NP} \in (\beta^A, \beta^R]$): Because $\beta^{NP} \geq \beta^P$, then according to (A8) we need to show that $\beta^* = \beta^{NP}$. Note that similarly to Case I, if $\beta_\omega(T) \in (\beta^A, \beta^R]$, then $d_M(T) = A \Leftrightarrow \beta_\omega(T) > \beta^{NP}$. Hence, it remains to prove that the manager also accepts the proposal if $\beta_\omega(T) > \beta^R$. Similarly to Case I, if $\beta_\omega(T) > \beta^R$, then $d_M(T) = A \Leftrightarrow \beta_\omega(T) > \beta^P$, which holds in this range because $\beta^P \leq \beta^{NP} \leq \beta^R < \beta_\omega(T)$. Thus, $\beta^* = \beta^{NP}$, as required. ■

Proof of Theorem 2, Case II - $\beta^R < \beta^P$: Suppose by way of contradiction that there exists a responsive equilibrium (ω_b, ω_g) with $\omega_g > \omega_b$. Since $\beta^R < \beta^P$, then according to Lemma B.2 $\beta^* = \beta^P$, and according to Lemma B.1 a proxy fight to approve the proposal is organized in equilibrium when $\beta_\omega(T) \in (\beta^R, \beta^P]$. Note that in this case, a shareholder's vote has the potential to change not only the manager's decision (if the posterior beliefs are around β^P), but also the activist's decision to initiate a proxy fight (if the posterior beliefs are around β^R). We will show that under the conditions of the theorem, the posterior beliefs in both of these events are sufficiently optimistic, inducing shareholders to vote in favor of the proposal. Let $\beta_\omega(N)$ and $\beta_\omega(0)$ be the highest and lowest posterior beliefs, respectively, that can be achieved for any $T \in [0, N]$ given the strategies ω. There are four cases to consider:

II.1 If $\beta^P > \beta_\omega(N) > \beta^R > \beta_\omega(0)$, then the manager always rejects the proposal and shareholders are only pivotal for the activist's decision to launch a proxy fight.

II.2 If $\beta_\omega(N) > \beta^P > \beta_\omega(0) > \beta^R$, then the activist always launches a proxy fight and shareholders are only pivotal for the manager's decision.

II.3 If $\beta_\omega(N) > \beta^P > \beta^R > \beta_\omega(0)$, then shareholders are pivotal for both the manager's and the activist's decisions.

II.4 In all other cases, the outcome of the vote does not change either the activist's or the manager's decision and hence the equilibrium is non-responsive by definition.

We prove the theorem for Case II.3, but it will become clear that Cases II.1 and II.2 follow by the same logic. Suppose that shareholders are pivotal for both decisions. Note that by voting for the proposal when there are T affirmative votes among other shareholders, the shareholder changes the probability that the proposal is ultimately accepted by Δ_T, where Δ_T takes the following values: if $T = T_\omega^R \equiv \lfloor \tau_\omega(\beta^R) \rfloor$, then $\Delta_T = \lambda$; if $T = T_\omega^P \equiv \lfloor \tau_\omega(\beta^P) \rfloor$, then $\Delta_T = 1 - \lambda$; otherwise, $\Delta_T = 0$. This is because for $T \leq T_\omega^R$ the proposal is never accepted, for $T \in (T_\omega^R, T_\omega^P]$ the proposal is accepted if and only if the proxy fight succeeds, which happens with probability λ, and for $T > T_\omega^P$ the proposal is accepted with probability one. Therefore, the expected value that a shareholder with signal s gains by voting for the proposal when the number of affirmative votes among the other $N-1$ shareholders, T_{N-1}, equals T is given by $\Delta_T [2 \Pr(G|T_{N-1} = T, s) - 1]$. It follows that a shareholder with signal s will vote affirmatively if and only if his expected relative benefit from doing so is positive:

$$\begin{aligned}\Lambda_\omega(s) &\equiv \Delta_{T_\omega^R}\left[2\Pr\left(G|T_{N-1} = T_\omega^R, s\right) - 1\right] + \Delta_{T_\omega^P}\left[2\Pr\left(G|T_{N-1} = T_\omega^P, s\right) - 1\right] \\ &= \lambda \Phi_{\omega, T_\omega^R}(s) + (1-\lambda)\Phi_{\omega, T_\omega^P}(s) \geq 0,\end{aligned}$$

where $\Phi_{\omega,T}(s)$ is given by (IA.3). Note that by the same logic, the relative benefit of voting affirmatively is given by $\Lambda_\omega(s) \equiv \lambda\Phi_{\omega,T_\omega^R}(s)$ in Case II.1 and by $\Lambda_\omega(s) \equiv (1-\lambda)\Phi_{\omega,T_\omega^P}(s)$ in Case II.2.

Before proving that no responsive equilibrium exists in this case, we point out two properties. First, recall that the proof of Lemma A.1 did not use any properties of T^*. Hence, it follows from the same proof that $\Phi_{\omega,T}(g) > \Phi_{\omega,T}(b)$ for $T \in \{T_\omega^R, T_\omega^P\}$ and hence $\Lambda_\omega(g) > \Lambda_\omega(b)$. In other words, the expected benefit from voting for the proposal relative to voting against it is strictly higher for the shareholder with a good signal than for the shareholder with a bad signal. Similarly to the proof of Theorem 1, it follows that there can only be three types of responsive equilibria: the equilibrium in pure strategies ($\omega_b = 0, \omega_g = 1$), and two types of mixed strategy equilibria ($\omega_b \in (0,1), \omega_g = 1$ and $\omega_b = 0, \omega_g \in (0,1)$).

Second, according to (A3),

$$\begin{aligned}\Phi_{\omega,T^*}(g) > 0 &\Leftrightarrow \frac{\Pr\left[T_{N,\omega} = T^*|B\right]}{\Pr\left[T_{N,\omega} = T^*|G\right]} < \frac{\rho}{1-\rho}\frac{1-\pi_B}{1-\pi_G} \\ \Phi_{\omega,T^*}(b) > 0 &\Leftrightarrow \frac{\Pr\left[T_{N,\omega} = T^*|B\right]}{\Pr\left[T_{N,\omega} = T^*|G\right]} < \frac{1-\rho}{\rho}\frac{1-\pi_B}{1-\pi_G}.\end{aligned}$$

Since $\frac{\Pr[T_{N,\omega}=T|B]}{\Pr[T_{N,\omega}=T|G]}$ is decreasing in T and $T_\omega^R < T_\omega^P$, then $\Phi_{\omega,T_\omega^R}(s) > 0$ implies $\Phi_{\omega,T_\omega^P}(s) > 0$.

Let us now prove that no responsive equilibrium exists under the conditions of the theorem if Case II.3 is realized. Recall that $k_A \geq \frac{\lambda - c_A}{\lambda - \underline{c}_A} \Leftrightarrow \frac{\beta^R}{1-\beta^R} \geq (\frac{\rho}{1-\rho})^2$. First, consider equilibria with $\omega_g = 1$ and $\omega_b \in [0,1)$. Repeating the argument in the proof of Theorem 1 and using (A4) and (A5), we conclude that since $\frac{\beta^R}{1-\beta^R} \geq (\frac{\rho}{1-\rho})^2$, then $\Phi_{\omega,T_\omega^R}(b) > 0$ for any $\omega = (\omega_b, 1)$ and any N. Therefore, $\Phi_{\omega,T_\omega^P}(b) > 0$ as well. Intuitively, when β^R is sufficiently high, the shareholder's beliefs conditional on being pivotal for the activist's decision are sufficiently optimistic to induce him to vote affirmatively even if his signal is bad. Because the manager's threshold is even higher than the activist's, the shareholder's beliefs conditional on being pivotal for the manager's decision are even more optimistic. It follows that $\Lambda_\omega(b) > 0$ and thus a shareholder with a bad signal strictly prefers to vote for the proposal, which contradicts $\omega_b < 1$. Hence, a responsive equilibrium of this type cannot exist.

Second, consider equilibria with $\omega_b = 0$ and $\omega_g \in (0,1)$. Repeating the argument in the proof of Theorem 1 and using (A6), we conclude that since $\frac{\beta^R}{1-\beta^R} \geq (\frac{\rho}{1-\rho})^2 > 1$, it follows that $\Phi_{\omega,T_\omega^R}(g) > 0$ and hence $\Phi_{\omega,T_\omega^P}(g) > 0$, implying that $\Lambda_\omega(g) > 0$. Thus, a shareholder with a good signal strictly prefers to vote affirmatively, which contradicts $\omega_g < 1$. Hence, a responsive equilibrium of this type cannot exist either. ■

Proof of Theorem 3, Case I - $\beta^R \geq \beta^P$: According to Lemmas B.1 and B.2, when a responsive equilibrium exists, $\beta^R \geq \beta^P$ implies that the manager's threshold beliefs are given by $\beta^* = \min\{\beta^R, \beta^{NP}\}$, corresponding to the threshold $T_\omega^* = \lfloor \tau_\omega(\beta^*) \rfloor$. Moreover, each shareholder is only pivotal for the manager's decision and hence his expected benefit from voting for the proposal relative to voting against it is $\Phi_{\omega,T_\omega^*}(s)$, which satisfies (A3). Recall that expression (A7) in the proof of Theorem 1 was derived for a general β^*. Therefore, we can replicate the proof of Theorem 1 and conclude that a responsive equilibrium exists if and only if (A7) holds for $\beta^* = \min\{\beta^R, \beta^{NP}\}$. Note, however, that $k_M \leq \frac{1}{2} + \frac{1}{2}(\frac{1-\rho}{\rho})^2 \Leftrightarrow \frac{\beta^{NP}}{1-\beta^{NP}} \geq (\frac{\rho}{1-\rho})^2$ and hence (A7) is violated for $\beta^{NP} \equiv \frac{1}{2k_M}$ under the conditions of the theorem. Thus, a responsive equilibrium exists if and only if 1) $\beta^* = \beta^R$ and 2) β^R satisfies (A7).

Recalling that $\beta^R = 1 + \frac{c_A/\lambda - 1}{2k_A}$, it is straightforward to show that for even N the condition (A7) that $\frac{\beta^R}{1-\beta^R} \in \left[(\frac{1-\rho}{\rho})^2, (\frac{\rho}{1-\rho})^2\right)$ is equivalent to condition (4) in the statement of the theorem. The condition $c_A < \lambda$ ensures that the interval in (4) is non-empty. Hence, it remains to ensure that $\beta^* = \beta^R$. According to (A8), $\beta^* = \beta^R$ if and only if $\beta^P \leq \beta^R \leq \beta^{NP}$, or equivalently, if and only if $\frac{\beta^P}{1-\beta^P} \leq \frac{\beta^R}{1-\beta^R} \leq \frac{\beta^{NP}}{1-\beta^{NP}}$. The first inequality is satisfied in Case I by assumption. The second inequality is satisfied because $\frac{\beta^R}{1-\beta^R} < (\frac{\rho}{1-\rho})^2 \leq \frac{\beta^{NP}}{1-\beta^{NP}}$ for $k_M \leq \frac{1}{2} + \frac{1}{2}(\frac{1-\rho}{\rho})^2$ and under condition (4).

We conclude that if $\beta^R \geq \beta^P$, $k_M \leq \frac{1}{2} + \frac{1}{2}(\frac{1-\rho}{\rho})^2$, and $c_A \in (\underline{c}_A, \lambda)$, then condition (4) is necessary and sufficient for the existence of a responsive equilibrium. In such equilibrium a proxy fight is never organized. ■

Proof of Theorem 3, Case II - $\beta^R < \beta^P$: There are two possibilities:

II.1 Suppose $k_M > \left(1 - \frac{c_M}{\underline{c}_M}\right)\left[\frac{1}{2} + \frac{1}{2}(\frac{1-\rho}{\rho})^2\right]$, and note that this condition is equivalent to $\frac{\beta^P}{1-\beta^P} < (\frac{\rho}{1-\rho})^2$. We show that in this case, condition (4) is sufficient for the existence of a pure strategy responsive equilibrium.

Recall that (4) is equivalent to $\frac{\beta^R}{1-\beta^R} \in \left[(\frac{1-\rho}{\rho})^2,\ (\frac{\rho}{1-\rho})^2\right)$ and hence $\beta^R < \beta^P$ implies that $\frac{\beta^P}{1-\beta^P} > (\frac{1-\rho}{\rho})^2$. We conclude that

$$\left(\frac{1-\rho}{\rho}\right)^2 < \frac{\beta^R}{1-\beta^R} < \frac{\beta^P}{1-\beta^P} < \left(\frac{\rho}{1-\rho}\right)^2. \tag{IA.5}$$

Consider a pure strategy responsive equilibrium: $(\omega_g, \omega_b) = (1, 0)$. In this equilibrium, $\frac{\beta_\omega(0)}{1-\beta_\omega(0)} = (\frac{1-\rho}{\rho})^N < (\frac{1-\rho}{\rho})^2$ and $(\frac{\rho}{1-\rho})^2 < (\frac{\rho}{1-\rho})^N = \frac{\beta_\omega(N)}{1-\beta_\omega(N)}$, which implies $\beta_\omega(0) < \beta^R < \beta^P < \beta_\omega(N)$ and hence that shareholders are pivotal with strictly positive probability for both the manager's and the activist's decisions. In particular, for $T \leq T^R_\omega \equiv \lfloor \tau_\omega(\beta^R) \rfloor$, the manager rejects the proposal and there is no proxy fight. For $T^R_\omega < T \leq T^P_\omega \equiv \lfloor \tau_\omega(\beta^P) \rfloor$, the manager rejects the proposal and the activist initiates a proxy fight. Hence, with probability λ the proposal is accepted. Last, for $T > T^P_\omega$ the manager accepts the proposal. Therefore, the expected benefit from voting for the proposal relative to voting against it for a shareholder with signal s is given by $\Lambda_\omega(s) \equiv \lambda \Phi_{\omega, T^R_\omega}(s) + (1-\lambda)\Phi_{\omega, T^P_\omega}(s)$. Shareholders will optimally vote according to their signals if and only if $\Lambda_\omega(b) \leq 0 \leq \Lambda_\omega(b)$. It follows that sufficient conditions for existence of a pure strategy equilibrium are $\Phi_{\omega, T^R_\omega}(b) \leq 0 \leq \Phi_{\omega, T^R_\omega}(g)$ and $\Phi_{\omega, T^P_\omega}(b) \leq 0 \leq \Phi_{\omega, T^P_\omega}(g)$. According to the proof of Theorem 1, condition (A4) for a general threshold belief β^* and $T^*_\omega = \lfloor \tau_\omega(\beta^*) \rfloor$ implies that $\Phi_{\omega, T^*_\omega}(b) \leq 0 \leq \Phi_{\omega, T^*_\omega}(g)$. Since both β^R and β^P satisfy condition (A4) according to (IA.5), we conclude that a pure strategy responsive equilibrium indeed exists.

II.2 Suppose $k_M \leq \left(1 - \frac{c_M}{\underline{c}_M}\right)\left[\frac{1}{2} + \frac{1}{2}(\frac{1-\rho}{\rho})^2\right]$. Note that this condition is equivalent to $\frac{\beta^P}{1-\beta^P} \geq (\frac{\rho}{1-\rho})^2$. We show that in this case, condition (5) in the statement of the theorem is sufficient for the existence of a responsive equilibrium in which a proxy fight occurs with positive probability. There are two possible cases.

   (a) Suppose $\frac{\beta^P}{1-\beta^P} \geq (\frac{\rho}{1-\rho})^N$. This implies that $\beta^P \geq \beta_\omega(N)$ for any ω. Hence, the manager always rejects the proposal and shareholders are never pivotal for his decision. The only threshold at which shareholders are pivotal is the activist's threshold T^R_ω: for $T \leq T^R_\omega$, the manager rejects the proposal and there is no proxy fight.

For $T > T_\omega^R$, the manager rejects the proposal but the activist organizes a proxy fight, and with probability λ the proposal is eventually accepted. Hence, the expected relative benefit from voting for the proposal for a shareholder with signal s is $\lambda\Phi_{\omega,T_\omega^R}(s)$. A sufficient condition for the existence of a pure strategy responsive equilibrium is therefore $\Phi_{\omega,T_\omega^R}(b) \leq 0 \leq \Phi_{\omega,T_\omega^R}(g)$. Note that condition (5) is equivalent to $1 < \frac{\beta^R}{1-\beta^R} < \frac{\rho}{1-\rho}$ and implies that condition (A4) is satisfied for $\beta^* = \beta^R$. Moreover, because $\frac{\beta_\omega(T)}{1-\beta_\omega(T)} = (\frac{\rho}{1-\rho})^{2T-N}$ when shareholders play pure strategies, (A4) also implies that $\beta_\omega(0) < \beta^R < \beta_\omega(N)$. Hence, by repeating the arguments in the proof of Theorem 1, there exists a responsive equilibrium in pure strategies, and in this equilibrium a proxy fight is organized with strictly positive probability.

(b) Suppose $(\frac{\rho}{1-\rho})^2 \leq \frac{\beta^P}{1-\beta^P} < (\frac{\rho}{1-\rho})^N$. We show that in this case, condition (5) is sufficient for the existence of a bad type mixing responsive equilibrium in which the manager always rejects the proposal and a proxy fight is initiated with strictly positive probability.

Consider bad type mixing equilibria: $\omega_b \in (0,1), \omega_g = 1$. Note that the maximal and minimal posterior beliefs following the nonbinding vote, $\beta_\omega(N)$ and $\beta_\omega(0)$, satisfy

$$\begin{aligned}\frac{\beta_\omega(N)}{1-\beta_\omega(N)} &= \left(\frac{\pi_G}{\pi_B}\right)^N = \left(\frac{(1-\rho)\,\omega_b+\rho}{\rho\omega_b+(1-\rho)}\right)^N \\ \frac{\beta_\omega(0)}{1-\beta_\omega(0)} &= \left(\frac{1-\rho}{\rho}\right)^N < 1,\end{aligned}$$

and hence as ω_b spans $(0,1)$, $\frac{\beta_\omega(N)}{1-\beta_\omega(N)}$ spans $\left(1,(\frac{\rho}{1-\rho})^N\right)$. Note also that if condition (5) holds, then $1 < \frac{\beta^R}{1-\beta^R} < \frac{\rho}{1-\rho} < (\frac{\rho}{1-\rho})^2 \leq \frac{\beta^P}{1-\beta^P}$ for any β^P and β^R in this range. Therefore, there exists a range of $\omega_b \in (0,1)$ over which $\frac{\beta_\omega(N)}{1-\beta_\omega(N)}$ spans the interval $(\frac{\beta^R}{1-\beta^R}, \frac{\beta^P}{1-\beta^P})$, or equivalently, over which $\log\frac{\pi_G}{\pi_B}$ spans $\left(\frac{1}{N}\log\frac{\beta^R}{1-\beta^R}, \frac{1}{N}\log\frac{\beta^P}{1-\beta^P}\right)$. For these values of ω_b, $\beta_\omega(N) < \beta^P$ and hence the manager always rejects the proposal and shareholders are only pivotal for the activist's decision to organize a proxy fight.

We next demonstrate that there exists a responsive equilibrium for some ω_b within this range. Consider a modification of the proof of Theorem 1. Because shareholders are only pivotal at the threshold $T_\omega^R = \lfloor \tau_\omega(\beta^R) \rfloor$, such an equilibrium exists and is responsive if and only if (1) $T_\omega^R \in [0, N-1]$ and (2) $\Phi_{\omega,T_\omega^R}(b) = 0$. Note that when $\omega_b \in (0,1)$, $\frac{1-\pi_B}{1-\pi_G} = \frac{\rho}{1-\rho}$ and according to (A3)

$$\Phi_{\omega,T_\omega^R}(b) = 0 \Leftrightarrow \frac{\Pr\left[T_{N,\omega} = T_\omega^R|B\right]}{\Pr[T_{N,\omega} = T_\omega^R|G]} = 1 \Leftrightarrow T_\omega^R = \frac{N\log\frac{\rho}{1-\rho}}{\log\frac{\pi_G}{\pi_B} + \log\frac{\rho}{1-\rho}}.$$

Since T_ω^R is an integer, this is only possible if the right-hand side is an integer. Since $\log \frac{\pi_G}{\pi_B}$ spans the interval $\left(\frac{1}{N}\log\frac{\beta^R}{1-\beta^R}, \frac{1}{N}\log\frac{\beta^P}{1-\beta^P}\right)$, the right-hand side can be equal to any integer $I \in \left(\frac{N\log\frac{\rho}{1-\rho}}{\frac{1}{N}\log\frac{\beta^P}{1-\beta^P}+\log\frac{\rho}{1-\rho}}, \frac{N\log\frac{\rho}{1-\rho}}{\frac{1}{N}\log\frac{\beta^R}{1-\beta^R}+\log\frac{\rho}{1-\rho}}\right) \equiv (I_1, I_2) \subseteq \left(\frac{N}{2}, N\right)$ for some ω_b in this range. Moreover, if $T_\omega^R \in (I_1, I_2)$, then the condition $T_\omega^R \in [0, N-1]$ (condition (1) above) is satisfied. Thus, this type of mixed strategy equilibria exists if and only if there exists a ω_b within this range such that both the left-hand side, T_ω^R, and the right-hand side are equal to some integer $I \in (I_1, I_2)$. Recall that $T_\omega^R = \left\lfloor \tau_\omega\left(\beta^R\right)\right\rfloor$ and $\tau_\omega\left(\beta^R\right)$ is given by (A2). Thus, when $\omega_g = 1$,

$$T_\omega^R = \left\lfloor \frac{N\log\frac{\rho}{1-\rho}}{\log\frac{\pi_G}{\pi_B}+\log\frac{\rho}{1-\rho}} + \frac{\log\frac{\beta^R}{1-\beta^R}}{\log\frac{\pi_G}{\pi_B}+\log\frac{\rho}{1-\rho}} \right\rfloor.$$

Since $\frac{N\log\frac{\rho}{1-\rho}}{\log\frac{\pi_G}{\pi_B}+\log\frac{\rho}{1-\rho}} = I = T_\omega^R$ and I is an integer, it must be that $\frac{\log\frac{\beta^R}{1-\beta^R}}{\log\frac{\pi_G}{\pi_B}+\log\frac{\rho}{1-\rho}} \in [0,1)$. Expressing $\log\frac{\pi_G}{\pi_B}$ through I, this is equivalent to the requirement that $\frac{I}{N}\frac{\log\frac{\beta^R}{1-\beta^R}}{\log\frac{\rho}{1-\rho}} \in [0,1)$ for some integer $I \in (I_1, I_2)$. Hence, a responsive equilibrium exists if and only if there exists $I \in (I_1, I_2)$ such that $\frac{I}{N}\frac{\log\frac{\beta^R}{1-\beta^R}}{\log\frac{\rho}{1-\rho}} \in [0,1)$, which is equivalent to $\log\frac{\beta^R}{1-\beta^R} \in \frac{N}{I}[0, \log\frac{\rho}{1-\rho})$. Clearly, the equivalent requirement is that the last inequality is satisfied for the lowest integer I in the interval (I_1, I_2), which is $\lfloor I_1 \rfloor + 1$. Because $\frac{\beta^R}{1-\beta^R} > 1$, it follows that $\log\frac{\beta^R}{1-\beta^R} > 0$. Because $\frac{\beta^R}{1-\beta^R} < \frac{\rho}{1-\rho}$ and $\lfloor I_1 \rfloor \leq I_1$, to show that $\log\frac{\beta^R}{1-\beta^R} < \frac{N\log\frac{\rho}{1-\rho}}{\lfloor I_1\rfloor+1}$ it is sufficient to show that $\log\frac{\rho}{1-\rho} < \frac{N\log\frac{\rho}{1-\rho}}{I_1+1} \Leftrightarrow I_1 < N-1$, where $I_1 = \frac{N\log\frac{\rho}{1-\rho}}{\frac{1}{N}\log\frac{\beta^P}{1-\beta^P}+\log\frac{\rho}{1-\rho}}$. Finally, because $(\frac{\rho}{1-\rho})^2 \leq \frac{\beta^P}{1-\beta^P}$, it is sufficient to show that $\frac{N\log\frac{\rho}{1-\rho}}{\frac{1}{N}\log\left(\frac{\rho}{1-\rho}\right)^2+\log\frac{\rho}{1-\rho}} < N-1 \Leftrightarrow N > 2$, which is always true in Case II.2(b). We conclude that there indeed exists a bad type mixing responsive equilibrium in which the manager always rejects the proposal and a proxy fight is initiated with strictly positive probability. ■

C. *Supplemental Proofs for Section III*

Proof of Proposition 2: Let $P(k_A, k_M, c_M, c_A, \omega)$ be the ex-ante probability that the proxy fight is organized in equilibrium. Recall from Lemmas B.1 and B.2 that $P > 0 \Leftrightarrow \left\lfloor \tau_\omega\left(\beta^R\right)\right\rfloor \equiv$

$T_\omega^R < T_\omega^P \equiv \lfloor \tau_\omega(\beta^P) \rfloor$. First, suppose that $P > 0$. Then P is given by

$$P = \Pr\left(T \in (T_\omega^R, T_\omega^P]\right) = 0.5 \sum_{T=T_\omega^R+1}^{T_\omega^P} \left[\Pr(T|G) + \Pr(T|B)\right].$$

Note that given the voting strategies $(\omega_b, \omega_g) = (0, 1)$, each of the terms

$$\Pr(T|G) + \Pr(T|B) = C_T^N \left[\pi_G^T (1-\pi_G)^{N-T} + \pi_B^T (1-\pi_B)^{N-T}\right]$$

only depends on N, T, and ρ. As shown in the proof of Proposition 1, T_ω^R weakly increases with k_A, while T_ω^P does not depend on k_A. Therefore, the terms $\Pr(T|G) + \Pr(T|B)$ in the sum remain constant but the number of terms weakly decreases with k_A, which implies that P weakly decreases with k_A. For the same reason, since T_ω^R weakly increases with c_A and T_ω^P does not depend on c_A, P weakly declines with c_A. Finally, since T_ω^P weakly declines with both k_M and c_M and T_ω^R does not depend on these parameters, P weakly declines with k_M and c_M. Hence, whenever $P > 0$, it weakly decreases with all four parameters.

It remains to show that P cannot increase from zero to positive probability as one of the parameters increases. Suppose that $P = 0$, which is satisfied if and only if $T_\omega^R \geq T_\omega^P$. As k_A or c_A increase, the left-hand side of the last inequality weakly increases and the right-hand side does not change, and hence the inequality continues to hold. Similarly, as k_M or c_M increase, the left-hand side does not change while the right-hand side weakly decreases, and hence P continues to be zero. Combining all the results, the probability of the proxy fight is weakly decreasing in k_M, k_A, c_A, and c_M. ■

D. Supplemental Proofs for Section IV

Proof of Lemma D.1: A shareholder with signal s votes affirmatively if and only if his posterior belief conditional on being pivotal, $\Pr(G|s, piv)$, satisfies

$$\Pr(G|s,piv) \geq \frac{1}{2} \Leftrightarrow \frac{1}{1 + \frac{1-\mu}{\mu}\frac{\Pr(s,piv|B)}{\Pr(s,piv|G)}} \geq \frac{1}{2} \Leftrightarrow (\frac{\pi_B}{\pi_G})^{\frac{N}{2}-1}(\frac{1-\pi_B}{1-\pi_G})^{\frac{N}{2}} \leq \frac{\mu}{1-\mu}\frac{\Pr(s|G)}{\Pr(s|B)},$$

since conditional on his being pivotal, there are $\frac{N}{2} - 1$ affirmative votes out of $N - 1$ total votes among the other shareholders. Note also that since $\rho > \frac{1}{2}$, we have that $\Pr(G|g, piv) > \Pr(G|b, piv)$. Using this inequality, a responsive equilibrium in pure strategies exists if and only if $\Pr(G|b, piv) \leq \frac{1}{2} \leq \Pr(G|g, piv) \Leftrightarrow 1 \leq \frac{\mu}{1-\mu} \leq (\frac{\rho}{1-\rho})^2$. Similarly, an equilibrium with $\omega_b = 0, \omega_g \in (0, 1]$ exists if and only if $\Pr(G|g, piv) = \frac{1}{2} \Leftrightarrow \omega_g = \frac{H-1}{H\rho+\rho-1}$, where $H = \left[\frac{\mu}{1-\mu}(\frac{\rho}{1-\rho})^{\frac{N}{2}}\right]^{\frac{2}{N}}$. It can be shown that $\frac{H-1}{H\rho+\rho-1} \in (0, 1] \Leftrightarrow (\frac{1-\rho}{\rho})^{\frac{N}{2}} < \frac{\mu}{1-\mu} \leq 1$. Finally, an equilibrium with $\omega_b \in [0,1), \omega_g = 1$ exists if and only if $\Pr(G|b, piv) = \frac{1}{2} \Leftrightarrow \omega_b = \frac{L\rho+\rho-1}{L\rho+\rho-L}$, where $L = \left[\frac{\mu}{1-\mu}(\frac{1-\rho}{\rho})^{\frac{N}{2}+1}\right]^{\frac{1}{\frac{N}{2}-1}}$. It can be shown that $\frac{L\rho+\rho-1}{L\rho+\rho-L} \in [0,1) \Leftrightarrow (\frac{\rho}{1-\rho})^2 \leq \frac{\mu}{1-\mu} < (\frac{\rho}{1-\rho})^{\frac{N}{2}+1}$. Combining the three cases, a responsive equilibrium exists if and only if $\frac{\mu}{1-\mu} \in ((\frac{1-\rho}{\rho})^{\frac{N}{2}}, (\frac{\rho}{1-\rho})^{\frac{N}{2}+1})$. ■

II. Extensions of the Model

A. Proposal Submission

The analysis in the main text ignores shareholders' decision of whether to submit a proposal to a vote. In this section we endogenize the process of proposal submission and demonstrate that it is the large shareholders who are more likely to submit proposals. Intuitively, only large shareholders have sufficient incentives to incur the various costs associated with the proposal submission.

A.1. Setup

Suppose there are two kinds of shareholders. There are $N-1$ small shareholders, each of which holds one share, and there is one large shareholder who holds $M \geq 1$ shares, where M is an integer. Each shareholder, regardless of his size, observes a private binary signal $s_i \in \{b, g\}$, as described in the basic model in Section I. All signals have the same precision ρ. We augment the basic model with an initial stage of proposal submission. In particular, once shareholders observe their private signals, each shareholder, large or small, can submit a proposal on which a nonbinding vote is held. Shareholders make their decisions to submit the proposal simultaneously, and the vote takes place if and only if at least one shareholder submits the proposal. Submitting a proposal, however, is costly. It requires time, effort, and legal advice. Therefore, submission of a proposal requires a personal cost $c > 0$. The cost is identical across all shareholders, large or small.[1]

If the proposal is submitted by at least one shareholder, a nonbinding vote is held as described in Section I. We assume that the manager can observe the vote of the large shareholder and of whoever submits the proposal. In addition, we focus attention on cases in which a responsive equilibrium at the voting stage exists; otherwise, not submitting a proposal is a strictly dominant strategy since the vote does not affect the final outcome. More specifically, we assume that $k_M > \frac{1}{2} + \frac{1}{2}(\frac{1-\rho}{\rho})^2$, which ensures that there always exists a responsive equilibrium in pure strategies (each shareholder votes affirmatively if and only if his signal is good).

[1] Shareholders incur the cost c by submitting a proposal even if the proposal is submitted by other shareholders as well.

Note that under this condition, there may also exist responsive equilibria in mixed strategies. However, we select the unique equilibrium in pure strategies, which is also the most informative of all responsive equilibria.[2]

A.2. Analysis

When shareholders consider whether to submit a proposal, they already observe their private information. Therefore, potentially, information could be revealed by the shareholder's decision of whether to submit a proposal. Due to signaling considerations, the game described above has multiple equilibria. We consider both symmetric and asymmetric equilibria at the submission stage, but to keep the analysis simple we focus our attention on pure strategy equilibria. In other words, for a given signal, each shareholder submits the proposal in equilibrium either with probability zero or with probability one. Finally, we focus on equilibria in which every shareholder whose private signal was revealed through his submission decision votes uninformatively at the voting stage (e.g., votes in favor of the proposal regardless of his private signal). Given uninformative voting, the manager optimally ignores the vote of such a shareholder, and given that the manager disregards his vote, it is optimal for this shareholder to vote uninformatively. Hence, these are indeed equilibrium strategies.

We denote by $x_s(m_g, m_b)$ the benefit per share from the proposal submission given the shareholder's own signal s and the information that $m_g \in \{0, 1, ..., N-1\}$ other shareholders have good signals while $m_b \in \{0, 1, ..., N-1\}$ other shareholders have bad signals, where $m_g + m_b \in \{0, 1, ..., N-1\}$. More specifically, $x_s(m_g, m_b)$ is the benefit in the absence of signaling considerations on the part of the shareholder, that is, his benefit from submission when the manager correctly infers his true signal s. In the proofs we show that the benefit from submitting the proposal increases with m_g, decreases with m_b, and is higher for shareholders with a good signal than for those with a bad signal. These results are intuitive given that the proposal adds value if and only if the state is good. Nevertheless, it is worth mentioning that the benefit from proposal submission stems partly from aggregating the dispersed information held by all shareholders of the company. Therefore, a priori, the benefit from proposal submission could be positive even if the shareholder observes a bad signal.

To rule out the situation in which even a completely uninformed small shareholder might have an incentive to submit the proposal to a vote, we make the following assumption. We assume that if no information on the value of the proposal is available, then the benefit per share from proposal submission does not outweigh the cost of submission.

ASSUMPTION IA.1: *The ex-ante benefit per share from the nonbinding vote is smaller than the cost of submission:*

$$\bar{x} \equiv \sum_{s=g,b} \Pr[s]\, x_s(0,0) < c.$$

[2] As in the main paper, for simplicity, we focus on the case when N is even.

We demonstrate in the proofs that if Assumption IA.1 holds, there are only three kinds of pure strategy equilibria at the submission stage: equilibria in which the proposal is never submitted, equilibria in which a single (either small or large) shareholder submits the proposal with positive probability, and equilibria in which one small shareholder submits the proposal if and only if his signal is good and the blockholder submits the proposal if and only if his signal is bad. Next we find conditions on M (the size of the large shareholder) under which each of these equilibria exist. The following proposition provides the characterization of the extended game.

PROPOSITION IA.1: *Suppose Assumption IA.1 holds. There exist* $M_3 \geq M_2 > M_1 > 0$ *such that:*

(*i*) *If* $M < M_1$*, there exists a unique equilibrium and in this equilibrium the proposal is never submitted by any shareholder.*

(*ii*) *If* $M \geq M_1$*, there exits an equilibrium in which the blockholder submits the proposal with strictly positive probability.*

(*iii*) *If* $M > M_2$*, then in any equilibrium the blockholder submits the proposal with strictly positive probability.*

(*iv*) *If* $M > M_3$*, there is no equilibrium in which at least one small shareholder submits the proposal with strictly positive probability.*

An immediate corollary follows from the above proposition.

COROLLARY IA.1: *If* $M > M_3$*, then in any equilibrium the large shareholder submits the proposal with strictly positive probability and small shareholders never submit the proposal.*

Intuitively, when the large shareholder has sufficient holdings in the firm, he has enough incentives to submit the proposal and benefit from the information aggregation during the nonbinding vote. At the same time, small shareholders free ride on the large shareholder and thereby save the cost of submission. Note, however, that we need the size of the blockholder to be sufficiently large to rule out equilibria whereby small shareholders submit the proposal with positive probability. Indeed, as follows from the proof of Proposition IA.1, if $M < M_2$ and $x_b(0,0) > c$, there exists an equilibrium in which a single small shareholder submits the proposal if and only if his signal is good, and all other shareholders never submit it.

The analysis of the model also suggests that the larger the size of the blockholder's stake, the less information is revealed through his decision to submit the proposal to a vote. Indeed, when the blockholder's stake is not very large, he finds submission beneficial only if his private signal about the value of the proposal is positive. However, when the blockholder's stake is sufficiently large, then submitting the proposal is profitable even if his private signal is negative. This is because the nonbinding vote efficiently aggregates other shareholders' information and thereby helps more informative decision making by the manager. As a result, submission of the proposal conveys positive news about the blockholder's signal when his stake is small, and does not convey any information when his stake is large. Formally,

COROLLARY IA.2: *There exist M^* and M^{**} with $M^{**} \geq M_3 \geq M^* > M_1$ such that if $M > M^{**}$, then there exists a unique equilibrium and in this equilibrium the blockholder submits the proposal with probability one. If $M^* > M > M_1$, then there is no equilibrium in which the blockholder submits the proposal with probability one, but there exists an equilibrium in which the blockholder submits the proposal with strictly positive probability smaller than one.*

Finally, note that when $M < M_1$, the proposal is never submitted by any shareholder even though there may be significant benefit from information aggregation during the vote if the number of shareholders is large. Thus, the collective action problem among shareholders may result in inefficiency when the proposal is not submitted although it is optimal for the vote to take place from the shareholders' aggregate point of view.

A.3. Proofs

To prove Proposition IA.1, we first prove the following claims:

IA.1 For any (m_g, m_b) as defined above,

(i) $x_g(m_g, m_b) > x_b(m_g, m_b)$, and

(ii) $x_s(m_g + 1, m_b) > x_s(m_g, m_b) > x_s(m_g, m_b + 1)$.

IA.2 Suppose that in a pure strategy equilibrium more than one shareholder submits a proposal with positive probability. Then no shareholder submits the proposal with probability one.

IA.3 There is no pure strategy equilibrium in which there is a small shareholder who submits the proposal if and only if his signal is bad.

IA.4 There is no equilibrium in which at least two shareholders submit the proposal if and only if their signal is good and all other shareholders do not submit the proposal.

IA.5 There is no equilibrium in which the blockholder submits the proposal if and only if his signal is bad, at least two small shareholders submit the proposal if and only if their signal is good, and all other small shareholders do not submit the proposal. If $x_g(0, 0) < c$, then there is also no equilibrium of this type in which only one small shareholder submits the proposal if and only if his signal is good and all other small shareholders do not submit the proposal.

As follows from these claims, there are only three kinds of pure strategy equilibria at the submission stage: equilibria in which one small shareholder submits the proposal if and only if his signal is good and the blockholder submits the proposal if and only if his signal is bad (which exists only if $x_g(0,0) \geq c$), equilibria in which a single (small or large) shareholder submits the proposal with positive probability, and equilibria in which the proposal is never submitted. Proposition IA.1 then demonstrates under which conditions on the size of the large shareholder each of these equilibria exist.

CLAIM IA.1: *For any* (m_g, m_b) *as defined above,* (i) $x_g(m_g, m_b) > x_b(m_g, m_b)$ and (ii) $x_s(m_g+1, m_b) > x_s(m_g, m_b) > x_s(m_g, m_b+1)$.

Proof of Claim IA.1: First, note that under the conditions of Theorem 1, there always exists a responsive equilibrium at the voting stage in which information is fully revealed. We focus on this equilibrium, which implies that shareholders whose information was not revealed by their submission decision will vote according to their signals. Moreover, in this equilibrium the endogenous majority rule T^* equals $N/2$ (so the proposal is accepted if and only if the number of good signals is $N/2+1$ or more).[3] Second, once m_g and m_b are revealed through the submission decisions, the posterior belief that the state is good of a shareholder with signal s is given by

$$\mu_s \equiv \Pr[G|m_g, m_b, s] = \begin{cases} \frac{1}{1+(\frac{1-\rho}{\rho})^{m_g-m_b+1}} & \text{if } s=g, \\ \frac{1}{1+(\frac{1-\rho}{\rho})^{m_g-m_b-1}} & \text{if } s=b. \end{cases} \quad \text{(IA.6)}$$

Moreover, if the proposal is submitted, the manager will accept it if and only if the number of good signals out of the remaining unknown $N-m_g-m_b-1$ signals together with the shareholder's own signal is at least T^*+1-m_g. Let $T_{N-m_g-m_b-1}$ be the number of good signals held by shareholders whose information the shareholder does not know. Then,

$$x_s(m_g, m_b) = \begin{cases} \mu_g \Pr\left[T_{N-m_g-m_b-1} + m_g \geq T^*|G\right] \\ -\left(1-\mu_g\right)\Pr\left[T_{N-m_g-m_b-1} + m_g \geq T^*|B\right] & \text{if } s=g, \\ \mu_b \Pr\left[T_{N-m_g-m_b-1} + m_g \geq T^*+1|G\right] \\ -\left(1-\mu_b\right)\Pr\left[T_{N-m_g-m_b-1} + m_g \geq T^*+1|B\right] & \text{if } s=b. \end{cases} \quad \text{(IA.7)}$$

This is because a shareholder with a good signal knows that if his signal is not revealed by his submission decision, then he will vote for the proposal, so that the manager will correctly infer his good signal in any case. Hence, only T^* affirmative votes among the other $N-1$ shareholders

[3] It can be verified from the proof of Theorem 1 for the pure strategy responsive equilibrium (we rule out $T^*_\omega = \frac{N}{2} - 1$ since $\beta^* \geq \frac{1}{2}$).

will be needed to induce the manager to accept the proposal. In contrast, a shareholder with a bad signal knows that the manager will always correctly infer his bad signal, and hence T^*+1 affirmative votes will be needed. To prove that $x_g(m_g,m_b) > x_b(m_g,m_b)$, note that

$$\begin{aligned} x_g(m_g,m_b) - x_b(m_g,m_b) &= \mu_g \Pr\left[T_{N-m_g-m_b-1} + m_g \geq T^* + 1|G\right] \\ &\quad - \left(1-\mu_g\right)\Pr\left[T_{N-m_g-m_b-1} + m_g \geq T^* + 1|B\right] \\ &\quad -\mu_b \Pr\left[T_{N-m_g-m_b-1} + m_g \geq T^* + 1|G\right] \\ &\quad + \left(1-\mu_b\right)\Pr\left[T_{N-m_g-m_b-1} + m_g \geq T^* + 1|B\right] \\ &\quad +\mu_g \Pr\left[T_{N-m_g-m_b-1} + m_g = T^*|G\right] \\ &\quad - \left(1-\mu_g\right)\Pr\left[T_{N-m_g-m_b-1} + m_g = T^*|B\right]. \end{aligned}$$

Because $-\left(1-\mu_g\right) + \left(1-\mu_b\right) = \mu_g - \mu_b$, by rearranging terms we get

$$\begin{aligned} x_g(m_g,m_b) - x_b(m_g,m_b) &= \left(\mu_g - \mu_b\right)\begin{pmatrix} \Pr\left[T_{N-m_g-m_b-1} + m_g \geq T^* + 1|G\right] \\ +\Pr\left[T_{N-m_g-m_b-1} + m_g \geq T^* + 1|B\right] \end{pmatrix} \\ &\quad +\mu_g \Pr\left[T_{N-m_g-m_b-1} + m_g = T^*|G\right] \\ &\quad - \left(1-\mu_g\right)\Pr\left[T_{N-m_g-m_b-1} + m_g = T^*|B\right]. \end{aligned}$$

Note that

$$\begin{aligned} \mu_g \Pr\left[T_{N-m_g-m_b-1} + m_g = T^*|G\right] &> \left(1-\mu_g\right)\Pr\left[T_{N-m_g-m_b-1} + m_g = T^*|B\right] \Leftrightarrow \\ \frac{\mu_g}{1-\mu_g} &> \frac{\Pr\left[T_{N-m_g-m_b-1} + m_g = T^*|B\right]}{\Pr\left[T_{N-m_g-m_b-1} + m_g = T^*|G\right]} \Leftrightarrow \\ \frac{1}{\left(\frac{1-\rho}{\rho}\right)^{m_g-m_b+1}} &> \frac{\left(1-\rho\right)^{T^*-m_g}\left(\rho\right)^{N-1-m_g-m_b-(T^*-m_g)}}{\rho^{T^*-m_g}\left(1-\rho\right)^{N-1-m_g-m_b-(T^*-m_g)}} \Leftrightarrow \\ T^* &> N/2-1. \end{aligned}$$

Thus, since $\mu_g > \mu_b$, we have that $x_g(m_g,m_b) > x_b(m_g,m_b)$, which proves part (i) of Claim IA.1.

To see part (ii) of Claim IA.1, note that the benefit from submission given $m = m_g + m_b$ signals is the expected benefit from submission given $m+1$ signals conditional on m_g, m_b. Hence, by isolating the unknown signal $\hat{s}$ of another shareholder, we get

$$x_s(m_g,m_b) = \Pr(\hat{s} = g|s,m_g,m_b)\, x_s(m_g+1,m_b) + \Pr(\hat{s} = b|s,m_g,m_b)\, x_s(m_g,m_b+1).$$

In other words, $x_s(m_g,m_b)$ is a weighted average of $x_s(m_g,m_b+1)$ and $x_s(m_g+1,m_b)$ with weights $\Pr(\hat{s} = g|s,m_g,m_b)$ and $1-\Pr(\hat{s} = g|s,m_g,m_b)$ strictly within the unit interval. Thus, it remains to show that $x_s(m_g+1,m_b) > x_s(m_g,m_b+1)$. Note that $x_b(m_g+1,m_b) =$

$x_g(m_g, m_b+1)$. This is because in both sides of the equality, the shareholder knows that there are overall m_b+1 bad signals and m_g+1 good signals. Given the first part of the claim, we get

$$x_g(m_g+1, m_b) > x_b(m_g+1, m_b) = x_g(m_g, m_b+1) > x_b(m_g, m_b+1),$$

and hence for $s \in \{b, g\}$ we have $x_s(m_g+1, m_b) > x_s(m_g, m_b+1)$ as required. ■

CLAIM IA.2: *Suppose that in a pure strategy equilibrium more than one shareholder submits a proposal with positive probability. Then no shareholder submits the proposal with probability one.*

Proof of Claim IA.2: Suppose by way of contradiction that there is an equilibrium in which one shareholder submits the proposal with probability one and at least one other shareholder submits the proposal with positive probability. There are four cases to consider:
Case 1: The second shareholder submits the proposal if and only if his signal is bad. Let us show that this shareholder has a strictly profitable deviation of not submitting the proposal when his signal is bad. The benefit from deviating is c. Since the proposal is always submitted in equilibrium by the first shareholder, the only cost from deviating is that his signal is misinterpreted by the manager, which may lead to inefficient decision making. In particular, if he deviates from his equilibrium strategy, his bad signal is interpreted as a good signal, and hence when (and only when) there are $T^* = N/2$ good signals among the other $N-1$ shareholders, the proposal is accepted even though it should be rejected. However, combined with the shareholder's bad signal, in this scenario there are exactly $N/2$ good signals out of N signals in the economy and hence the shareholder is indifferent between approval and rejection of the proposal. Therefore, the cost of misinterpretation for a shareholder with a bad signal is exactly zero and deviation is always beneficial.
Case 2: The second shareholder is small and submits the proposal if and only if his signal is good. We show that this shareholder has a strictly profitable deviation of not submitting the proposal if his signal is good. The benefit of deviating is c. Since the proposal is always submitted in equilibrium by the first shareholder, the only cost from deviating is that his good signal is misinterpreted by the manager as bad and the proposal can be overrejected. Note that this misinterpretation only matters for the final decision when there are exactly $N/2$ good signals among the other $N-1$ shareholders and hence the proposal is rejected though it should be accepted. The following two steps demonstrate that deviation is optimal:

- Step 1 - calculating the shareholder's expected loss when his good signal is misinterpreted to be bad: The loss occurs only when there are exactly $N/2$ good signals among the other $N-1$ shareholders. Given that the shareholder has a good signal, the probability of this event is given by

$$\begin{aligned}
& \Pr[G|s=g]\, C_{N/2}^{N-1} \rho^{N/2} (1-\rho)^{N-1-N/2} + \Pr[B|s=g]\, C_{N/2}^{N-1} (1-\rho)^{N/2} \rho^{N-1-N/2} \\
= \; & C_{N/2}^{N-1} [\rho(1-\rho)]^{N/2-1} \left[\rho^2 + (1-\rho)^2\right].
\end{aligned}$$

The loss from rejecting the proposal when there are exactly $N/2+1$ good signals out of N is given by

$$\begin{aligned}
\Pr[G|N/2+1] - \Pr[B|N/2+1] &= 2\Pr[G|N/2+1] - 1 \\
&= \frac{2}{1+(\frac{1-\rho}{\rho})^2} - 1 \\
&= \frac{2\rho - 1}{\rho^2 + (1-\rho)^2} > 0.
\end{aligned}$$

Overall, the expected loss from deviation is

$$\begin{aligned}
L &\equiv C_{N/2}^{N-1} [\rho(1-\rho)]^{N/2-1} \left[\rho^2 + (1-\rho)^2\right] \frac{2\rho-1}{\rho^2+(1-\rho)^2} \\
&= C_{N/2}^{N-1} [\rho(1-\rho)]^{N/2-1} (2\rho - 1) > 0.
\end{aligned}$$

Hence, to show that deviation is beneficial, we need to show that

$$c - L > 0 \Leftrightarrow C_{N/2}^{N-1} [\rho(1-\rho)]^{N/2-1} (2\rho-1) < c.$$

- Step 2 - showing that $L < \bar{x}$: Recall that according to Assumption IA.1, $\bar{x} < c$, where $\bar{x}$ is the expected benefit from submitting the proposal without any private information. Hence, it is sufficient to show that $L < \bar{x}$. Because decision making is based on efficient aggregation of signals ($T^* = N/2$), the proposal is accepted upon submission when the number of good signals is at least $N/2+1$. For any number of good signals above $N/2+1$, the benefit from the proposal approval is higher than the benefit when there are exactly $N/2+1$ good signals, which equals $\frac{2\rho-1}{\rho^2+(1-\rho)^2}$ as calculated above. Hence,

$\bar{x} \geq \frac{2\rho-1}{\rho^2+(1-\rho)^2} \sum_{t=N/2+1}^{N} \Pr[t]$, where t is the number of good signals. Note that

$$\begin{aligned}
\sum_{t=N/2+1}^{N} \Pr[t] &= \frac{1}{2} \sum_{t=N/2+1}^{N} C_t^N \left[\rho^t (1-\rho)^{N-t} + \rho^{N-t} (1-\rho)^t\right] \\
&= \frac{1}{2} \left(\sum_{t=N/2+1}^{N} C_t^N \left[\rho^t (1-\rho)^{N-t}\right] + 1 - \left(1 - \sum_{t=N/2+1}^{N} C_t^N \left[\rho^{N-t} (1-\rho)^t\right]\right)\right) \\
&= \frac{1}{2} \left(\sum_{t=N/2+1}^{N} C_t^N \left[\rho^t (1-\rho)^{N-t}\right] + 1 - \sum_{t=0}^{N/2} C_t^N \left[\rho^{N-t} (1-\rho)^t\right]\right) \\
&= \frac{1}{2} \left(1 - C_{N/2}^N \left[\rho(1-\rho)\right]^{N/2}\right) = \frac{1}{2} - C_{N/2}^{N-1} \left[\rho(1-\rho)\right]^{N/2},
\end{aligned}$$

and hence it is sufficient to show that

$$\begin{aligned}
L &< \left[\frac{1}{2} - C_{N/2}^{N-1} \left[\rho(1-\rho)\right]^{N/2}\right] \frac{2\rho-1}{\rho^2+(1-\rho)^2} \Leftrightarrow \\
\frac{1}{2} &> C_{N/2}^{N-1} \left[\rho(1-\rho)\right]^{N/2-1} \left[1-\rho(1-\rho)\right].
\end{aligned}$$

Letting $y = \rho(1-\rho) \in \left[0, \frac{1}{4}\right]$, it is sufficient to show that

$$\frac{1}{2} > C_{N/2}^{N-1} \max_{y \in \left[0, \frac{1}{4}\right]} \left\{y^{N/2-1} - y^{N/2}\right\}.$$

One can show that for any $N \geq 3$, the maximum is achieved at $y = 1/4$. Hence, we need to show that $2^{N-1} > 3C_{N/2}^{N-1}$. Note that $2^{N-1} = \sum_{i=0}^{N-1} C_i^{N-1}$ and

$$\begin{aligned}
C_{N/2}^{N-1} &= C_{N/2-1}^{N-1} = \frac{(N-1)!}{(N/2-1)!\,(N/2)!} \\
C_{N/2-2}^{N-1} &= C_{N/2+1}^{N-1} = \frac{(N-1)!}{(N/2+1)!\,(N/2-2)!}.
\end{aligned}$$

Since $2^{N-1} > C_{N/2}^{N-1} + C_{N/2-1}^{N-1} + C_{N/2-2}^{N-1} + C_{N/2+1}^{N-1} = 2C_{N/2}^{N-1} + 2C_{N/2-2}^{N-1}$, it is sufficient to show that $C_{N/2}^{N-1} < 2C_{N/2-2}^{N-1}$, which always holds for $N > 6$.

Case 3: The second shareholder submits the proposal regardless of his signal. Since there is only one large shareholder, there is at least one small shareholder who submits the proposal with probability one. By deviating and not submitting the proposal, the shareholder gains c. Deviation from submission, however, becomes an off-equilibrium event. Because the shareholder wants efficient information aggregation, he incurs the maximum possible costs of deviation if his signal is misinterpreted. However, as was argued in the first and second cases, the cost from misinterpretation of the signal is strictly smaller than c for both a good and a bad signal. Hence, it is always optimal to deviate under any off-equilibrium beliefs.

Case 4: The second shareholder is large and submits the proposal if and only if his signal is good. Since the second shareholder is the large shareholder, then it must be the case that a small shareholder (the first one) submits the proposal with probability one. We will show that the small shareholder has incentives to deviate and not submit the proposal if he has a bad signal. First, note that there is no other small shareholder who submits the proposal with positive probability. This follows from the above three cases. Suppose that the small shareholder has a bad signal. If the small shareholder submits the proposal, he gets $x_b(0,0) - c < \bar{x} - c$, which by Assumption IA.1 is negative. If the shareholder deviates and does not submit the proposal, then the following two scenarios are possible. If the large shareholder has a bad signal as well, the proposal is not submitted and the payoff is zero. If the large shareholder has a good signal, the proposal is submitted. The worst-case scenario in terms of off-equilibrium beliefs is that the small shareholder's signal is interpreted as good. But, repeating the argument in Case 1, the loss from the resulting misinterpretation is zero. Hence, the payoff in this scenario is nonnegative. Combining the two scenarios, deviation gives a nonnegative payoff and hence deviation is optimal.

■

CLAIM IA.3: *There is no pure strategy equilibrium in which there is a small shareholder who submits the proposal if and only if his signal is bad.*

Proof of Claim IA.3: Let $m_s \geq 0$ be the number of shareholders who submit the proposal if and only if they observe signal s in equilibrium. Suppose by way of contradiction that at least one of the m_b shareholders is small. The relative benefit of the small shareholder with a bad signal from submitting the proposal relative to not submitting it is given by

$$\Pr\left(T_{m_b+m_g-1} = m_b - 1|b\right) x_b\left(m_b - 1, m_g\right) - c, \tag{IA.8}$$

where $\Pr\left(T_{m_b+m_g-1} = m_b - 1|b\right)$ is the probability that the other $m_b + m_g - 1$ shareholders who are supposed to submit the proposal with positive probability in equilibrium do not submit the proposal (this is because similar to Case 1 in the proof of Claim IA.2, shareholders with a bad signal have no cost of misinterpretation of their signal if they decide not to submit the proposal as they are supposed to). For $m_b = 1$ we get

$$\Pr\left(T_{m_g} = 0|b\right) x_b\left(0, m_g\right) \leq x_b\left(0, m_g\right) \leq x_b\left(0, 0\right) < \bar{x} < c.$$

Hence, if we show that (IA.8) decreases in m_b, the claim holds for any m_b, m_g because the small shareholder has incentives to deviate and not submit the proposal. To see that (IA.8) decreases in m_b note that

$$\begin{aligned}\Pr\left(T_{m_b+m_g-1} = m_b - 1|b\right) &= (1-\rho)\,\rho^{m_b-1}(1-\rho)^{m_g} + \rho\,(1-\rho)^{m_b-1}\rho^{m_g} \\ &= \rho^{m_b-1}(1-\rho)^{m_g+1} + (1-\rho)^{m_b-1}\rho^{m_g+1} \\ &= [(1-\rho)\,\rho]^{m_g+1}\left[\rho^{m_b-m_g-2} + (1-\rho)^{m_b-m_g-2}\right].\end{aligned}$$

Let

$$f(m_b) \equiv \Pr\left(T_{m_b+m_g-1} = m_b - 1|b\right) x_b(m_b - 1, m_g) / [(1-\rho)\,\rho]^{m_g+1}.$$

Given (IA.6) and (IA.7), it can be shown that

$$\begin{aligned}f(m_b) &= \rho^{m_b-2-m_g}\Pr\left[T_{N-m_b-m_g} \geq N/2 - m_b + 2|G\right] \\ &\quad - (1-\rho)^{m_b-2-m_g}\Pr\left[T_{N-m_b-m_g} \geq N/2 - m_b + 2|B\right] \\ &= \rho^{m_b-2-m_g}\sum_{i=N/2-m_b+2}^{N-m_b-m_g} C_i^{N-m_b-m_g}\rho^i(1-\rho)^{N-m_b-m_g-i} \\ &\quad - (1-\rho)^{m_b-2-m_g}\sum_{i=N/2-m_b+2}^{N-m_b-m_g} C_i^{N-m_b-m_g}(1-\rho)^i\rho^{N-m_b-m_g-i} \\ &= \sum_{i=N/2-m_b+2}^{N-m_b-m_g} C_i^{N-m_b-m_g}\left[\begin{array}{c}\rho^{i+m_b-2-m_g}(1-\rho)^{N-m_b-m_g-i} \\ -(1-\rho)^{i+m_b-2-m_g}\rho^{N-m_b-m_g-i}\end{array}\right].\end{aligned}$$

It remains to prove that $f(m_b) \geq f(m_b+1)$. Note that if $m_b \geq N/2+2$, then $f(m_b) = \rho^{m_b-2-m_g} - (1-\rho)^{m_b-2-m_g}$ and $f(m_b) \geq f(m_b+1) \Leftrightarrow m_b - m_g \geq 3$, which holds if $m_b \geq N/2+2$. If $m_b < N/2+2$, then $f(m_b) \geq f(m_b+1)$ if and only if

$$\begin{aligned}&\sum_{i=N/2-m_b+2}^{N-m_b-m_g} C_i^{N-m_b-m_g}\left[\begin{array}{c}\rho^{i+m_b-2-m_g}(1-\rho)^{N-m_b-m_g-i} \\ -(1-\rho)^{i+m_b-2-m_g}\rho^{N-m_b-m_g-i}\end{array}\right] \geq \\ &\sum_{i=N/2-m_b-1+2}^{N-m_b-1-m_g} C_i^{N-m_b-1-m_g}\left[\begin{array}{c}\rho^{i+1+m_b-2-m_g}(1-\rho)^{N-m_b-m_g-i-1} \\ -(1-\rho)^{i+1+m_b-2-m_g}\rho^{N-m_b-m_g-i-1}\end{array}\right].\end{aligned}$$

Note that

$$\begin{aligned}&\sum_{i=N/2-m_b-1+2}^{N-m_b-1-m_g} C_i^{N-m_b-1-m_g}\left[\begin{array}{c}\rho^{i+1+m_b-2-m_g}(1-\rho)^{N-m_b-m_g-i-1} \\ -(1-\rho)^{i+1+m_b-2-m_g}\rho^{N-m_b-m_g-i-1}\end{array}\right] \\ = &\sum_{i=N/2-m_b+2}^{N-m_b-m_g} C_{i-1}^{N-m_b-1-m_g}\left[\begin{array}{c}\rho^{i+m_b-2-m_g}(1-\rho)^{N-m_b-m_g-i} \\ -(1-\rho)^{i+m_b-2-m_g}\rho^{N-m_b-m_g-i}\end{array}\right].\end{aligned}$$

Hence, $f(m_b) \geq f(m_b+1)$ if and only if

$$\sum_{i=N/2-m_b+2}^{N-m_b-m_g} \left[C_i^{N-m_b-m_g} - C_{i-1}^{N-m_b-1-m_g} \right] \left[\begin{array}{c} \rho^{i+m_b-2-m_g} (1-\rho)^{N-m_b-m_g-i} \\ -(1-\rho)^{i+m_b-2-m_g} \rho^{N-m_b-m_g-i} \end{array} \right] \geq 0.$$

Since $C_i^{N-m_b-m_g} = C_i^{N-m_b-m_g-1} + C_{i-1}^{N-m_b-m_g-1}$, then $f(m_b) \geq f(m_b+1)$ if and only if

$$\sum_{i=N/2-m_b+2}^{N-m_b-m_g-1} C_i^{N-m_b-m_g-1} \left[\begin{array}{c} \rho^{i+m_b-2-m_g} (1-\rho)^{N-m_b-m_g-i} \\ -(1-\rho)^{i+m_b-2-m_g} \rho^{N-m_b-m_g-i} \end{array} \right] \geq 0.$$

Note that each component in the summation is positive because

$$\rho^{i+m_b-2-m_g} (1-\rho)^{N-m_b-m_g-i} - (1-\rho)^{i+m_b-2-m_g} \rho^{N-m_b-m_g-i} > 0 \Leftrightarrow i > N/2 - m_b + 1.$$

Therefore, $f(m_b) \geq f(m_b+1)$ indeed holds. ■

CLAIM IA.4: *There is no equilibrium in which at least two shareholders submit the proposal if and only if their signal is good and all other shareholders do not submit the proposal.*

Proof of Claim IA.4: Consider an equilibrium in which there are $m \geq 2$ shareholders who submit the proposal if and only if their signal is good and all other shareholders do not submit the proposal. There is at least one small shareholder among m. Consider his decision to submit the proposal given a good signal. Let $\lambda(m)$ be the probability that the other $m-1$ shareholders have a bad signal. The benefit of a small shareholder from submission is

$$X(m) \equiv \lambda(m) x_g(0, m-1) + (1-\lambda(m)) L(m) - c,$$

where $L(m) > 0$ is the cost that the manager misinterprets the shareholder's good signal as bad, conditional on the shareholder having a good signal and knowing that out of the $m-1$ shareholders, at least one of them has a good signal as well. Recall that the cost of misinterpretation is incurred if and only if there are exactly $N/2$ good signals among the other $N-1$ shareholders. A direct calculation implies that

$$\begin{aligned} X(m) &= \Pr[G|g] \left(\begin{array}{c} \Pr[T_{m-1} = 0|G] \Pr[T_{N-m} \geq N/2|G] \\ + \sum_{i=1}^{m-1} \Pr[T_{m-1} = i|G] \Pr[T_{N-m} = N/2 - i|G] \end{array} \right) \\ &\quad - \Pr[B|g] \left(\begin{array}{c} \Pr[T_{m-1} = 0|B] \Pr[T_{N-m} \geq N/2|B] \\ + \sum_{i=1}^{m-1} \Pr[T_{m-1} = i|B] \Pr[T_{N-m} = N/2 - i|B] \end{array} \right). \end{aligned}$$

Rearranging,

$$
\begin{aligned}
X(m) &= \rho\left(\begin{array}{c}(1-\rho)^{m-1}\sum_{i=N/2}^{N-m}C_i^{N-m}\rho^i(1-\rho)^{N-m-i}\\ +\sum_{i=1}^{m-1}\left[C_i^{m-1}\rho^i(1-\rho)^{m-1-i}\right]\left[C_{N/2-i}^{N-m}\rho^{N/2-i}(1-\rho)^{N-m-(N/2-i)}\right]\end{array}\right)\\
&\quad-(1-\rho)\left(\begin{array}{c}\rho^{m-1}\sum_{i=N/2}^{N-m}C_i^{N-m}(1-\rho)^i\rho^{N-m-i}\\ +\sum_{i=1}^{m-1}\left[C_i^{m-1}(1-\rho)^i\rho^{m-1-i}\right]\left[C_{N/2-i}^{N-m}(1-\rho)^{N/2-i}\rho^{N-m-(N/2-i)}\right]\end{array}\right)\\
&= \left(\sum_{i=N/2}^{N-m}C_i^{N-m}\rho^{i+1}(1-\rho)^{N-1-i}+\rho^{N/2+1}(1-\rho)^{N/2-1}\sum_{i=1}^{m-1}C_i^{m-1}C_{N/2-i}^{N-m}\right)\\
&\quad-\left(\sum_{i=N/2}^{N-m}C_i^{N-m}(1-\rho)^{i+1}\rho^{N-1-i}+(1-\rho)^{N/2+1}\rho^{N/2-1}\sum_{i=1}^{m-1}C_i^{m-1}C_{N/2-i}^{N-m}\right)\\
&= \sum_{i=N/2}^{N-m}C_i^{N-m}\left[\rho^{i+1}(1-\rho)^{N-1-i}-(1-\rho)^{i+1}\rho^{N-1-i}\right]\\
&\quad+\left[\rho^{N/2+1}(1-\rho)^{N/2-1}-(1-\rho)^{N/2+1}\rho^{N/2-1}\right]\sum_{i=1}^{m-1}C_i^{m-1}C_{N/2-i}^{N-m}\\
&= \sum_{i=N/2+1}^{N-m}C_i^{N-m}\left[\rho^{i+1}(1-\rho)^{N-1-i}-(1-\rho)^{i+1}\rho^{N-1-i}\right]\\
&\quad+(2\rho-1)\left[\rho(1-\rho)\right]^{N/2-1}\left(C_{N/2}^{N-m}+\sum_{i=1}^{m-1}C_i^{m-1}C_{N/2-i}^{N-m}\right),
\end{aligned}
$$

where the derivation above holds for any $m \leq N$. Note that by Vandermonde's identity, $\sum_{i=0}^{N/2} C_i^{m-1}C_{N/2-i}^{N-m} = C_{N/2}^{N-1}$, and since $C_i^{m-1} = 0$ for $i > m-1$, $\sum_{i=1}^{m-1} C_i^{m-1}C_{N/2-i}^{N-m} = C_{N/2}^{N-1} - C_{N/2}^{N-m}$. Therefore,

$$X(m) = \sum_{i=N/2+1}^{N-m} C_i^{N-m}\left[\rho^{i+1}(1-\rho)^{N-1-i} - (1-\rho)^{i+1}\rho^{N-1-i}\right] + (2\rho-1)\left[\rho(1-\rho)\right]^{N/2-1} C_{N/2}^{N-1}.$$

Note that since $\rho^{i+1}(1-\rho)^{N-1-i} - (1-\rho)^{i+1}\rho^{N-1-i} \geq 0$ for $i \geq N/2+1$ and $C_i^{N-m} \geq C_i^{N-m-1}$, $X(m)$ is decreasing in m. Therefore, to prove that the small shareholder wants to deviate and not submit the proposal, it is sufficient to show that $X(2) < c$. Note that

$$X(2) = \sum_{i=N/2+1}^{N-2} C_i^{N-2}\left[\rho^{i+1}(1-\rho)^{N-1-i} - (1-\rho)^{i+1}\rho^{N-1-i}\right] + (2\rho-1)\left[\rho(1-\rho)\right]^{N/2-1} C_{N/2}^{N-1}.$$

Recall that

$$\bar{x} = \frac{1}{2} \sum_{i=N/2+1}^{N} C_i^N \left[\rho^i (1-\rho)^{N-i} - (1-\rho)^i \rho^{N-i} \right] < c.$$

Thus, it is sufficient to show that $X(2) \leq \bar{x}$. Note that $C_i^N = C_{i-2}^{N-2} + 2C_{i-1}^{N-2} + C_i^{N-2}$ and hence

$$\begin{aligned}
\bar{x} &= \sum_{i=N/2+1}^{N} C_{i-1}^{N-2} \left[\rho^i (1-\rho)^{N-i} - (1-\rho)^i \rho^{N-i} \right] + \frac{1}{2} \sum_{i=N/2+1}^{N} C_{i-2}^{N-2} \left[\rho^i (1-\rho)^{N-i} - (1-\rho)^i \rho^{N-i} \right] \\
&\quad + \frac{1}{2} \sum_{i=N/2+1}^{N} C_i^{N-2} \left[\rho^i (1-\rho)^{N-i} - (1-\rho)^i \rho^{N-i} \right].
\end{aligned}$$

Consider the first term and let $j = i - 1$. Then

$$\begin{aligned}
\bar{x} &= \sum_{j=N/2}^{N-1} C_j^{N-2} \left[\rho^{j+1} (1-\rho)^{N-j-1} - (1-\rho)^{j+1} \rho^{N-j-1} \right] \\
&\quad + \frac{1}{2} \sum_{i=N/2+1}^{N} C_{i-2}^{N-2} \left[\rho^i (1-\rho)^{N-i} - (1-\rho)^i \rho^{N-i} \right] + \frac{1}{2} \sum_{i=N/2+1}^{N} C_i^{N-2} \left[\rho^i (1-\rho)^{N-i} - (1-\rho)^i \rho^{N-i} \right] \\
&= \sum_{i=N/2+1}^{N-2} C_i^{N-2} \left[\rho^{i+1} (1-\rho)^{N-1-i} - (1-\rho)^{i+1} \rho^{N-1-i} \right] + (2\rho - 1) \left[\rho (1-\rho) \right]^{N/2-1} C_{N/2}^{N-2} \\
&\quad + \frac{1}{2} \sum_{i=N/2+1}^{N} C_{i-2}^{N-2} \left[\rho^i (1-\rho)^{N-i} - (1-\rho)^i \rho^{N-i} \right] + \frac{1}{2} \sum_{i=N/2+1}^{N} C_i^{N-2} \left[\rho^i (1-\rho)^{N-i} - (1-\rho)^i \rho^{N-i} \right].
\end{aligned}$$

Thus,

$$\begin{aligned}
\bar{x} - X(2) &= (2\rho - 1)\left[\rho(1-\rho)\right]^{N/2-1}\left(C_{N/2}^{N-2} - C_{N/2}^{N-1}\right) \\
&\quad + \frac{1}{2}\sum_{i=N/2+1}^{N} C_{i-2}^{N-2}\left[\rho^{i}(1-\rho)^{N-i} - (1-\rho)^{i}\rho^{N-i}\right] \\
&\quad + \frac{1}{2}\sum_{i=N/2+1}^{N} C_{i}^{N-2}\left[\rho^{i}(1-\rho)^{N-i} - (1-\rho)^{i}\rho^{N-i}\right] \\
&= -(2\rho - 1)\left[\rho(1-\rho)\right]^{N/2-1} C_{N/2-1}^{N-2} \\
&\quad + \sum_{i=N/2+1}^{N}\left(\frac{C_{i}^{N-2} + C_{i-2}^{N-2}}{2}\right)\left[\rho^{i}(1-\rho)^{N-i} - (1-\rho)^{i}\rho^{N-i}\right] \\
&= -\left(\frac{C_{N/2-1}^{N-2} - C_{N/2+1}^{N-2}}{2}\right)(2\rho - 1)\left[\rho(1-\rho)\right]^{N/2-1} \\
&\quad + \sum_{i=N/2+2}^{N}\left(\frac{C_{i}^{N-2} + C_{i-2}^{N-2}}{2}\right)\left[\rho^{i}(1-\rho)^{N-i} - (1-\rho)^{i}\rho^{N-i}\right] \\
&\geq \left[\sum_{i=N/2+2}^{N}\left(\frac{C_{i}^{N-2} + C_{i-2}^{N-2}}{2}\right) - \left(\frac{C_{N/2-1}^{N-2} - C_{N/2+1}^{N-2}}{2}\right)\right](2\rho - 1)\left[\rho(1-\rho)\right]^{N/2-1}.
\end{aligned}$$

The last inequality holds because $\rho^{i}(1-\rho)^{N-i} - (1-\rho)^{i}\rho^{N-i}$ increases in i. It remains to show that

$$\sum_{i=N/2+2}^{N}\left(C_{i}^{N-2} + C_{i-2}^{N-2}\right) > C_{N/2-1}^{N-2} - C_{N/2+1}^{N-2}.$$

Note that

$$\begin{aligned}
\sum_{i=N/2+2}^{N}\left(C_{i}^{N-2} + C_{i-2}^{N-2}\right) &= \sum_{i=N/2+2}^{N} C_{i}^{N-2} + \sum_{i=N/2}^{N-2} C_{i}^{N-2} \\
&= \sum_{i=N/2+2}^{N-2} C_{i}^{N-2} + \sum_{i=N/2}^{N-2} C_{i}^{N-2} \\
&= \sum_{i=0}^{N/2-4} C_{i}^{N-2} + \sum_{i=N/2}^{N-2} C_{i}^{N-2} \\
&= 2^{N-2} - C_{N/2-3}^{N-2} - C_{N/2-2}^{N-2} - C_{N/2-1}^{N-2}.
\end{aligned}$$

Thus, we require that

$$\begin{aligned} 2^{N-2} - C^{N-2}_{N/2-3} - C^{N-2}_{N/2-2} - C^{N-2}_{N/2-1} &> C^{N-2}_{N/2-1} - C^{N-2}_{N/2+1} \Leftrightarrow \\ 2^{N-2} &> 2C^{N-2}_{N/2-1} + C^{N-2}_{N/2-2}. \end{aligned}$$

Note that

$$\begin{aligned} 2^{N-2} &> C^{N-2}_{N/2-1} + C^{N-2}_{N/2-2} + C^{N-2}_{N/2} + C^{N-2}_{N/2+1} + C^{N-2}_{N/2-3} \\ &> C^{N-2}_{N/2-1} + C^{N-2}_{N/2-2} + 3C^{N-2}_{N/2+1}. \end{aligned}$$

Thus, it is sufficient to require that

$$C^{N-2}_{N/2-1} + C^{N-2}_{N/2-2} + 3C^{N-2}_{N/2+1} > 2C^{N-2}_{N/2-1} + C^{N-2}_{N/2-2} \Leftrightarrow 3C^{N-2}_{N/2+1} > C^{N-2}_{N/2-1},$$

which holds for any $N > 9$. Numerically it can also be shown that the proposition holds for $N > 6$. ■

CLAIM IA.5: *There is no equilibrium in which the blockholder submits the proposal if and only if his signal is bad, at least two small shareholders submit the proposal if and only if their signal is good, and all other small shareholders do not submit the proposal. If* $x_g(0,0) < c$*, then there is also no equilibrium of this type in which only one small shareholder submits the proposal if and only if his signal is good and all other small shareholders do not submit the proposal.*

Proof of Claim IA.5: Consider an equilibrium in which there are $m \geq 1$ small shareholders who submit the proposal if and only if their signal is good and the blockholder submits the proposal if and only if his signal is bad. All other shareholders do not submit the proposal. Consider the relative benefit $X(m)$ of a small shareholder with a good signal from submitting the proposal relative to not submitting it. Below we show that $X(m) < c$ for $m \geq 2$ and hence the small shareholder has incentives to deviate and not submit the proposal. A direct calculation implies that

$$\begin{aligned} X(m) &= \Pr[G|g] \begin{pmatrix} \rho \Pr[T_{m-1} = 0|G] \Pr[T_{N-m-1} \geq N/2 - 1|G] \\ + (1-\rho) \sum_{i=0}^{m-1} \Pr[T_{m-1} = i|G] \Pr[T_{N-m-1} = N/2 - i|G] \\ +\rho \sum_{i=1}^{m-1} \Pr[T_{m-1} = i|G] \Pr[T_{N-m-1} = N/2 - i - 1|G] \end{pmatrix} \\ &\quad - \Pr[B|g] \begin{pmatrix} (1-\rho) \Pr[T_{m-1} = 0|B] \Pr[T_{N-m-1} \geq N/2 - 1|B] \\ +\rho \sum_{i=0}^{m-1} \Pr[T_{m-1} = i|B] \Pr[T_{N-m-1} = N/2 - i|B] \\ + (1-\rho) \sum_{i=1}^{m-1} \Pr[T_{m-1} = i|B] \Pr[T_{N-m-1} = N/2 - i - 1|B] \end{pmatrix} \end{aligned}$$

$$
= \rho \begin{pmatrix} \rho(1-\rho)^{m-1}\sum_{i=N/2-1}^{N-m-1} C_i^{N-m-1}\rho^i(1-\rho)^{N-m-1-i} \\ +(1-\rho)\sum_{i=0}^{m-1}\left[C_i^{m-1}\rho^i(1-\rho)^{m-1-i}\right]\left[C_{N/2-i}^{N-m-1}\rho^{N/2-i}(1-\rho)^{N-m-1-(N/2-i)}\right] \\ +\rho\sum_{i=1}^{m-1}\left[C_i^{m-1}\rho^i(1-\rho)^{m-1-i}\right]\left[C_{N/2-i-1}^{N-m-1}\rho^{N/2-i-1}(1-\rho)^{N-m-1-(N/2-i-1)}\right] \end{pmatrix}
$$

$$
-(1-\rho)\begin{pmatrix} (1-\rho)\rho^{m-1}\sum_{i=N/2-1}^{N-m-1} C_i^{N-m-1}(1-\rho)^i\rho^{N-m-1-i} \\ +\rho\sum_{i=0}^{m-1}\left[C_i^{m-1}(1-\rho)^i\rho^{m-1-i}\right]\left[C_{N/2-i}^{N-m-1}(1-\rho)^{N/2-i}\rho^{N-m-1-(N/2-i)}\right] \\ +(1-\rho)\sum_{i=1}^{m-1}\left[C_i^{m-1}(1-\rho)^i\rho^{m-1-i}\right]\left[C_{N/2-i-1}^{N-m-1}(1-\rho)^{N/2-i-1}\rho^{N-m-1-(N/2-i-1)}\right] \end{pmatrix}
$$

$$
\begin{aligned}
= & \sum_{i=N/2-1}^{N-m-1} C_i^{N-m-1}\rho^{i+2}(1-\rho)^{N-2-i} \\
& +\rho^{N/2+1}(1-\rho)^{N/2-1}\left[\sum_{i=0}^{m-1} C_i^{m-1}C_{N/2-i}^{N-m-1}+\sum_{i=1}^{m-1} C_i^{m-1}C_{N/2-i-1}^{N-m-1}\right] \\
& -\sum_{i=N/2-1}^{N-m-1} C_i^{N-m-1}(1-\rho)^{i+2}\rho^{N-2-i} \\
& -(1-\rho)^{N/2+1}\rho^{N/2-1}\left[\sum_{i=0}^{m-1} C_i^{m-1}C_{N/2-i}^{N-m-1}+\sum_{i=1}^{m-1} C_i^{m-1}C_{N/2-i-1}^{N-m-1}\right] \\
= & \sum_{i=N/2-1}^{N-m-1} C_i^{N-m-1}\left[\rho^{i+2}(1-\rho)^{N-2-i}-(1-\rho)^{i+2}\rho^{N-2-i}\right] \\
& +\left[(1-\rho)\rho\right]^{N/2-1}(2\rho-1)\left[\sum_{i=0}^{m-1} C_i^{m-1}C_{N/2-i}^{N-m-1}+\sum_{i=1}^{m-1} C_i^{m-1}C_{N/2-i-1}^{N-m-1}\right].
\end{aligned}
$$

Note that by Vandermonde's identity, $\sum_{i=0}^{N/2} C_i^{m-1}C_{N/2-i}^{N-m-1} = C_{N/2}^{N-2}$ and hence

$$
\begin{aligned}
\sum_{i=0}^{m-1} C_i^{m-1}C_{N/2-i}^{N-m-1}+\sum_{i=1}^{m-1} C_i^{m-1}C_{N/2-i-1}^{N-m-1} &= C_{N/2}^{N-2}+C_{N/2-1}^{N-2}-C_{N/2-1}^{N-m-1} \\
&= C_{N/2}^{N-1}-C_{N/2-1}^{N-m-1}.
\end{aligned}
$$

Thus,

$$\begin{aligned} X(m) &= \sum_{i=N/2-1}^{N-m-1} C_i^{N-m-1}\left[\rho^{i+2}(1-\rho)^{N-2-i}-(1-\rho)^{i+2}\rho^{N-2-i}\right] \\ &\quad +[(1-\rho)\rho]^{N/2-1}(2\rho-1)\left[C_{N/2}^{N-1}-C_{N/2-1}^{N-m-1}\right] \\ &= \sum_{i=N/2}^{N-m-1} C_i^{N-m-1}\left[\rho^{i+2}(1-\rho)^{N-2-i}-(1-\rho)^{i+2}\rho^{N-2-i}\right] \\ &\quad +[(1-\rho)\rho]^{N/2-1}(2\rho-1)C_{N/2}^{N-1}. \end{aligned}$$

Note that $X(m)$ is decreasing in m. Below we show that $X(2)\leq\bar{x}$, which implies that $X(m)<c$ for all $m\geq 2$. When $m=2$,

$$X(2)=\sum_{i=N/2}^{N-3} C_i^{N-3}\left[\rho^{i+2}(1-\rho)^{N-2-i}-(1-\rho)^{i+2}\rho^{N-2-i}\right]+[(1-\rho)\rho]^{N/2-1}(2\rho-1)C_{N/2}^{N-1},$$

where

$$\begin{aligned} \bar{x} &= \frac{1}{2}\sum_{i=N/2+1}^{N} C_i^N\left[\rho^i(1-\rho)^{N-i}-(1-\rho)^i\rho^{N-i}\right] \\ &= \frac{1}{2}\sum_{i=N/2+1}^{N}\left(C_i^{N-1}+C_{i-1}^{N-1}\right)\left[\rho^i(1-\rho)^{N-i}-(1-\rho)^i\rho^{N-i}\right] \\ &= \frac{1}{2}\sum_{i=N/2+1}^{N}\left(C_i^{N-2}+2C_{i-1}^{N-2}+C_{i-2}^{N-2}\right)\left[\rho^i(1-\rho)^{N-i}-(1-\rho)^i\rho^{N-i}\right] \\ &= \frac{1}{2}\sum_{i=N/2+1}^{N}\left(C_i^{N-3}+3C_{i-1}^{N-3}+3C_{i-2}^{N-3}+C_{i-3}^{N-3}\right)\left[\rho^i(1-\rho)^{N-i}-(1-\rho)^i\rho^{N-i}\right]. \end{aligned}$$

Note that

$$\begin{aligned} \sum_{i=N/2+1}^{N} C_{i-2}^{N-3}\left[\rho^i(1-\rho)^{N-i}-(1-\rho)^i\rho^{N-i}\right] &= \sum_{i=N/2}^{N-3} C_i^{N-3}\left[\rho^{i+2}(1-\rho)^{N-2-i}-(1-\rho)^{i+2}\rho^{N-2-i}\right] \\ &\quad +C_{N/2-1}^{N-3}[(1-\rho)\rho]^{N/2-1}(2\rho-1), \end{aligned}$$

and hence it is sufficient to show that

$$\frac{1}{2}\sum_{i=N/2+1}^{N}\left(C_i^{N-3}+3C_{i-1}^{N-3}+C_{i-2}^{N-3}+C_{i-3}^{N-3}\right)\left[\rho^i\left(1-\rho\right)^{N-i}-\left(1-\rho\right)^i\rho^{N-i}\right]$$
$$+C_{N/2-1}^{N-3}\left[\left(1-\rho\right)\rho\right]^{N/2-1}\left(2\rho-1\right)\geq\left[\left(1-\rho\right)\rho\right]^{N/2-1}\left(2\rho-1\right)C_{N/2}^{N-1}.$$

Because $\rho^i\left(1-\rho\right)^{N-i}-\left(1-\rho\right)^i\rho^{N-i}\geq\left[\left(1-\rho\right)\rho\right]^{N/2-1}\left(2\rho-1\right)$ for $i\geq N/2+1$, a sufficient condition for this is

$$\frac{1}{2}\sum_{i=N/2+1}^{N}\left(C_i^{N-3}+3C_{i-1}^{N-3}+C_{i-2}^{N-3}+C_{i-3}^{N-3}\right)+C_{N/2-1}^{N-3}\geq C_{N/2}^{N-1},$$

which holds for any $N\geq 6$. This proves the claim for $m\geq 2$.

Hence, the only possible equilibrium of this type is with $m=1$. Next, we show that $X\left(1\right)\leq x_g\left(0,0\right)$ and hence this type of equilibrium does not exist even for $m=1$ if $x_g\left(0,0\right)<c$. Note that

$$X\left(1\right)=\sum_{i=N/2}^{N-2}C_i^{N-2}\left[\rho^{i+2}\left(1-\rho\right)^{N-2-i}-\left(1-\rho\right)^{i+2}\rho^{N-2-i}\right]+\left[\left(1-\rho\right)\rho\right]^{N/2-1}\left(2\rho-1\right)C_{N/2}^{N-1},$$

and

$$\begin{aligned}
x_g\left(0,0\right) &= \sum_{i=N/2}^{N-1}C_i^{N-1}\left[\rho\rho^i\left(1-\rho\right)^{N-1-i}-\left(1-\rho\right)\left(1-\rho\right)^i\rho^{N-1-i}\right]\\
&= \sum_{i=N/2}^{N-1}\left(C_i^{N-2}+C_{i-1}^{N-2}\right)\left[\rho^{i+1}\left(1-\rho\right)^{N-1-i}-\left(1-\rho\right)^{i+1}\rho^{N-1-i}\right]\\
&= \sum_{i=N/2}^{N-1}C_i^{N-2}\left[\rho^{i+1}\left(1-\rho\right)^{N-1-i}-\left(1-\rho\right)^{i+1}\rho^{N-1-i}\right]\\
&\quad+\sum_{i=N/2}^{N-2}C_i^{N-2}\left[\rho^{i+2}\left(1-\rho\right)^{N-2-i}-\left(1-\rho\right)^{i+2}\rho^{N-2-i}\right]\\
&\quad+C_{N/2-1}^{N-2}\left[\left(1-\rho\right)\rho\right]^{N/2-1}\left(2\rho-1\right).
\end{aligned}$$

Thus,

$$\begin{aligned} x_g(0,0) &\geq X(1) \Leftrightarrow \\ \sum_{i=N/2}^{N-1} C_i^{N-2}\left[\rho^{i+1}(1-\rho)^{N-1-i} - (1-\rho)^{i+1}\rho^{N-1-i}\right] &\geq [(1-\rho)\rho]^{N/2-1}(2\rho-1)\left[C_{N/2}^{N-1} - C_{N/2-1}^{N-2}\right]. \end{aligned}$$

Because $\rho^{i+1}(1-\rho)^{N-1-i} - (1-\rho)^{i+1}\rho^{N-1-i} \geq [(1-\rho)\rho]^{N/2-1}(2\rho-1)$ for $i \geq N/2$, it is sufficient to show that $\sum_{i=N/2}^{N-1} C_i^{N-2} \geq C_{N/2}^{N-1} - C_{N/2-1}^{N-2}$, which holds for any $N \geq 4$. ■

Proof of Proposition IA.1: Let

$$\begin{aligned} M_1 &\equiv \frac{c}{x_g(0,0)} \\ M_3 &\equiv \frac{c}{2\rho(1-\rho)x_g(0,1) - L} \\ M_2 &\equiv \min\left\{\frac{c}{\left((1-\rho)^2+\rho^2\right)x_b(0,1)}, M_3\right\}. \end{aligned}$$

It can be verified (given Assumption IA.1 and Claim IA.1) that $M_2 > \max\{1, M_1\}$. Recall that in any equilibrium in which the proposal is submitted with strictly positive probability, either a single shareholder (small or large) submits the proposal with positive probability or the large shareholder submits the proposal if and only if his signal is bad and a single small shareholder submits the proposal if and only if his signal is good. We refer to the first equilibrium as type I and to the second one as type II. We prove each part of the proposition separately.

(*i*) Suppose $M < M_1$. Note that because $M \geq 1$, this implies that $M_1 > 1$ and hence $x_g(0,0) < c$. We argue that both types of equilibria do not exist. Type I does not exist since the benefit from submitting the proposal is bounded from above by $x_g(0,0) - c < 0$ for the small shareholder and by $Mx_g(0,0) - c < M_1 x_g(0,0) - c = 0$ for the large shareholder. Thus, not submitting the proposal, and thereby avoiding the submission costs, is a profitable deviation. Type II does not exist according to Claim IA.5 because $x_g(0,0) < c$. Next, we show that an equilibrium in which no shareholder submits the proposal exists. To see why, note that even under the most favorable off-equilibrium beliefs (beliefs that correctly interpret the shareholder's signal), submission yields a payoff bounded from above by $x_g(0,0) - c < 0$ for the small shareholder and by $Mx_g(0,0) - c < M_1 x_g(0,0) - c = 0$ for the large shareholder. Thus, not submitting the proposal is indeed optimal for any shareholder and any signal.

(*ii*) Suppose $M \geq M_1$. First, if $\frac{c}{x_b(0,0)} > M \geq M_1$, there exists an equilibrium in which the blockholder submits the proposal if and only if his signal is good and small shareholders do not submit the proposal. Indeed, the blockholder's benefit from submitting the proposal relative to not submitting it is $Mx_s(0,0) - c$, which is positive if and only if his signal is good. Small shareholders will not deviate and submit the proposal in this equilibrium since their relative benefit from deviation (which is maximized when their signal is not misinterpreted) is bounded from above by $\Pr(b|s)\, x_s(0,1) - c \le x_g(0,1) - c < \bar{x} - c$, which is strictly negative by Assumption IA.1. Second, if $M \ge \frac{c}{x_b(0,0)}$, there exists an equilibrium in which the blockholder submits the proposal with probability one and other shareholders do not submit the proposal. Indeed, the blockholder's relative benefit from submission is strictly positive regardless of his signal. Small shareholders have no incentives to deviate and submit the proposal since by doing so, they can only lose due to the misinterpretation of their signal.

(*iii*) Suppose $M > M_2$. The claim is proved if we rule out two kinds of equilibria - those in which some small shareholders submit the proposal with positive probability but the large shareholder does not, and the one where the proposal is never submitted. Suppose that the first kind of equilibrium exists. The only case we have not ruled out by Claims IA.2 to IA.4 is the case in which a single small shareholder submits the proposal in equilibrium with positive probability. Given Assumption IA.1 and Claim IA.3, this shareholder must submit the proposal if and only if he observes a good signal. Let us show that the large shareholder has incentives to deviate and submit the proposal in this equilibrium for at least some signal. Submission of the proposal is an off-equilibrium event. The smallest benefit from deviation is realized when the large shareholder's signal is misinterpreted. Recall that the cost of misinterpretation is positive (and given by L because the cost is incurred regardless of the small shareholder's signal) only for a blockholder with a good signal, and is zero for a blockholder with a bad signal. Thus, the off-equilibrium beliefs that minimize the benefit from deviation for both types of signal are those for which submission of the proposal is interpreted as having a bad signal. Hence, the large shareholder finds it optimal to deviate under the worst off-equilibrium beliefs if and only if

$$M \max\left\{\left((1-\rho)^2 + \rho^2\right) x_b(0,1), 2\rho(1-\rho)\, x_g(0,1) - L\right\} > c,$$

which is equivalent to the requirement that $M > M_2$. Hence, if $M > M_2$, then for any off-equilibrium beliefs the large shareholder finds it optimal to deviate for at least one type of signal. Thus, there is no such equilibrium. It remains to rule out the possibility that there exists an equilibrium in which the proposal is never submitted. If this were an equilibrium, the large shareholder with a good signal would find it optimal to deviate even under the worst off-equilibrium beliefs if $M > \frac{c}{x_g(0,0)-L}$. Using Claim IA.1 and the observation that $x_g(0,0) - L > x_b(0,0)$, it is easy to show that $M_2 > \frac{c}{x_g(0,0)-L}$ and hence $M > M_2 > \frac{c}{x_g(0,0)-L}$.[4] Therefore, this equilibrium does not exist in this region either.

[4] Indeed, $x_g(0,0) - L$ is the benefit from the proposal of a shareholder with a good signal who decides to

(*iv*) Suppose $M > M_3$. Note that type I equilibrium with a single small shareholder submitting the proposal does not exist by the reasoning of part (*iii*) because $M_3 \geq M_2$. Hence, the only possible equilibrium where the small shareholder submits with positive probability is that of type II. However, type II equilibrium exists only if the large shareholder finds it optimal to submit the proposal if and only if his signal is bad, that is, if and only if

$$\begin{aligned} \left((1-\rho)^2+\rho^2\right)x_b(0,1) &> \frac{c}{M} > 2\rho(1-\rho)x_g(0,1) - L \Leftrightarrow \\ M_3 &> M > \frac{c}{\left((1-\rho)^2+\rho^2\right)x_b(0,1)}. \end{aligned}$$

Thus, since $M > M_3$, this equilibrium does not exist either.

■

Proof of Corollary IA.2: Given the proof of part (*iv*) of Proposition IA.1, if $M > M_3$, then in any equilibrium the large shareholder submits the proposal with strictly positive probability and small shareholders never submit the proposal. According to part (*ii*), at least one such equilibrium exists. Note, however, that an equilibrium in which the blockholder submits the proposal if and only if his signal is bad never exists. Indeed, a necessary condition for such an equilibrium to exist is that $x_b(0,0) > c/M > x_g(0,0) - L$. But, given the argument in the proof of part (*iii*) of Proposition IA.1, $x_g(0,0) - L > x_b(0,0)$ and hence this cannot be an equilibrium. If in addition $M > \frac{c}{x_b(0,0)}$, then there also does not exist an equilibrium in which the blockholder submits the proposal if and only if his signal is good. This is because the blockholder with a bad signal has strict incentives to submit the proposal when $Mx_b(0,0) > c$ (recall that the cost of misinterpretation is zero for a shareholder with a bad signal). Thus, if $M > M^{**} \equiv \max\left\{M_3, \frac{c}{x_b(0,0)}\right\}$, there exists a unique equilibrium and in this equilibrium the blockholder submits the proposal with probability one, which proves the first claim of the corollary.

For the second claim of the corollary, let $M^* \equiv \min\left\{M_3, \frac{c}{x_b(0,0)}\right\}$. It is easy to verify that $M^* > M_1$. Suppose that $M^* > M > M_1$. First, because $M < \frac{c}{x_b(0,0)}$, there is no equilibrium in which the blockholder submits the proposal with probability one (the blockholder with a bad signal is better off deviating and not submitting the proposal because his cost of misinterpretation is zero regardless of the off-equilibrium beliefs). Second, according to the proof of part (*ii*) of Proposition IA.1, there exists an equilibrium in which the blockholder submits the proposal if and only if his signal is good, which completes the proof. ■

vote against the proposal in spite of his signal, while $x_b(0,0)$ is the benefit of a shareholder with a bad signal who will vote against the proposal. In both cases the proposal is accepted if and only if at least $N/2+1$ among the other $N-1$ shareholders have good signals. The inequality then holds because a shareholder with a good signal is more optimistic than a shareholder with a bad signal both about the proposal and the probability that it will be accepted.

B. Managerial Retaliation

The main analysis has focused on a common value setting: the only way a shareholder's vote affects his utility is by the vote's indirect effect on firm value through the approval or rejection of the proposal. This effect is common for all shareholders of the firm. However, shareholders could also have private values from voting, that is, they could derive utility directly from voting in a particular way.

An important concern that may affect shareholder voting behavior in situations in which the proposal on the agenda is not supported by the manager is retaliation by the manager. Anecdotal evidence indicates that managers occasionally punish shareholders for voting against them either by constraining their access to valuable information about the company or by terminating their business relations with the company. For example, in his letter written to the SEC in 2002 on proxy voting disclosure, John Bogle, founder and former CEO of the Vanguard Group, states that "...votes against management may jeopardize the retention of clients of 401K and pension accounts." Consistent with this view, a 2004 New York Times article notes that "while several mutual funds vote in favor of options expending, Fidelity does not, perhaps because Fidelity is the record keeper for Intel's 401(k) plan, which held eight Fidelity funds worth 1 billion at the end of 2003" (September 12, 2004). Ashraf, Jayaraman, and Ryan (2010) show that mutual funds with pension-related business ties with a company are less likely to vote for shareholder-initiated proposals regarding executive compensation than other mutual funds. Moreover, the likelihood that the fund votes for shareholder proposals negatively depends on the annual fees received from the firm for pension fund services. Davis and Kim (2007) find no evidence that mutual fund proxy voting depends on whether a firm is a client. However, they do find that the more business ties a fund family has overall, the less likely it is to vote in favor of shareholder proposals that are opposed by management. Matvos and Ostrovsky (2010) find evidence consistent with mutual funds being worried about retaliation from management, making them more reluctant to vote against management, unless a large number of other funds vote the same way.

While empirical evidence on managerial retaliation is not conclusive, in theory retaliation may play an important role in shareholders' formation of voting strategies. Moreover, the cost of voting against management need not necessarily be related directly to the manager's actions. For example, by voting against the manager, a shareholder posits that he is a troublemaker and may deter other companies' managers from cooperating with him in the future. Finally, the costs of opposing management may arise due to a personal bias some shareholders might have towards voting with the manager and supporting him. We believe that some interesting insights can be drawn in such a setting and therefore address this possibility.

Suppose that prior to the vote it is common knowledge that those shareholders who vote for the shareholder proposal will incur a fixed monetary cost. In this setting, shareholders trade off the benefits of voting according to their private information and the costs of opposing management. As the following proposition demonstrates, the presence of retaliation costs may improve information aggregation in voting.

PROPOSITION IA.2: *For any $k_M > \frac{1}{2} + \frac{1}{2}\left(\frac{1-\rho}{\rho}\right)^N$ there exists a level of opposition costs such that a responsive equilibrium in pure strategies exists.*

Proof: We prove the result for even N. Let q be the shareholder's personal cost of voting for the proposal. A shareholder will vote affirmatively if and only if $\Phi_{\omega,T^*_\omega}(s) > q$. Let us prove that for any $k_M > \frac{1}{2} + \frac{1}{2}(\frac{1-\rho}{\rho})^N$, there exists $q > 0$ such that the strategy profile $(\omega_b, \omega_g) = (0, 1)$ constitutes a responsive equilibrium.

If $\omega = (0, 1)$, then according to Lemma 2 and expression (A2), the endogenous threshold is given by

$$T^*_{0,1} = \left\lfloor \frac{N}{2} + \frac{\log \frac{\beta^*}{1-\beta^*}}{2 \log \frac{\rho}{1-\rho}} \right\rfloor,$$

where $\beta^* = \frac{1}{2k_M}$. Shareholders are pivotal with a strictly positive probability if and only if $T^*_{0,1} \in [0, N-1]$, which is equivalent to $\frac{\log \frac{\beta^*}{1-\beta^*}}{2 \log \frac{\rho}{1-\rho}} \in [-\frac{N}{2}, \frac{N}{2})$. Note that $k_M \in (0,1] \Rightarrow \beta^* \geq \frac{1}{2}$, and since $\rho > \frac{1}{2}$, it is guaranteed that $\frac{\log \frac{\beta^*}{1-\beta^*}}{2 \log \frac{\rho}{1-\rho}} \geq 0$ and hence $T^*_{0,1} \geq \frac{N}{2}$. Thus, $T^*_{0,1} \in [0, N-1]$ if and only if $\frac{\log \frac{\beta^*}{1-\beta^*}}{2 \log \frac{\rho}{1-\rho}} < \frac{N}{2}$, which is equivalent to $k_M > \frac{1}{2} + \frac{1}{2}\left(\frac{1-\rho}{\rho}\right)^N$, which is satisfied by assumption. We conclude that $T^*_{0,1} \in \left[\frac{N}{2}, N-1\right]$. If $T^*_{0,1} \in \left[\frac{N}{2}, N-1\right]$, Lemma A.1 tells us that $\Phi_{\omega,T^*_\omega}(g) > \Phi_{\omega,T^*_\omega}(b)$ and that $\Phi_{\omega,T^*_\omega}(g) > 0 \Leftrightarrow \frac{\Pr\left[T_{N,\omega}=T^*_\omega|B\right]}{\Pr\left[T_{N,\omega}=T^*_\omega|G\right]} < \left(\frac{\rho}{1-\rho}\right)^2 \Leftrightarrow T^*_\omega > \frac{N}{2} - 1$. Since $T^*_{0,1} \geq \frac{N}{2}$, we conclude that $\Phi_{\omega,T^*_\omega}(g) > 0$. Therefore, for any $q \in \left(\max\left\{0, \Phi_{\omega,T^*_\omega}(b)\right\}, \Phi_{\omega,T^*_\omega}(g)\right)$, voting strategies $(\omega_b, \omega_g) = (0, 1)$ are optimal and hence a responsive equilibrium in pure strategies exists. ■

The intuition for this result is as follows. As we show earlier, when the manager's interests are misaligned with those of shareholders, then, conditional on being pivotal, shareholders are optimistic about the proposal and prefer to vote affirmatively regardless of their private information. The cost of opposing the manager makes shareholders more reluctant to vote for the proposal and counterbalances this effect. Thus, the presence of moderate costs of opposition has a surprising positive effect: it encourages shareholders to vote according to their signals and improves information aggregation in voting.

Note that, to some extent, the retaliation mechanism and the opportunism in the activist's behavior are two interchangeable forces that sustain a responsive equilibrium. Interestingly, while the activist is required to oppose management in order for a responsive equilibrium to exist, managerial retaliation should force shareholders to support management.

References

Ashraf, Rasha, Narayanan Jayaraman, and Harley E. Ryan Jr., 2010, Do pension-related business ties influence mutual fund proxy voting? Evidence from shareholder proposals on executive compensation, Working paper, Georgia State University.

Davis, Gerald F., and E. Han Kim, 2007, Business ties and proxy voting by mutual funds, *Journal of Financial Economics* 85, 552-570.

Matvos, Gregor, and Michael Ostrovsky, 2010, Heterogeneity and peer effects in mutual fund proxy voting, *Journal of Financial Economics* 98, 90-112.